Review problems for the Ergodic Theory course.

1. Find the asymptotic of the series \( \sum_{n=1}^{\infty} \sin^2(n\sqrt{2}) \).

2. Let \( a_n = [10^{G_n}(\pi)] \) where \( G \) denotes Gauss transformation and \([\cdot]\) denotes integer part. How often \( a_n \) will be equal to 1?

3. Let \( A = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \). Find \( A^{100} \) with accuracy 0.001.

4. In DNA sequences of some particular species written in ACGT letters the following rules were found:

   • After A it may appear A or C with equal probabilities.
   • After C in 1/3 of cases it appear T and in another 2/3 it appeared G.
   • After G it always appear T.
   • After T with equal probabilities it appear A or C.

   Find the probability to get a sequence which contains both words AA and TC (not necessarily one next to another).

5. Compute topological entropy of the transformation \( T x = \{7x\} \) on \([0, 1]\).

6. Two professors used coding procedure to model some dynamical system on a unit interval. Professor A obtained the shift on the space of sequences on four symbols with probabilities \((\frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{1}{15})\) and Professor B obtained the shift on the space of sequences on three symbols with probabilities \((\frac{1}{4}, \frac{1}{8}, \frac{5}{8})\). Can it happen that they model the same dynamical system?