MATH 603 Homework on Generalized Inverses

(1) Review the definition of block matrix multiplication on Meyer, p. 111 and do Exercises 3.6.1 and 3.6.2.

(2) Let $\mathbf{E_A}$ denote the unique reduced row echelon form obtained from an $m \times n$ matrix $\mathbf{A}$ by using row operations. Suppose that $\text{Rk}(\mathbf{A}) = r$. Let $\mathbf{B}$ be the $m \times r$ matrix consisting of the basic columns of $\mathbf{A}$ and let $\mathbf{C}$ be the $r \times n$ matrix consisting of the non-zero rows of $\mathbf{E_A}$. Prove that

(a) $\mathbf{A} = \mathbf{BC}$

(b) $\text{Rk}(\mathbf{B}) = r$

(c) $\text{Rk}(\mathbf{C}) = r$

(d) $\mathbf{B}^T\mathbf{B}$ and $\mathbf{CC}^T$ are both invertible matrices.

(3) Find the matrices $\mathbf{B}$ and $\mathbf{C}$ for the matrix $\mathbf{A}$ in Exercise 2.2.1 (a).

(4) Find the matrices $\mathbf{B}$ and $\mathbf{C}$ for the matrix

$$\mathbf{A} = \begin{pmatrix} \mathbf{D}_{r\times r} & \mathbf{E}_{r\times(n-r)} \\ \mathbf{0}_{(m-r)\times r} & \mathbf{0}_{(m-r)\times(n-r)} \end{pmatrix}$$

where $\mathbf{D}$ is invertible.

(5) Now do Exercise 4.5.20. Hint: For most parts you'll find it helpful to express all matrices in terms of $\mathbf{B}$ and $\mathbf{C}$.

(6) Calculate $\mathbf{A}^\dagger$ for the matrix $\mathbf{A}$ given in problem (3) above.

(7) Calculate $\mathbf{A}^\dagger$ for the matrix $\mathbf{A}$ given in problem (4) above in the special case that $\mathbf{E} = \mathbf{0}$.