SC Design
Facility Location
Sections 4.1, 4.2
Chapter 5 and 6
Outline

- Frequency decomposition of activities
- A strategic framework for facility location
- Multi-echelon networks
- Analytical methods for location
Frequency Decomposition

- SCs are enormous
- It is hard to make all decisions at once
- Integration by smart decomposition
- Frequency decomposition yields several sets of decisions such that each set is integrated within itself
Frequency Decomposition

- Low frequency activity, ~ once a year, high fixed cost
  - Capacity expansion budget
- Moderate frequency activity, ~ once a month
  - Specific machines to purchase
- High frequency activity, ~ once a day, low fixed cost
  - What to produce
The Cost-Response Time Frontier

Cost

Response Time

Hi

Low

Central Raw Material and Custom production

Custom production with raw material at suppliers

7-Eleven

Regional

Central

Sam’s Club

Local FG

Mix

Regional FG

Local WIP

Central FG

Central WIP

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Service and Number of Facilities

Response Time

Number of Facilities
Where inventory needs to be for a one week order response time - typical results --> 1 DC
Where inventory needs to be for a 5 day order response time - typical results --> 2 DCs
Where inventory needs to be for a 3 day order response time - typical results --> 5 DCs
Where inventory needs to be for a next day order response time - typical results --> 13 DCs
Where inventory needs to be for a same day / next day order response time - typical results --> 26 DCs
Costs and Number of Facilities

No economies of scale in shipment size
Economies of scale in inbound shipping
Cost Build-up as a function of facilities

- Percent Service
- Level Within
- Promised Time
- Facilities
- Inventory
- Transportation
- Labor
Network Design Decisions

- Facility function: Plant, DC, Warehouse
  - Where to locate functions, e.g. packaging
- Facility location
- Capacity allocation
- Market and supply allocation
  - Who serves whom
Factors Influencing Network Design Decisions

◆ Strategic

Facilities

Global Customers

Offshore

<reduce tariffs>

VW plants in Mexico
Serving Latin America

Regional Customers

Source

<low-cost>

Nike plants in Korea

Server

<local-content>

Suzuki’s Indian venture
Maruti Udyog

Contributor

<customization>

Maruti Udyog

Outpost facility

<Learn local skills>

Facilities in Japan

Lead facility

Contribution

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<customization>

Maruti Udyog
Factors Influencing Network Design Decisions

◆ Technological,
  – availability and economies of scale (fixed operational costs)
◆ Macroeconomic,
  – Tariffs, exchange rate volatility, economic volatility
◆ Political, stability
◆ Infrastructure, electricity, phone lines, suppliers
◆ Competitive
  – Negative externalities, see the next slide
  – Positive externalities
    » Nissan in India
    » Toyota City
    » Shopping Malls
    » Telecom corridor
◆ Logistics and facility costs
Negative externality: Market Splitting by Hotelling’s Model

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(a)</td>
<td>(1-a-b)</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>(b)</td>
</tr>
</tbody>
</table>

Suppose customers (preferences) are uniformly distributed over \([0,1]\)

How much does firm at \(a\) get, how about firm at \(b\)?

If \(a\) locates first, where should \(b\) locate?

If \(a\) estimates how \(b\) will locate in response to \(a\)’s location, where should \(a\) locate?
Analytical Models for SC Design

- **Objective functions**
  - Private sector vs. Public sector. Equity?

- **Demand allocation**
  - Distance vs. Price vs. Quality
  - Recall Hotelling

- **Demand pattern over a geography**
  - Discrete vs. Continuous

- **Feasibility check**
  - Ante vs. Post

- **Distances**
  - Euclidean vs. Rectilinear
  - Triangular inequality
Network Optimization Models

◆ Allocating demand to production facilities
◆ Locating facilities and allocating capacity

Key Costs:
• Fixed facility cost
• Transportation cost
• Production cost
• Inventory cost
• Coordination cost

Which plants to establish? How to configure the network?
Demand Allocation Model: Transportation Problem

Which market is served by which plant?
Which supply sources are used by a plant?

