Optimal Level of Product Availability

Chapter 12 of Chopra

Outline

- Determining optimal level of product availability
  - Single order in a season
  - Continuously stocked items
- Ordering under capacity constraints
- Levers to improve supply chain profitability

Mattel, Inc. & Toys “R” Us

Mattel [who introduced Barbie in 1959 and run a stock out for several years then on] was hurt last year by inventory cutbacks at Toys “R” Us, and officials are also eager to avoid a repeat of the 1998 Thanksgiving weekend. Mattel had expected to ship a lot of merchandise after the weekend, but retailers, wary of excess inventory, stopped ordering from Mattel. That led the company to report a $500 million sales shortfall in the last weeks of the year ... For the crucial holiday selling season this year, Mattel said it will require retailers to place their full orders before Thanksgiving. And, for the first time, the company will no longer take reorders in December, Ms. Barad said. This will enable Mattel to tailor production more closely to demand and avoid building inventory for orders that don't come.

- Wall Street Journal, Feb. 18, 1999

Key Questions

- How much should Toys R Us order given demand uncertainty?
- How much should Mattel order?
- Will Mattel's action help or hurt profitability?
- What actions can improve supply chain profitability?

How much to order? Parkas at L.L. Bean

Demand D_i	Probabability p_i	Cumulative Probability of demand being this size or less, F()	Probability of demand greater than this size, 1-F()
4	.01	.01	.99
5	.02	.03	.97
6	.04	.07	.93
7	.08	.15	.85
8	.09	.24	.76
9	.11	.35	.65
10	.16	.51	.49
11	.20	.71	.29
12	.11	.82	.18
13	.10	.92	.08
14	.04	.96	.04
15	.02	.98	.02
16	.01	.99	.01
17	.01	1.00	.00

Parkas at L.L. Bean

Cost per parka = c = $45

Sale price per parka = p = $100

Discount price per parka = $50

Holding and transportation cost = $10

Salvage value per parka = s = $40

Profit from selling parka = p-c = 100-45 = $55

Cost of overstocking = c-s = 45-40 = $5

Parkas at L.L. Bean

- Expected demand = 10 (‘00) parkas
- Expected profit from ordering 10 (‘00) parkas = $499

- Approximate Expected profit from ordering 1(‘00) extra parkas if 10(’00) are already ordered

$$= 100.\mathbf{55.P(D>=1100)} - 100.\mathbf{5.P(D<1100)}$$

Parkas at L.L. