ANTI-MIDTERM (SOLUTIONS)

1. Solve the equation.

$$x + \sqrt{14 - 2x} = 3$$

SOLUTION.

Isolate the square root,

$$\sqrt{14 - 2x} = 3 - x$$

Square,

$$14 - 2x = 9 - 6x + x^2,$$

$$x^2 - 6x + 9 + 2x - 14 = 0$$

Solve the quadratic equation,

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0 \qquad \text{or} \qquad x = \frac{4 \pm \sqrt{4^2 - (4)(1)(-5)}}{2} = \frac{4 \pm 6}{2} = 2 \pm 3$$

$$x = 5 \;\text{ or }\; x = -1 \qquad\qquad x = 5 \;\text{ or }\; x = -1$$

Check,

$$5 + \sqrt{14 - 2(5)} \neq 3 \quad \textit{(not a solution)}$$

$$-1 + \sqrt{14 - 2(-1)} = 3 \quad \textit{(solution)}$$

Answer: $\boxed{x = -1}$.

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2. Solve the inequality and express your solution using the interval notation.

$$|-7x + 5| \geq 2$$

SOLUTION.

$$\begin{array}{rcc} \textit{Either} & -7x + 5 \geq 2 \quad \text{or} & -7x + 5 \leq -2 \\ & -7x \geq -3 & -7x \leq -7 \\ & x \leq \dfrac{3}{7} & x \geq 1 \end{array}$$

Answer: $\boxed{\left(-\infty, \frac{3}{7}\right] \;\cup\; [1, +\infty)}$.

3. Four dollars are paid by nickels and quarters. Forty coins are used overall. How many nickels are used?

SOLUTION.

Let x be the number of nickels. Then the number of quarters is $(40-x)$, and the total amount paid is

$$5x + 25(40 - x) = 400 \quad \textit{(400 cents or 4 dollars)}$$

Solve this linear equation,

$$5x + 1000 - 25x = 400$$

$$20x = 600$$

$$x = 30$$

Check: $(5)(30) + (25)(40 - 30) = 400$ *(correct).*

Answer: $\boxed{\textit{30 nickels}}$.

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4. Write an equation of the line **P** passing through the point $(3, -2)$ that is perpendicular to the line **Q** given as $2x + 3y = 12$. Draw the graph of this line on the given Cartesian coordinate system.

SOLUTION. Obtain the slope-intercept formula of line **Q**,

$$3y = -2x + 12, \qquad y = -\frac{2}{3} + 4$$

Thus, line **Q** *has the slope* $-\frac{2}{3}$. *Line* **P** *is perpendicular to line* **Q**, *therefore, its slope is*

$$m = -\frac{1}{-2/3} = \frac{3}{2}$$

Also, it passes through the point $(3, -2)$. *Then, its equation (slope-point formula) is*

$$y - (-2) = \frac{3}{2}(x - 3)$$

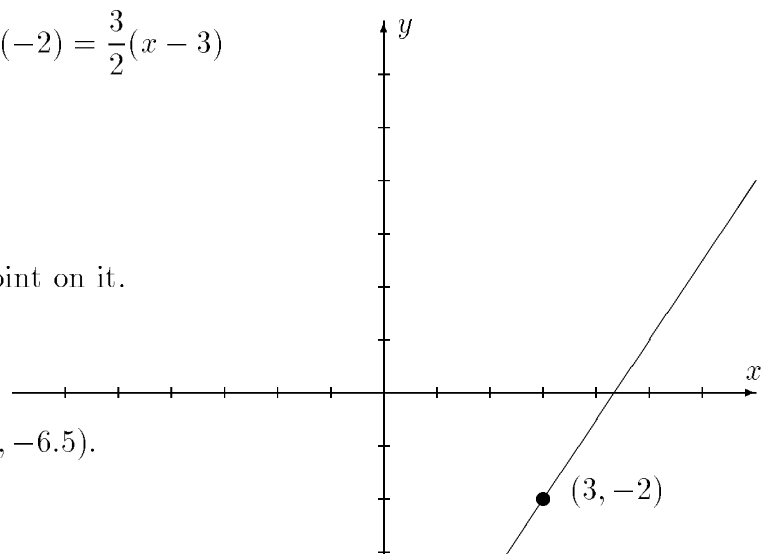
Answer: $\boxed{y + 2 = \frac{3}{2}(x - 3)}$.

To draw the line, we need the second point on it.
Let $x = 0$ and find the y-intercept,

$y = \frac{3}{2}(0 - 3) - 2 = -6.5$

Now we have two points, $(3, -2)$ and $(0, -6.5)$.

See the line on the graph.



5. Find an equation of the circle with the diameter having endpoints $(-4, 1)$ and $(3, -6)$.

SOLUTION.

The center of this circle is the midpoint between the given two points,

$$\left(\frac{(-4)+3}{2}\ ,\ \frac{1+(-6)}{2}\right) = (-0.5\ ,\ -2.5)$$

The radius is the half-distance between the given points,

$$r = \frac{1}{2}\,d = \frac{1}{2}\sqrt{(-4-3)^2 + (1+6)^2} = \frac{1}{2}\sqrt{98}$$

so that

$$r^2 = \frac{1}{4}(98) = 24.5$$

Answer: $\boxed{(x+0.5)^2 + (y+2.5)^2 = 24.5}$

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6. A \$1,000 investment in a certain project gives a profit of \$52. A \$1,400 investment in the same project gives a profit of \$68.

(a) Use these data to write a linear relationship between the invested amount and the profit.

(b) Your financial advisor suggests to invest \$1,900 into this project. According to your results, what profit should be expected from this investment?

SOLUTION.

(a) Use the two-point formula,

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-52}{x-1000} = \frac{68-52}{1400-1000}$$

$$\frac{y-52}{x-1000} = 0.04$$

$$y - 52 = 0.04(x-1000)$$

$$y - 52 = 0.04x - 40$$

$$y = 0.04x + 12$$

Answer: $\boxed{\frac{y-52}{x-1000} = 0.04, \quad or \quad y - 52 = 0.04(x-1000), \quad or \quad y = 0.04x + 12}$

(b) Substitute $x = 1900$ and calculate y,

$$y = (0.04)(1900) + 12 = 88$$