Experiment 2: Linear Systems

This experiment examines the characterization of a system by its impulse response and transfer function and the relationship between them. The design and implementation of filters is also discussed here. Specifically, second order passive and active low pass filters are considered.

1 Introduction

In communication systems signals are applied to devices (systems) to be amplified, modulated, transmitted and filtered. In order to determine the effect of such systems over an input signal, it is necessary to describe the systems in a mathematical form.

1.1 Impulse response of a system

Just like a signal is described by a function of t, a linear system is described in the time domain by the signal (function) that results at the output when an impulse $\delta(t)$ is applied to the input. This function is referred to as the *impulse response* of the system $h(t)$. The output signal $y(t)$ resulting of applying the signal $x(t)$ to the system $h(t)$ is given by the *convolution* operation between $x(t)$ and $h(t)$,

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda \equiv x(t) * h(t) \tag{1}$$

1.2 Transfer function of a system

By taking the Fourier transform of both sides of (1), the linear system relates the input and output spectra by

$$Y(f) = X(f)H(f) \tag{2}$$

or

$$H(f) = \frac{Y(f)}{X(f)} \tag{3}$$

where $H(f)$ is the *transfer function* of the system. Therefore, the impulse response and the transfer function are a Fourier transform pair $h(t) \leftrightarrow H(f)$. The tranfer function is generally a complex quantity that can be expressed in terms of its magnitude and phase angle:

$$H(f) = |H(f)| \exp(j\angle H(f)) \tag{4}$$

The function $|H(f)|$ describes the way the system affects (amplifying or attenuating) the magnitude of the input frequency components, and therefore is referred to as the *magnitude response* of the system. Similarly, $\angle H(f)$ describes the phase variations of the frequency components at the output with respect to the input, and it is called the *phase response* of the system.

Since an impulse signal is physically non-existent, the transfer function (also called frequency response) of a linear system can be determined by applying a sinusoid sweeping the frequency range of interest and measuring the ratio of the output and input amplitudes, as well as the phase difference for each value of frequency. In the lab, the virtual instrument RESPONSE.VI uses this approach to determine the frequency response of a system.

1.3 Practical filters

Noticing from equation (2) that a linear system acts as a frequency selective device, a system (network) can be designed such that its transfer function shows the amplification or attenuation of a desired range of frequencies. This idea leads to the concept of *filtering*. Filters are commonly described as *lowpass*, *highpass*, *bandpass* or *stopband*, depending of the location of the range of frequencies they allow to pass without attenuation. The main parameter of a filter is the *half-power bandwidth*, that is determined by the range of frequencies in which the power of the output signal is more than one half the power of the input signal.

In practice, there are two forms of implementing a filter, *passive* filters which do not require external power to perform the filtering operation, and *active* filters which do require external power. The convenience of the latter is that they can produce amplification of the input signal in the bandpass. Practical filters can be implemented from polynomial functions of frequency ($H(f)$) of order one to about order eight. These functions can be derived by using different approximations such as Butterworth, Chebyshev, elliptic, etc. Filter theory is an extensive subject, in this lab we will consider only second-order Butterworth lowpass filters of both passive and active types. For the interested reader in the subject an introductory document (filters.pdf) on filters is provided in the compressed .zip file.

Dealing specifically with lowpass filters, the half-power bandwidth is determined by the *cut-off frequency* f_c of the filter. We may think of f_c as the frequency at which the filter begins gradually attenuating the frequency components above it. When the magnitude response is plotted in decibels (dB), the cut-off frequency is at the point where the response is 3 dB lower than the *maximum* magnitude of the response. In the lab, we will deal with magnitude response plots in decibels.

1.3.1 Passive LPF

Consider the passive lowpass filter (LPF) of Fig. 2 in the PSPICE schematic. This is a second-order LPF whose cut-off frequency is given by

$$f_c \approx \frac{1}{10RC} \tag{5}$$

provided that the conditions shown in Fig. 2 are met. A value of $R > 500\,\Omega$ must be selected in order to avoid a small input impedance, the larger the value of R the better.

