Math 5302, Homework 6

John Zweck

In this homework, all sets are subsets of $\mathbb{R}^n$.

1. Prove that an open ball is an open set.

2. Prove that if $A \subseteq \mathbb{R}^n$ is compact then A is bounded in that there is an $R > 0$ for that for all $x, y \in A$ we have $d(x, y) < R$.

3. Let $B(x, r)$ be the open ball centered at x with radius r. Prove that $B(x, r) \subseteq B(x', r')$ if and only if $d(x, x') \leq r' - r$.

4. Prove that $(A \cap B)^\circ = A^\circ \cap B^\circ$.

5. Provide an example of sets, A and B, for which $(A \cup B)^\circ \neq A^\circ \cup B^\circ$

6. Give an example of disjoint closed sets, A and B, of $\mathbb{R}^2$ for which there does *not* exist an $\epsilon > 0$ so that for all $x \in A$ and $y \in B$ we have $d(x, y) \geq \epsilon$.