Ergodic Theory. Homework 5.

(due $\sim$ 03.04.2015)

1. Prove that cylindrical σ-algebra for the sequence representation of continued fractions generates any Borel set on the intervasl $[0, 1]$.

2. Let $f(x) : S^1 \to S^1$ is a continuous transformation of a unit circle. Prove that there exists a lifting of f: a continuous transformation $F(x) : \mathbb{R} \to \mathbb{R}$ such that $f([x]) = [F(x)]$. Prove that this transformation is unique up to an additive constant, i.e. for any two such transformations F_1 and F_2 there exists a constant C such that $F_1(x) = F_2(x) + C$.

3. Let F be a lifting of f. Define degree of a transformation by a formula $deg(f) = F(x + 1) - F(x)$. Prove that if f is homeomorphism then $deg(f) = \pm 1$.

4. Prove that if seqence a_n is such that $a_{m+k} \leqslant a_m + a_k + 1$ then $\limsup a_n/n = \liminf a_n/n$.

5. Define rotation number for $f : S^1 \to S^1$ as $\omega_f = \lim(F^n(x) - F(x))/n$. Prove that if $\omega_f = p/q$ then $\omega_{f^q} = 0$.