RANDOM VARIABLES

Random variable = function of an outcome

$$X = f(outcome)$$

$$S \rightarrow \begin{cases} \text{real numbers} \\ \text{integers} \\ (0,1) \\ (0,+\infty) \\ \text{etc.} \end{cases}$$

$$(\text{domain} \rightarrow \text{range})$$

Random variable is a quantity that depends on chance.

A value of a random variable becomes known once an experiment is completed and its outcome is obtained.

Example: toss 3 fair coins.

Let X = number of heads

Possible values: $\{0, 1, 2, 3\}$, and

$$\begin{aligned} P\{X = 0\} &= 1/8 \\ P\{X = 1\} &= 3/8 \\ P\{X = 2\} &= 3/8 \\ P\{X = 3\} &= 1/8 \end{aligned}$$

(Same model for good/defective items, pass/fail, girl/boy = *Bernoulli trials*)

Summarize:

x	Probability mass function pmf $p_X(x) = P\{X = x\}$
0	1/8
1	3/8
2	3/8
3	1/8
$TOTAL$	1

Properties:

$$0 \leq p(x) \leq 1$$

$$\sum_x p(x) = 1$$

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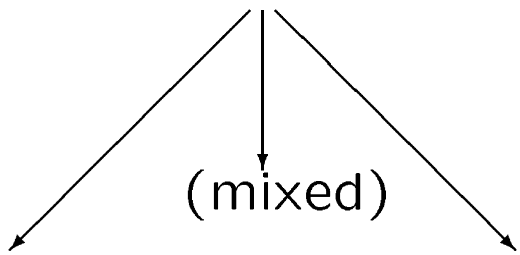

DISCRETE	CONTINUOUS
Finite or countable number of values	The whole interval of possible values
Examples	
number of jobs in a queue	execution time
	waiting time
number of errors	temperature
number of failures $(0, 1, 2, ...)$	height, weight
	intensity
proportion of failures $(0, \frac{1}{N}, \frac{2}{N}, ...)$	distance
	miles per gallon
high jump	long jump

Distribution of a random variable X
$=$ collection of probabilities

$$P\{X \in A\} = \sum_{x \in A} p_X(x)$$

Examples: $P\{X = 3\}$, $\{X > 10\}$,
$P\{X$ is an even number $\}$

Cumulative distribution function (CDF) of X is

$$F_X(x) = P\{X \leq x\} = \sum_{y \leq x} p_X(y)$$

Properties:
$F(x)$ is nondecreasing
Jumps by $p(x)$ at the point x
$F(-\infty) = 0, \quad F(+\infty) = 1$

Computing probabilities:

$$P\{a < X \leq b\} = F(b) - F(a)$$

Random vectors and joint distribution

If X, Y = random variables, then

(X, Y) = random vector

It has a **joint pmf**

$$\begin{aligned} p(x,y) &= P\{(X,Y)=(x,y)\} \\ &= P\{X=x \cap Y=y\} \end{aligned}$$

From $p(x,y)$, the marginal pmf of X and Y are

$$\begin{aligned} p_X(x) &= P\{X=x\} = \sum_y p(x,y) \\ p_Y(y) &= P\{Y=y\} = \sum_x p(x,y) \end{aligned}$$

From $p_X(x)$ and $p_Y(y)$, in general, one cannot compute $p(x, y)$.

Independence

Random variables X and Y are **independent** if

$$p(x, y) = p_X(x)p_Y(y)$$

i.e., $\{X = x\}$ and $\{Y = y\}$ are independent events for **all** x and y.

So,

- to show independence, verify this equality for all x and y;

- to show dependence, find one pair (x, y) violating it