1.3.2 Active LPF

In Fig. 3 it is shown a second-order active LPF whose cut-off frequency is given by

$$f_c = \frac{1}{2\pi RC} \tag{6}$$

and whose amplification factor K (called *gain*), expressed in decibels, of the frequencies below f_c is

$$K_{dB} = 20\log_{10}\left(1 + \frac{R_a}{R_b}\right) \tag{7}$$

2 Prelab instructions

To perform the prelab PSPICE will be required. Enclosed in brackets is the number of points assigned to each question/plot.

1. Consider the RLC circuit of Fig. 1. A very narrow rectangular pulse approximating an impulse is applied to the circuit to obtain an approximation of the impulse response. Notice that the 7 Ω resistor represents the internal resistance of the inductor. Simulate the transient response of this circuit. The parameters of the applied pulse are: Ampl=10 V, pulse width = 250 ns. Plot the results from 0 to 50 μs with a print step of 0.01 μs. [3].

2. Obtain the Fourier transform of the simulated output by selecting `Fourier` in the `Trace` menu. Then in the `Plot` menu, select `Axis settings...` and set the `Y Axis` scale to `log`. The resulting plot although is not in dBV resembles the plot you will see in the lab. This is the plot that must be included in the prelab. [3].

3. Consider the passive lowpass filter of Fig. 2. Using R=1 kΩ and C=0.1 μF, (a) compute its cut-off frequency f_c [1]. (b) Plot the magnitude response by simulating an AC sweep of `Decade` type from 100 Hz to 10 kHz. After the simulation is completed double-click on the legend in the lower left corner (something like V(C2:2)) to open the `Modify trace` window. Here, enter 20*log10(Vo/Vi) with Vo and Vi sustituted by the proper node variables representing the output and input voltages. This way you will create a plot of the magnitude response (not the output voltage) in decibels [4]. In the lab you will get similar plots. [5 points total].

4. Additionally, plot the phase response. Modify the trace expression as before but now entering P(Vo) sustituting Vo with the proper node variable representing the output voltage. The resulting plot is in degrees and is comparable to the one that will be obtained in the lab. [4].

5. Consider the active LPF of Fig. 3. Use R=1.6 kΩ, R_a=33 kΩ, R_b=56 kΩ, and C=0.01 μF. Notice that the 0.1 μF capacitor connected across the power terminals of the op-amp does not influence the frequency response, it will be used during the lab for decoupling of the power supplies to help reduce noise. Calculate the (a) cut-off frequency f_c [1] and (b) the gain (in dB) K [1]. (c) Obtain plots of the magnitude [4] and phase [4] responses in the frequency range from 1 kHz to 100 kHz following the procedures of Prelab 2.3 and 2.4. [10 points total].

3   Lab procedure

GENERAL INSTRUCTIONS:

- Load the virtual instrument TIMEFREQ.VI by double-clicking the shortcut located in the computer desktop.

- **After taking each plot make sure to ask your TA to verify that your results are correct. This also serves to monitor your progress and performance.** Your TA have the correct plots from actual circuits so you can compare yours.

3.1   Impulse response and transfer function

In this section an approximation to the impulse response of an RLC circuit is obtained experimentally. The input impulse function to the circuit is approximated by a very narrow rectangular pulse. The magnitude of the transfer function is obtained as well by taking the Fourier transform of the output signal.

1. Assemble the circuit of Fig. 1 using the top function generator (FG1) as the signal source. Remember that the 7 Ω resistor is the internal resistance of the inductor, so disregard it for purposes of assembling.

2. Connect the channel 1 probe in parallel with the capacitor and the channel 2 probe in parallel with the top function generator (FG1).

3. Set the following parameters in the FG1: Square wave, Ampl=5 V, Offset=2.5 V, Freq=500 kHz. Press Shift Offset and set the duty cycle to 20%. Set the FG in burst mode by pressing Shift Burst.

