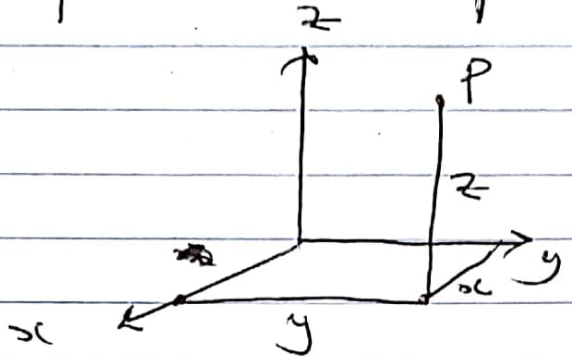
# 15.8, 15.9 CYLINDRICAL + SPHERICAL COORDS
(NO INTEGRATION YET!)

## RECTANGULAR COORDS

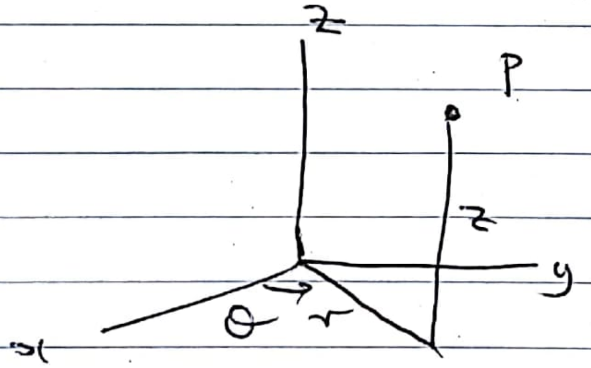
Represent point P in space as $(x, y, z)$




Coordinate Planes: $x = c$, $y = c$, $z = c$.

## CYLINDRICAL COORDS

Represent point P using $(r, \theta, z)$




$r \geq 0$

$0 \leq \theta < 2\pi$

$z \in \mathbb{R}$

$\boxed{\text{CYL} \rightarrow \text{RECT}}$

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$\boxed{\text{RECT} \rightarrow \text{CYL}}$

$r = \sqrt{x^2 + y^2}$

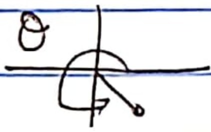
$z = z$

$\boxed{\text{1ST ATTEMPT}}$: $\theta = \arctan\left(\frac{y}{x}\right)$ + ADJUST QUADRANT.

Since tan has period $\pi$ we have

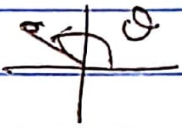
$$\theta = \arctan\left(\frac{y}{x}\right) + k\pi \quad \text{for some integer } k.$$

**Q** $(x,y) = (1,-1)$




**A** $\theta = \arctan(-1) + k\pi = -\pi/4 + k\pi \overset{k=2}{=} \frac{7\pi}{4}$

**Q** $(x,y) = (-1,1)$

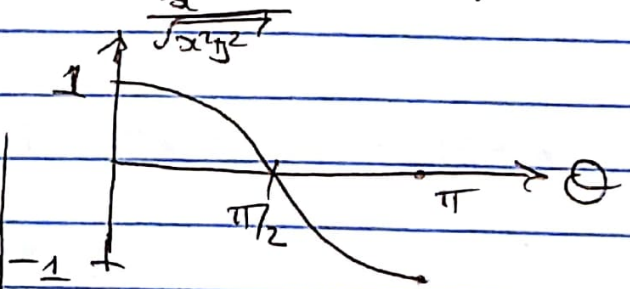



**A** $\theta = \arctan(-1) + k\pi = -\pi/4 + k\pi \overset{k=1}{=} 3\pi/4$

$\boxed{\text{2ND ATTEMPT:}}$ Choice of Quadrant is simpler using arccos.

