Experiment 3: Techniques of Circuit Analysis

1 Objectives

The objective of this experiment is to analyze resistive circuits in DC employing the node-voltage method, the mesh-current method, source transformations and the Thévenin and Norton equivalents. Experimental results will allow the verification of the theoretical analysis.

2 Introduction and Test Circuits

In Experiment 2 the analysis of simple circuits was carried by employing Kirchhoff’s laws and Ohm’s law. This approach can be used for all circuits. However, for circuits with more elements and structurally more complicated a systematic approach is preferable. The techniques of circuit analysis studied here provide an aid in the analysis of more complex circuits.

2.1 Techniques of circuits analysis

The purpose of circuit analysis is to determine the current in each of the $b$ branches where the current is unknown. The formulation of $b$ independent equations with these currents as variables is achieved by applying Kirchhoff’s laws. In a circuit with $n$ nodes, $n - 1$ equations are formulated by applying Kirchhoff’s current law (KCL) to any set of $n - 1$ nodes and the remainder $b - (n - 1)$ equations can be written by applying Kirchhoff’s voltage law (KVL) to that number of meshes in the circuit. In general, $n_e \leq n$ and $b_e \leq b$, where $n_e$ and $b_e$ are the number of essential nodes and essential branches, respectively. Therefore, the formulation of the independent equations is achieved in terms of essential nodes and branches. However, by introducing new variables (node voltages and mesh currents), the circuit can be described with just $n_e - 1$ equations or just $b_e - (n_e - 1)$ equations. A brief description of each method is presented next.

2.1.1 Node-Voltage Method

The node-voltage method permits the description of a circuit in terms of $n_e - 1$ equations. Figure 3-1 shows a circuit suitable for analysis with the node-voltage method. In this circuit three essential nodes can be identified ($n_e = 3$), therefore two ($n_e - 1$) node-voltage equations describe the circuit. To select the set of $n_e - 1$ nodes to perform the analysis, one of the essential nodes is selected as a reference node. The node with the most branches is usually chosen. The equations are then written by applying KCL to each nonreference node expressing the branch currents in terms of the node voltages. Once the equation system is solved and the node voltages are known, all the branch currents can be calculated.

2.1.2 Mesh-Current Method

The mesh-current method permits the description of a circuit in terms of $b_e - (n_e - 1)$ equations. Figure 3-2 shows a circuit suitable for analysis with the mesh-current method. In this circuit four essential nodes ($n_e = 4$) and six essential branches ($b_e = 6$) can be identified, therefore three ($b_e - (n_e - 1)$) mesh-current equations describe the circuit. The equations are formulated by applying KVL to each mesh, expressing...
the voltages across the elements on the mesh in terms of the mesh currents. Once the equation system is
solved and the mesh currents are known, all the branch currents (and any other parameter of interest) can
be calculated.

2.2 Techniques for circuit simplification

In Experiment 2 was demonstrated the use of series-parallel reductions of resistive circuits in order to simplify
the circuit analysis. In this section, source transformations and Thévenin and Norton equivalent circuits are
discussed as additional methods to simplify the circuit analysis.

2.2.1 Source transformations

A source transformation, shown in Fig. 3-3, allows the replacement of a voltage source in series with a resistor
by a current source in parallel with the same resistor, or vice versa. In order for these two circuits to be
equivalent, the voltage drop and the current drawn by any load at nodes a and b must be the same. The
relationship between $V_s$ and $I_s$ is simply:

$$I_s = \frac{V_s}{R} \quad (3-1)$$

The above equation holds for the special cases when there is a resistor $R_p$ in parallel with the voltage source
or a resistor $R_s$ in series with the current source. In both cases the resistance has no effect on the equivalent
circuit (see Fig. 3-4). It can be shown that by applying source transformations on the circuit of Fig. 3-5, an
equivalent circuit with respect to the voltage $v_o$ consisting of a current source and a resistor can be found.

2.2.2 Thévenin and Norton equivalents

For purpose of analysis, it is often desirable to replace, at a specific pair of terminals (nodes) of a circuit,
the set of interconnected elements (resistors and power sources) “behind” the pair of terminals, by a single
resistor and a single power source. Thévenin and Norton equivalents are techniques of circuit simplification
that describe the behavior of a circuit at a specific pair of terminals.