4. Capture the *input* signal/spectrum to the RLC circuit by executing TIMEFREQ.VI with the following parameters: Channel=2, TS=10u (meaning microseconds), FS=5000 (enter this number as it is, it is already given in kHz), Save data=ON, a proper file name. Run the VI by pressing Ctrl-R. [5 points, plot P1].

5. Now capture the *output* signal/spectrum from the RLC circuit by executing TIMEFREQ.VI with the following parameters: Channel=1, TS=100u, FS=5000, Save data=ON, a proper file name. Run the VI by pressing Ctrl-R. [5 points, P2].

   Notice that the frequency span seems to be much larger than what is required, as all the significant spectral components (above -60 dBV) are located below 500 kHz, however this is necessary to obtain a clear spectrum from the oscilloscope.

3.2   Frequency response

In this section the complete frequency response of the RLC circuit is obtained. Specifically, both magnitude and phase responses are obtained by applying a sinusoid at different frequencies (an AC sweep like in PSPICE) and measuring the magnitude ratio and phase difference of the input and output of the circuit.

1. Minimize the TIMEFREQ.VI window and load the virtual instrument RESPONSE.VI by double-clicking the shortcut located in the computer desktop.

2. Obtain the frequency response of the RLC circuit by executing RESPONSE.VI with the following parameters: Lowfreq=50k (you may enter "50k"), Upperfreq=500k, Amplitude=10, Offset=0, points/decade=20, Save file=ON, a proper file name. The VI with this number of points/decade will take a few minutes to complete. You should see that the frequency of the FG is changing. [5 points, P3].

3.3   Lowpass Filters

In this section you will assemble and test the filters of Prelab 2.3-2.5. By applying an AC sweep like in the previous section, the frequency responses of both filters will be obtained.

3.3.1   Passive LPF

1. Using FG1 as the signal source, assemble the passive filter of Fig. 2. Connect the channel 1 probe to the output and the channel 2 probe to the input.

2. Obtain the frequency response of the passive LPF circuit by executing RESPONSE.VI with the following parameters: Lowfreq=100, Upperfreq=10k, Amplitude=10, Offset=0, points/decade=10, Save file=ON, a proper file name. [5 points, P4].

3.3.2   Active LPF

1. Set the power supply outputs to ±15 V and turn it off. Using FG1 as the signal source, assemble the active filter of Fig. 3. Connect the channel 1 probe to the output (at pin 6) and the channel 2 probe to the input.

2. Turn on the power supply and obtain the frequency response of the active LPF circuit by executing RESPONSE.VI with the following parameters: Lowfreq=1k, Upperfreq=100k, Amplitude=8, Offset=0, points/decade=10, Save file=ON, a proper file name. [5 points, P5].

   You may observe an abrupt change in the phase response plot around 100 kHz, this is because *the plot* (not the phase) is restricted to ±180°, and -180°=180°.

3.3.3   Filtering of a signal

In this section a signal composed by two sinusoids is lowpass filtered with the active filter of the previous section. The effect of the filter on the signal is observed in both the time and frequency domains.

1. If the FG is in remote mode (indicated by `Rmt` in the front panel) press $\boxed{\texttt{Shift}}$ to set it in local mode. Set Ampl=8 V and Freq=1 kHz. Connect the bottom FG (FG2) to the input of the filter along with the top FG (FG1). Enter the following parameters in FG2: Ampl=8 V, Freq=40 kHz.

2. Maximize the TIMEFREQ.VI window and enter the following parameters: Channel=2, TS=2m, FS=50, Save data=ON, a proper file name. Run the VI by pressing Ctrl-R. This will allow the capture of the input signal/spectrum to the filter. [5 points, P6].

   As you can see, the input signal is the sum of two sinusoids with the same amplitude.

3. Now observe the effect of the LPF on the input signal by capturing the output signal. Run the VI changing only Channel=1 and a different file name. [5 points, P7].

