

Modeling Random Phenomena and Making Decisions under Uncertainty

Uncertainty in computer environment

- Arrival of jobs
- Execution time
- Memory requirement
- Failure of components
- Exposure to viruses
- Errors in codes
- Etc., etc., etc.

Random phenomena elsewhere

- Economy:
 - stock prices
 - number of jobs
 - price of oil

- Environment
 - temperature
 - pollution
 - natural disasters

- Going to UTD
 - number of green lights
 - available parking

- This class
 - quiz problems
 - time spent on each topic
 - grades

This course: **Quantify uncertainty, model uncertainty, make decisions**

Probability

= chance (common sense)

= odds (gambling)

= long-term proportion (relative frequency)

= likelihood (forecasting)

= finite measure (mathematics)

Probability
= function of an event = $P(E)$

Domain: events

Event = combination of *outcomes*

Consider an experiment, results are outcomes,

Sample space $S = \{\text{all possible outcomes}\}$

Event $E =$ a set of outcomes

= subset of S ($E \subset S$)

Range: $[0, 1]$

“Every event” E has probability $P(E)$,

$$0 \leq P(E) \leq 1$$

If

$$E = \{O_1, \dots, O_n\} = \{\text{outcomes}\}$$

then

$$P(E) = \sum_{k=1}^n P(O_k) = P(O_1) + \dots + P(O_n)$$

An empty event \emptyset is an event,

$$P(\emptyset) = 0$$

Also, $P(S) = 1$.

Set operations

E_1, E_2, \dots, E_n - events (sets of outcomes)

Union of E_1, \dots, E_n is an event

- consists of all outcomes of E_1, \dots, E_n
- occurs if any of E_1, \dots, E_n occurs

$$\{E_1 \cup \dots \cup E_n\} = \{E_1 \text{ or } \dots \text{ or } E_n\}$$

Intersection of E_1, \dots, E_n is an event

- consists of common outcomes of E_1, \dots, E_n
- occurs if each E_1, \dots, E_n occurs

$$\{E_1 \cap \dots \cap E_n\} = \{E_1 \text{ and } \dots \text{ and } E_n\}$$

Complement of E is an event

- consists of outcomes that are not in E
- occurs if E does not occur

$$\bar{E} = \{ \text{not } E \}$$

Disjoint, or mutually exclusive events

- cannot occur together
- $A \cap B = \emptyset$

Exhaustive events

- their union is Sample Space
- at least one occurs for sure
- $A \cup B \cup C = S$

Basic rules of Probability

- $0 \leq P(E) \leq 1$ for all E
- $P(\emptyset) = 0$ and $P(S) = 1$

- Disjoint events:

$$P(E_1 \cup \dots \cup E_n) = P(E_1) + \dots + P(E_n)$$

- Any events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Complement rule

$$P(E) + P(\bar{E}) = 1$$

(disjoint and exhaustive)

$$\Rightarrow P(\bar{E}) = 1 - P(E)$$

Question:

What should I compute: $P(E)$ or $P(\bar{E})$?

Independence

- Independent events:

$$P(A \cap B) = P(A)P(B)$$

$$P(E_1 \cap \dots \cap E_n) = P(E_1) \times \dots \times P(E_n)$$

Questions:

Can disjoint events be independent?

Can exhaustive events be independent?