Given m demand points, j=1..m
with demands D_j

Given n supply points, i=1..n
with capacity K_i

Send supplies from supply points to demand points
x_{ij} = Quantity shipped from plant site i to customer j

Each unit of shipment from supply point I to demand point j costs c_{ij}

\[ \text{Min} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \]

\[ \text{s.t.} \]

\[ \sum_{i=1}^{n} x_{ij} = D_j \]
\[ \sum_{j=1}^{m} x_{ij} \leq K_i \]
\[ x_{ij} \geq 0 \]

<See Excel File>
Plant Location with Multiple Sourcing

Which market is served by which plant?
Which supply sources are used by a plant?

None of the plants are open, a cost of $f_i$ is paid to open plant $i$.

At most $k$ plants will be opened.

$y_i = 1$ if plant is located at site $i$, $0$ otherwise.

$x_{ij}$ = Quantity shipped from plant site $i$ to customer $j$.

How does cost change as $k$ increases?

\[
\begin{align*}
\text{Min} & \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \\
\text{s.t.} & \\
\sum_{i=1}^{n} x_{ij} &= D_j \\
\sum_{j=1}^{m} x_{ij} &\leq K_i y_i \\
\sum_{i=1}^{n} y_i &\leq k \\
y_i &\in \{0,1\}
\end{align*}
\]
Plant Location with Single Sourcing

Which market is served by which plant? Which supply sources are used by a plant?

None of the plants are open, a cost of $f_i$ is paid to open plant $i$.

At most $k$ plants will be opened.

$y_i = 1$ if plant is located at site $i$, 0 otherwise.

$x_{ij} = 1$ if market $j$ is supplied by factory $i$, 0 otherwise.

How does cost change as $k$ increases?

$$\begin{align*}
\text{Min} & \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m} D_{ij} c_{ij} x_{ij} \\
\text{s.t.} & \sum_{i=1}^{n} x_{ij} = 1 \\
& \sum_{j=1}^{m} D_{ij} x_{ij} \leq K_i y_i \\
y_i \in \{0,1\}
\end{align*}$$
**Gravity Methods for Location**

**Ton Mile-Center Solution**

Given \( n \) delivery locations, \( i=1..n \),

\( x_i, y_i : \) Coordinates of delivery location \( i \)

\( d_i : \) Distance to delivery location \( i \)

\( F_i : \) Annual tonnage to delivery location \( i \)

Locate a warehouse at \((x,y)\)

\[
d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}
\]

\[
\text{Min}_{x,y} \sum_{i=1}^{n} F_i \sqrt{(x_i-x)^2 + (y_i-y)^2}
\]

\[
x = \frac{\sum_{i=1}^{n} x_i F_i}{\sum_{i=1}^{n} d_i}
\]

\[
y = \frac{\sum_{i=1}^{n} y_i F_i}{\sum_{i=1}^{n} d_i}
\]

<Show Excel File>
Gravity Methods for Location

Change the distance

\[ d_i = (x - x_i)^2 + (y - y_i)^2 \]

\[ \text{Min} \sum_{x, y} F_i [(x_i - x)^2 + (y_i - y)^2] \]

Given \( n \) delivery locations, \( i=1..n \),
\( x_i, y_i \): Coordinates of delivery location \( i \)
\( d_i \): Distance to delivery location \( i \)
\( F_i \): Annual tonnage to delivery location \( i \)

Locate a warehouse at \((x,y)\)

\[
\begin{align*}
    x &= \frac{\sum_{i=1}^{n} x_i F_i}{\sum_{i=1}^{n} F_i} \\
    y &= \frac{\sum_{i=1}^{n} y_i F_i}{\sum_{i=1}^{n} F_i}
\end{align*}
\]
## AppliChem Demand Allocation