   Please shut down properly the computer and equipment.

4   Analysis

This section contains questions regarding the obervations you made during the lab. Enclosed in brackets is the number of points assigned to each question.

1. Include in your report all the plots obtained during the lab. Present the spectrum of plot P2 in the range from 0 to 500 kHz only. Make sure to label properly all the plots. The number of points assigned to each plot is specified in the lab procedure. Refer to Appendix B section 3.4 for instructions regarding the plotting of experimental results.

2. About plot P1, and considering the Fourier transform pair $A\delta(t) \leftrightarrow A$, explain why this narrow rectangular pulse is a good approximation to the impulse function in the range of frequencies from 0 to 1000 kHz. [2].

3. (a) Estimate the coordinates (kHz,dBV) of the point with the highest magnitude in the spectrum of P2. [2]. (b) Compare this frequency value with the theoretical resonant frequency of this RLC circuit given by $f_o = (2\pi\sqrt{LC})^{-1}$. [2]. [4 points total].

4. From plot P3: (a) estimate the coordinates (kHz,dB) of the point with the highest magnitude and compare it with the previous answer [2]. (b) Measure the phase shift at this frequency [2]. (c) What type of filter is this circuit? (lowpass, highpass, bandpass, or stopband) [2]. [6 points total].

5. Measure the magnitude (in dB) at 1 kHz (r_1) and 40 kHz (r_{40}) of the magnitude response of the active LPF in plot P5. [2].

6. The signal of plot P6 was applied to the input of the active LPF, (a) measure the magnitude (in dBV) of the *input* spectral components at 1 kHz (m_{i1}) and 40 kHz (m_{i40}) in plot P6. [2]. (b) Measure the magnitude (in dBV) of the *output* spectral components at 1 kHz (m_{o1}) and 40 kHz (m_{o40}) in plot P7. [2]. (c) Write the equation that relates r_x, m_{ix} and m_{ox} where x is any frequency. [2]. [6 points total].

7. From plot P4 (a) measure the magnitude (in dB) of the passive LPF transfer function at 100 Hz. [2]. (b) Calculate the amplitude (in V) of the output signal if an input signal of 100 Hz with amplitude=$\sqrt{2}$ V is applied to this filter. [2]. [4 points total].

Vo

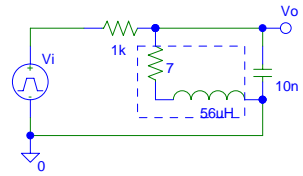Vi
1k   7

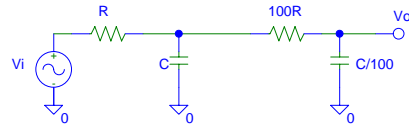56µH   10n

0

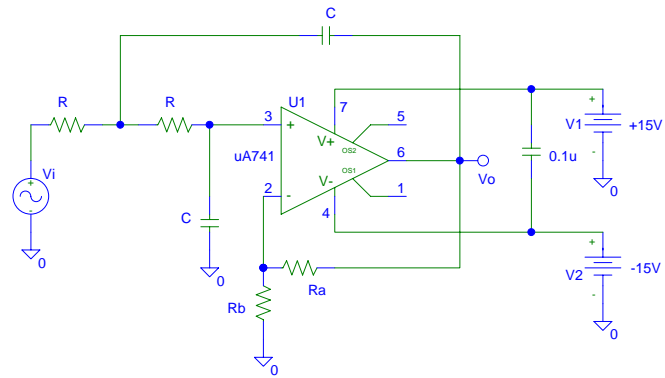Fig. 1 RLC Test circuit for impulse response.

R   100R   Vo

Vi
C   C/100

0   0   0

Fig. 2 Passive LPF.

C

R   R   U1
3   7   5
+
V+   OS2
uA741   6
2   OS1   Vo
V-
1
C   4

Rb   Ra

0

Fig. 3 Active LPF.

V1   +15V
0.1u

V2   -15V
0

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