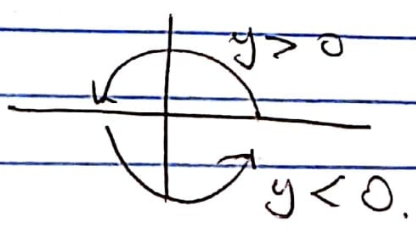
$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\theta = \arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right) + 2k\pi \quad \text{as cos has period } 2\pi$$




__FORMULA__ IF $(x,y) \neq (0,0)$ THEN

$$\theta = \begin{cases} \arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right) & \text{IF } y \geq 0 \\ 2\pi - \arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right) & \text{IF } y < 0. \end{cases}$$

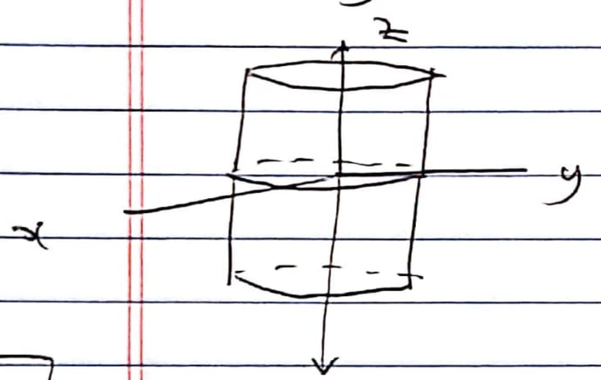



CHECK IT FOR YOURSELF!!

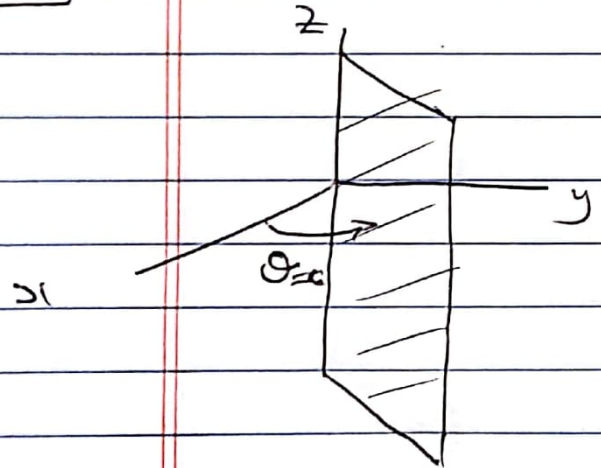
## COORDINATE SURFACES

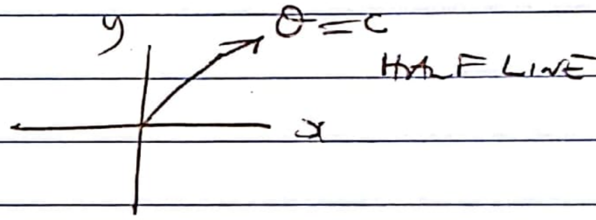
$\boxed{z=c}$ HORIZONTAL PLANE

$\boxed{r=c} \iff x^2+y^2=c^2$ CYLINDER RADIUS c.




$\boxed{\theta=c}$ HALF PLANE.



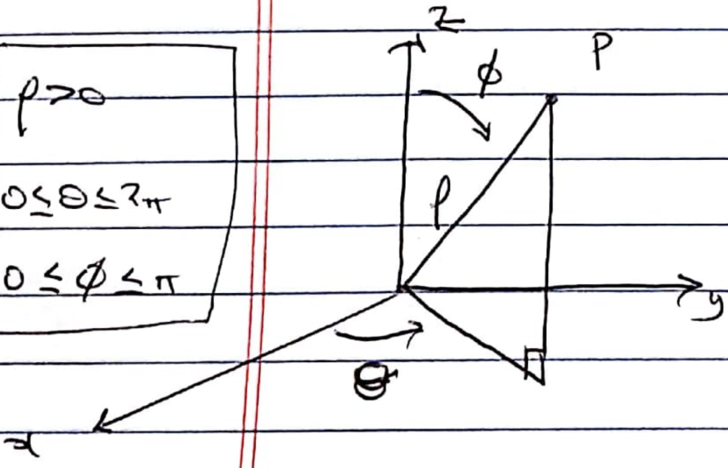

SHIFT THIS LINE UP + DOWN z-AXIS TO GET ½-PLANE.