<table>
<thead>
<tr>
<th>From</th>
<th>Mexico</th>
<th>Canada</th>
<th>Venezuela</th>
<th>Frankfurt</th>
<th>Gary</th>
<th>Sunchem</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>$81</td>
<td>$92</td>
<td>$136</td>
<td>$101</td>
<td>$96</td>
<td>$101</td>
<td>220</td>
</tr>
<tr>
<td>Canada</td>
<td>$147</td>
<td>$78</td>
<td>$135</td>
<td>$98</td>
<td>$88</td>
<td>$97</td>
<td>37</td>
</tr>
<tr>
<td>Venezuela</td>
<td>$172</td>
<td>$106</td>
<td>$96</td>
<td>$120</td>
<td>$111</td>
<td>$117</td>
<td>45</td>
</tr>
<tr>
<td>Frankfurt</td>
<td>$115</td>
<td>$71</td>
<td>$110</td>
<td>$59</td>
<td>$74</td>
<td>$77</td>
<td>470</td>
</tr>
<tr>
<td>Gary, Indiana</td>
<td>$143</td>
<td>$77</td>
<td>$134</td>
<td>$91</td>
<td>$71</td>
<td>$90</td>
<td>185</td>
</tr>
<tr>
<td>Sunchem</td>
<td>$222</td>
<td>$129</td>
<td>$205</td>
<td>$145</td>
<td>$136</td>
<td>$116</td>
<td>50</td>
</tr>
<tr>
<td>Demand</td>
<td>30</td>
<td>26</td>
<td>160</td>
<td>200</td>
<td>264</td>
<td>119</td>
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</tbody>
</table>
Applichem Demand Allocation (1982)

<table>
<thead>
<tr>
<th>Capacity</th>
<th>220</th>
<th>Mexico</th>
<th>30</th>
<th>Mexico</th>
</tr>
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<tbody>
<tr>
<td>37</td>
<td>Canada</td>
<td>32</td>
<td>26</td>
<td>Canada</td>
</tr>
<tr>
<td>45</td>
<td>Venezuela</td>
<td>11</td>
<td>45</td>
<td>Latin America</td>
</tr>
<tr>
<td>470</td>
<td>Frankfurt</td>
<td>115</td>
<td>200</td>
<td>Europe</td>
</tr>
<tr>
<td>185</td>
<td>Gary</td>
<td>185</td>
<td>36</td>
<td>U.S.A</td>
</tr>
<tr>
<td>50</td>
<td>Sunchem</td>
<td>119</td>
<td>Japan</td>
<td></td>
</tr>
</tbody>
</table>

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Applichem Production Network 1982
(with duties)

Annual Cost = $72,916,400
Applichem Production Network 1982 (without duties)

Annual Cost = 66,328,100
1981 Network

Mexico → Mexico
Canada → Canada
Venezuela → Latin America
Frankfurt → Europe
Gary → U.S.A
Sunchem → Japan

Annual Cost = $79,598,500
1981 Network (Sunchem Closed)

Mexico  ->  Mexico
Canada  ->  Canada
Venezuela  ->  Latin America
Frankfurt  ->  Europe
Gary  ->  U.S.A
Sunchem  ->  Japan

Annual Cost = $82,246,800
### Cash Flows From Sunchem Plant

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimal ($ Million)</strong></td>
<td>60.562</td>
<td>68.889</td>
<td>75.999</td>
<td>79.887</td>
<td>79.598</td>
<td>72.916</td>
</tr>
<tr>
<td><strong>Sunchem Closed</strong></td>
<td>60.721</td>
<td>68.889</td>
<td>77.503</td>
<td>80.999</td>
<td>82.247</td>
<td>72.916</td>
</tr>
<tr>
<td><strong>Difference</strong></td>
<td>0.159</td>
<td>0.000</td>
<td>1.504</td>
<td>1.112</td>
<td>2.649</td>
<td>0.000</td>
</tr>
</tbody>
</table>
## Value of Adding 0.1 M Pounds Capacity (1982)

Shadow (dual) prices from LP tells you where to invest.

