MATH 2415 Calculus of Several Variables

Fall-2019

PLTL Packet# 2(Sec 12.4-12.5)

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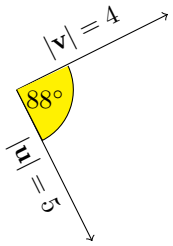
1. Given $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$
   (a) Find the cross product $\mathbf{u} \times \mathbf{v}$.
   (b) Find a vector that is orthogonal to $\mathbf{u}$ and $\mathbf{v}$ both.
   (c) Find a vector that is orthogonal to $\mathbf{u}$ and $\mathbf{v}$ both and has length π.
   (d) Let θ $(0 \leq \theta \leq \pi)$ be the angle between $\mathbf{u}$ and $\mathbf{v}$, find $\sin\theta$.
   (e) Find the area of parallelogram with adjacent sides represented by $\mathbf{u}$ and $\mathbf{v}$.

2. Given $P(1,1,1),\ Q(3,-2,5),\ R(4,1,4)$ three points in space.
   (a) Find a vector that orthogonal to the plane through P, Q, R.
   (b) Find the area of the $\triangle PQR$.
   (c) Let P, Q, R, S be four corners of a parallelogram. Find the area of the parallelogram.

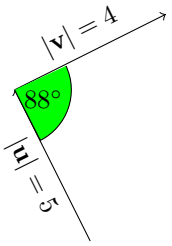
3. Given three vectors $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{v} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$, and $\mathbf{w} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.
   (a) Calculate the scalar triple product $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.
   (b) Find the volume of the parallelopiped whose adjacent edges are represented by the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$.
   (c) Are the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ coplanar? Explain.

4. Find $|\mathbf{u} \times \mathbf{v}|$ and determine whether $\mathbf{u} \times \mathbf{v}$ is directed into the page or out of the page.

   (a)



   (b)



5. Given $P(1,1,1), Q(3,-2,5), R(4,1,4), S(3,6,1)$ points in space,
   (a) find the volume of the parallelopiped with adjacent edges PQ, PS, and PR.
   (b) find the volume of the parallelopiped with adjacent edges SR, SQ, and SP.

6. Find the vector equation and parametric equations of the following lines.

   (a) passing through the point $(2, 3, -2)$ and parallel to the vector $\langle 3, -2, 5\rangle$.

   (b) passing through the points $(2, 3, -2)$ and $(3, -2, 5)$.

   (c) passing through the point $(2, 3, -2)$ and parallel to the vector $\langle 3, 0, 5\rangle$.

   (d) passing through the points $(2, 3, -2)$ and $(3, 3, 5)$.

   (e) passing through the point $(2, 3, -2)$ and perpendicular to the plane $2x + 3y + 5z = 0$.

   (f) passing through the point $(2, 3, -2)$ and perpendicular to the plane $2x + 3y + 5z = 10$.

   (g) passing through the point $(2, 3, -2)$ and perpendicular to the plane $3y + 5z = 0$.

7. Find the point at which each of the line in Q.N.#6 intersects $yz-$plane. Also, find the point where each of the line intersects $y-$axis.

8. Find the vector equation and parametric equations of the line segment joining the points $(2, 3, -2)$ and $(3, -2, 5)$. Determine whether the line segment intersects each of the following planes. If yes, find the point of intersection. If no, explain.

   (a) $2x + 3y + 5z = 10$

   (b) $2x + 3y + 5z = 110$