## SPHERICAL COORDS $(\rho, \theta, \phi)$

$\rho \geq 0$

$0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \pi$




$\rho$ = DISTANCE FROM ORIGIN

$\theta$ = Angle in xy-plane from x axis (as in Cyl Coords)

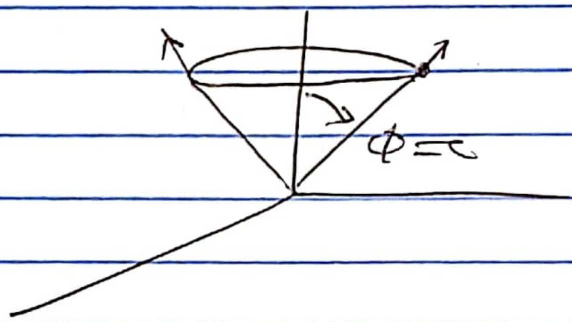
$\phi$ = Drop Angle from +z Axis.

## COORD SURFACES

$\boxed{\rho = c}$ Sphere radius c.

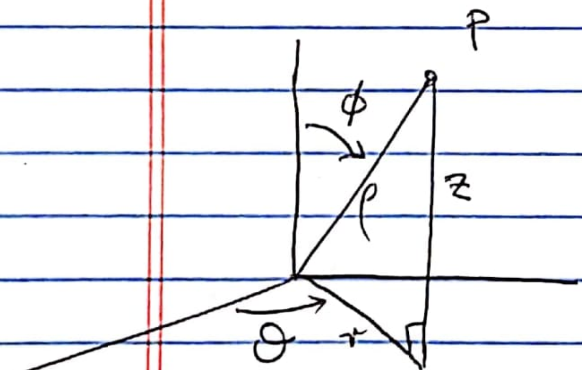
$\boxed{\theta = c}$ 1/2 - PLANE as in Cyl Coords

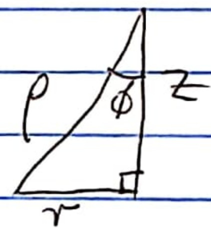
$\boxed{\phi = c}$ CONE




All points on cone have same drop angle.

## SPH → RECT (VIA CYL)




FROM $\triangle$:

$$r = \rho \sin\phi$$
$$z = \rho \cos\phi$$

So

| RECT | CYL | SPH |
|---|---|---|
| $x =$ | $r\cos\theta =$ | $\rho\sin\phi\cos\theta$ |
| $y =$ | $r\sin\theta =$ | $\rho\sin\phi\sin\theta$ |
| $z =$ | $z =$ | $\rho\cos\phi$ |

$\boxed{\text{RECT} \Rightarrow \text{SPH}}$

$\rho = \sqrt{x^2+y^2+z^2}$ (Distance from origin)

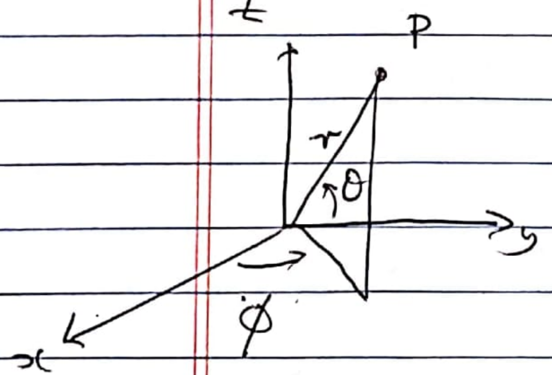
$\theta$. As for CYL.

$\phi$ Use $z = \rho\cos\phi$ to get

$$\phi = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

Gives $0 \le \phi \le \pi$ ✓✓

PHYSICS CONVENTION (OR Why You Shouldn't Memorize Formulae)




ONE OPTION:

Use $(r, \phi, \theta)$

$\phi$: AZIMUTH ANGLE

$\theta$: ELEVATION ANGLE ABOVE HORIZON ($z=0$)

$-\pi/2 \le \theta \le \pi/2$.