<table>
<thead>
<tr>
<th>Location</th>
<th>Shadow price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>$0</td>
</tr>
<tr>
<td>Canada</td>
<td>$8,300</td>
</tr>
<tr>
<td>Venezuela</td>
<td>$36,900</td>
</tr>
<tr>
<td>Frankfurt</td>
<td>$22,300</td>
</tr>
<tr>
<td>Gary</td>
<td>$25,200</td>
</tr>
<tr>
<td>Sunchem</td>
<td>$0</td>
</tr>
</tbody>
</table>

Should be evaluated as an option and priced accordingly.
Chapter 6

Network Design in an Uncertain Environment
A tree representation of uncertainty

- One way to represent Uncertainty is binomial tree
- Up by 1 down by -1 move with equal probability

\[
\text{Normal}(0, T\sigma^2)
\]

\[
\sigma^2 = (1)^2 (0.5) + (-1)^2 (0.5) = 1
\]

<Show Applet>
Decision tree

– One column of nodes for each time period
– Each node corresponds to a future state
  » What is in a state?
    ♦ Price, demand, inflation, exchange rate, your OPRE 6366 grade
– Each path corresponds to an evolution of the states into the future
– Transition from one node to another determined by probabilities
– Pieces of optimal paths must be optimal
  » Find shorter and optimal paths starting from period T and work backwards in time to period 0.
Evaluating Facility Investments: AM Tires. Section 6.5 of Chopra.

<table>
<thead>
<tr>
<th>Plant</th>
<th>Dedicated Plant</th>
<th>Flexible Plant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed Cost</td>
<td>Fixed Cost</td>
</tr>
<tr>
<td></td>
<td>Variable Cost</td>
<td>Variable Cost</td>
</tr>
<tr>
<td>US</td>
<td>$1 M/year</td>
<td>$1.1 M/year</td>
</tr>
<tr>
<td>100,000</td>
<td>$15/tire</td>
<td>$15/tire</td>
</tr>
<tr>
<td>Mexico</td>
<td>4 M pesos/year</td>
<td>4.4 M pesos/year</td>
</tr>
<tr>
<td>50,000</td>
<td>110 pesos/tire</td>
<td>110 pesos/tire</td>
</tr>
</tbody>
</table>

U.S. Demand = 100,000; Mexico demand = 50,000. 1US$ = 9 pesos

Demand goes up or down by 20 percent with probability 0.5 and exchange rate goes up or down by 25 per cent with probability 0.5.
AM Tires

Period 0

Period 1

Period 2

RU=100
RM=50
E=9

RU=120
RM=60
E=11.25

RU=120
RM=60
E=6.75

RU=120
RM=40
E=11.25

RU=120
RM=40
E=6.75

RU=80
RM=60
E=11.25

RU=80
RM=60
E=6.75

RU=80
RM=40
E=11.25

RU=80
RM=40
E=6.75

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E=6.75

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E=6.75

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E=6.75

RU=80
RM=40
E=6.75

RU=80
RM=40
E=6.75

RU=80
RM=40
E=6.75

RU=144
RM=72
E=14.06

RU=144
RM=72
E=8.44

RU=144
RM=72
E=14.06

RU=144
RM=72
E=8.44

RU=144
RM=48
E=14.06

RU=144
RM=48
E=8.44

RU=96
RM=72
E=14.06

RU=96
RM=72
E=8.44

RU=96
RM=48
E=14.06

RU=96
RM=48
E=8.44
AM Tires

Four possible capacity scenarios:
- Both dedicated
- Both flexible
- U.S. flexible, Mexico dedicated
- U.S. dedicated, Mexico flexible

For each node solve the demand allocation model.
AM Tires: Demand Allocation for RU = 144; RM = 72, E = 14.06

<table>
<thead>
<tr>
<th>Source</th>
<th>Destination</th>
<th>Variable cost</th>
<th>Shipping cost</th>
<th>E</th>
<th>Sale price</th>
<th>Margin($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>U.S.</td>
<td>$15</td>
<td>0</td>
<td>14.06</td>
<td>$30</td>
<td>$15</td>
</tr>
<tr>
<td>U.S.</td>
<td>Mexico</td>
<td>$15</td>
<td>$1</td>
<td>14.06</td>
<td>240 pesos</td>
<td>$1.1</td>
</tr>
<tr>
<td>Mexico</td>
<td>U.S.</td>
<td>110 pesos</td>
<td>$1</td>
<td>14.06</td>
<td>$30</td>
<td>$21.2</td>
</tr>
<tr>
<td>Mexico</td>
<td>Mexico</td>
<td>110 pesos</td>
<td>0</td>
<td>14.06</td>
<td>240 pesos</td>
<td>$9.2</td>
</tr>
</tbody>
</table>

\[
\text{Max } \sum_{i=1}^{n} \sum_{j=1}^{m} m_{ij}x_{ij} \text{ such that }
\]

\[
\sum_{i=1}^{n} x_{ij} \leq D_j
\]

\[
\sum_{j=1}^{m} x_{ij} \leq K_i
\]

\[
x_{ij} \geq 0
\]