Thévenin equivalent

A Thévenin equivalent circuit is composed by an independent voltage source $V_{Th}$ in series with a resistor
$R_{Th}$, which replaces an interconnection of sources and resistors. The value of $V_{Th}$ is the open circuit voltage
at the pair of terminals in the original circuit. $R_{Th}$ is given by the ratio of $V_{Th}$ and the short circuit current
$i_{sc}$ observed at the pair of terminals when a short circuit is placed across them. Thus, $R_{Th} = V_{Th}/i_{sc}$.

Fig. 3-6 shows a circuit that can be replaced by its Thévenin equivalent at terminals $a$ and $b$. Determination
of $V_{Th}$ and $R_{Th}$ can be simplified by applying source transformations.

Norton equivalent

A Norton equivalent circuit is composed by an independent current source $I_N$ in parallel with a resistor $R_N$.
The Norton equivalent can be derived from the Thévenin equivalent simply by making a source transforma-
tion. Therefore, the Norton current equals the short-circuit current at the terminals of interest, $I_N = i_{sc}$,
and the Norton resistance is identical to the Thévenin resistance, $R_N = R_{Th}$. The circuit shown in Fig. 3-7
can be easily replaced by its Norton equivalent at terminals $a$ and $b$. Notice that source transformations can
be applied to calculate $I_N$ and $R_N$.  

15
3 Preparation

In preparation for the lab compute the required quantities.

1. Node-voltage method. Consider the circuit in Figure 3-1 and the component values shown in Table 3-1. Use the node-voltage method to complete the entries corresponding to the theoretical values. Do not use source transformations. Include your calculations in a separate sheet.

2. Mesh-current method. Consider the circuit in Figure 3-2 and the component values shown in Table 3-2. Use the mesh-current method to complete the entries corresponding to the theoretical values. Include the calculations in a separate sheet.

3. Source transformations. Consider the circuit in Figure 3-5 and the component values shown in Table 3-3. Use a series of source transformations to find an equivalent circuit (with respect to $v_o$) consisting of a current source and a resistor. Complete the entries in Table 3-3 corresponding to the theoretical values. Use $R'_1$ and $R'_3$ for the calculation of $v'_o$. Include the calculations and circuit simplifications in a separate sheet.

4. Thévenin equivalent. Consider the circuit in Figure 3-6 and the component values shown in Table 3-4. Find the Thévenin equivalent and complete the entries corresponding to the theoretical values. Include the calculations in a separate sheet.

5. Norton equivalent. Consider the circuit in Figure 3-7 and the component values shown in Table 3-5. Find the Norton equivalent and complete the entries corresponding to the theoretical values. Include the calculations in a separate sheet.

4 Procedure

This part of the experiment requires assembling the resistive circuits presented in the previous section and measuring data from all of them. Set the current compliance value of the voltage source to 20 mA. The default voltage compliance value (21 V) for the current source is appropriate for this experiment.

1. Node-voltage method. Assemble the circuit in Figure 3-1 with the component values shown in Table 3-1. Take measurements to complete the entries corresponding to the experimental values.

2. Mesh-current method. Assemble the circuit in Figure 3-2 with the component values shown in Table 3-2. Take measurements to complete the entries corresponding to the experimental values.

3. Source transformations. Assemble the circuit in Figure 3-5 with the component values shown in Table 3-3. Take measurements to complete the entries corresponding to the experimental values. Replace $R_1$ by $R'_1$ and $R_3$ by $R'_3$ to measure $v'_o$.

4. Thévenin equivalent. Assemble the circuit in Figure 3-6 with the component values shown in Table 3-4. Take measurements to complete the entries corresponding to the experimental values. Connect the DMM in current measurement mode across terminals $a$ and $b$ in order to measure $i_{sc}$.

5. Norton equivalent. Assemble the circuit in Figure 3-7 with the component values shown in Table 3-5. Connect the DMM in current measurement mode across terminals $a$ and $b$ in order to measure $I_N$. Measure $V_{Th}$ in order to determine $R_N$. 

16
5 Analysis

This section is intended for the analysis and comparison of the experimental and theoretical results. Answer all the questions.

1. Calculate the error percentage between the measured and theoretical data and complete all the corresponding entries in Tables 3-1 through 3-5. The error percentage is given by

\[
\%\text{error} = \frac{d_{th} - d_m}{d_{th}} \times 100
\]

where \(d_{th}\) and \(d_m\) are the theoretical and measured data respectively.

