Review problems for the Ergodic Theory course.

1. Find the asymptotic of the series $\sum\limits_{n=1}^{\infty} \sin^2(n\sqrt{2})$.

2. Let $a_n = [10^{G^n(\pi)}]$ where $G$ denotes Gauss transformation and $[\cdot]$ denotes integer part. How often $a_n$ will be equal to 1?

3. Let $A = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$. Find $A^{100}$ with accuracy 0.001.

4. In DNA sequences of some particular species written in $ACGT$ letters the following rules were found:

   - After $A$ it may appear $A$ or $C$ with equal probabilities.
   - After $C$ in 1/3 of cases it appear $T$ and in another 2/3 it appeared $G$.
   - After $G$ it always appear $T$.
   - After $T$ with equal probabilities it appear $A$ or $C$.

   Find the probability to get a sequence which contains both words $AA$ and $TC$ (not necessarily one next to another).

5. Compute topological entropy of the transformation $Tx = \{7x\}$ on $[0, 1]$.

6. Two professors used coding procedure to model some dynamical system on a unit interval. Professor A obtained the shift on the space of sequences on four symbols with probabilities $(\frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{1}{15})$ and Professor B obtained the shift on the space of sequences on three symbols with probabilities $(\frac{1}{4}, \frac{1}{8}, \frac{5}{8})$. Can it happen that they model the same dynamical system?