Compare this formulation to the Transportation problem.
AM Tires: Demand Allocation for RU = 144; RM = 72, E = 14.06

The computations in the book go beyond my understanding now. Profit without fixed costs = Objective value of optimization = $1,138,000. The number associated with Node (RU=144, RM=72, E=14.06) must be $1,138,000. Fixed costs should not be deducted now. They are incurred in year 0 so must be deducted in year 0.
Facility Decision at AM Tires

Make profit computations for the first year nodes one by one:
Compute the profit for a node and add to that
$(0.9)(1/8)(\text{Sum of the profits of all 8 nodes connected to the current one})$

<table>
<thead>
<tr>
<th>Plant Configuration</th>
<th>NPV</th>
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<tbody>
<tr>
<td>United States</td>
<td>Mexico</td>
</tr>
<tr>
<td>Dedicated</td>
<td>Dedicated</td>
</tr>
<tr>
<td>Flexible</td>
<td>Dedicated</td>
</tr>
<tr>
<td>Dedicated</td>
<td>Flexible</td>
</tr>
</tbody>
</table>
Capacity Investment Strategies

- Single sourcing
- Hedging Strategy
  - Risk management?
  - Match revenue and cost exposure
- Flexible Strategy
  - Excess total capacity in multiple plants
  - Flexible technologies

- More will be said in aggregate planning chapter
Summary

- Frequency decomposition
- Factors influencing facility decisions
- A strategic framework for facility location
- Gravity methods for location
- Network-LP-IP optimization models
- Value capacity as a real option
Location Allocation Decisions

Which plants to establish? Which warehouses to establish? How to configure the network?
p-Median Model

Inputs:
A set of feasible plant locations, indexed by j
A set of markets, indexed by i

D_i demand of market i
No capacity limitations for plants
At most p plants are to be opened

d_ij distance between market i and plant j

y_j = 1 if plant is located at site j,
    0 otherwise
x_{ij} = 1 if market i is supplied from plant site j,
    0 otherwise

\[
\begin{align*}
\text{Min} & \sum_i D_i \sum_j d_{ij} x_{ij} \\
\text{s.t.} & \\
\sum_j y_j &= p \\
x_{i,j} &\leq y_j \quad \text{for all } i, j \\
\sum_j x_{ij} &\leq 1 \quad \text{for all } i \\
x_{ij}, y_j &\in \{0,1\} \quad \text{for all } i, j
\end{align*}
\]
p-Center Model

Replace the objective function in p-Median problem with Min Max \{d_{ij}x_{ij} : i \text{ is a market assigned to plant } j\}

We are minimizing maximum distance between a market and a plant
Or say minimizing maximum distance between fire stations and all the houses served by those fire stations. An example with p=3 stations and 9 houses:
p-Covering Model

\( x_i = 1 \) if demand point \( i \) is covered, 0 otherwise
\( y_j = 1 \) if facility \( j \) is opened, 0 otherwise
\( N_i \) facilities associated with demand point \( i \)
If \( j \) is in \( N_i \), \( j \) can serve \( i \)

Can you read constraint (*) in English?

\[
\begin{align*}
\text{Max} \sum_i D_i x_i \\
\text{s.t.} \\
\sum_{j \in N_i} y_j \geq x_i \quad \text{for all } i \quad (*) \\
\sum_j y_j = p \\
x_i, y_j \in \{0,1\} \quad \text{for all } i, j
\end{align*}
\]
Other Models

◆ p-Choice Models
  – Criteria to choose the server: distance, price?
◆ Models with multiple decision makers
  – Franchise model