2. Comment on the overall agreement between the theoretical and experimental results. Consider that the resistor values have a tolerance of up to \(\pm 10\%\).

3. From the results in Table 3-3, compare \(v_o\) and \(v'_o\). What is the effect of changing the value of \(R_1\) and \(R_3\)?

4. Show that the voltage \(v\) in the circuit of Fig. 3-1 can be found by applying source transformations, include the calculations. Compare the complexity of the node-voltage method (used in the Preparation section) and the source transformations approach for this particular problem.

<table>
<thead>
<tr>
<th>Param</th>
<th>Unit</th>
<th>Theor</th>
<th>Param</th>
<th>Unit</th>
<th>Theor</th>
<th>Exper</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_1)</td>
<td>kΩ</td>
<td>1.0</td>
<td>(V_{R_1})</td>
<td>(V)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_2)</td>
<td>kΩ</td>
<td>2.2</td>
<td>(V_{R_2})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_3)</td>
<td>kΩ</td>
<td>5.6</td>
<td>(V_{R_3})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_4)</td>
<td>kΩ</td>
<td>6.8</td>
<td>(V_{R_4})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_5)</td>
<td>kΩ</td>
<td>8.2</td>
<td>(V_{R_5})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V_s)</td>
<td>V</td>
<td>10</td>
<td>(v)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(I_s)</td>
<td>mA</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-1: Node-voltage method.

<table>
<thead>
<tr>
<th>Param</th>
<th>Unit</th>
<th>Theor</th>
<th>Param</th>
<th>Unit</th>
<th>Theor</th>
<th>Exper</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_1)</td>
<td>kΩ</td>
<td>1.0</td>
<td>(V_{R_1})</td>
<td>(V)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_2)</td>
<td>kΩ</td>
<td>2.2</td>
<td>(V_{R_2})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_3)</td>
<td>kΩ</td>
<td>5.6</td>
<td>(V_{R_3})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_4)</td>
<td>kΩ</td>
<td>6.8</td>
<td>(V_{R_4})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_5)</td>
<td>kΩ</td>
<td>8.2</td>
<td>(V_{R_5})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V_s)</td>
<td>V</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-2: Mesh-current method.

<table>
<thead>
<tr>
<th>Param</th>
<th>Unit</th>
<th>Theor</th>
<th>Param</th>
<th>Unit</th>
<th>Theor</th>
<th>Exper</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_s)</td>
<td>V</td>
<td>10</td>
<td>(v)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(I_s)</td>
<td>mA</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-3: Source transformations.
### Table 3-4: Thévenin equivalent.

<table>
<thead>
<tr>
<th>Param</th>
<th>$V_s$</th>
<th>$I_s$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>Param</th>
<th>Unit</th>
<th>Theor</th>
<th>Exper</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>V</td>
<td>mA</td>
<td>kΩ</td>
<td>V</td>
<td></td>
<td>$V_{Th}$</td>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theor</td>
<td>10</td>
<td>10</td>
<td>1.0</td>
<td>2.2</td>
<td>5.6</td>
<td>$I_{sc}$</td>
<td>mA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$R_{Th}$</td>
<td>kΩ</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3-5: Norton equivalent.

<table>
<thead>
<tr>
<th>Param</th>
<th>$I_s$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$R_4$</th>
<th>Param</th>
<th>Unit</th>
<th>Theor</th>
<th>Exper</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>mA</td>
<td>kΩ</td>
<td>V</td>
<td></td>
<td></td>
<td>$V_{Th}$</td>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theor</td>
<td>10</td>
<td>1.0</td>
<td>2.2</td>
<td>5.6</td>
<td>6.8</td>
<td>$I_N$</td>
<td>mA</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$R_N$</td>
<td>kΩ</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-1: Test circuit for node-voltage method.

Figure 3-2: Test circuit for mesh-current method.

Figure 3-3: Source transformations.
Figure 3-4: Equivalent circuits containing a resistor in parallel with a voltage source or in series with a current source.

Figure 3-5: Test circuit for source transformations.

Figure 3-6: Test circuit for Thévenin equivalent.

Figure 3-7: Test circuit for Norton equivalent.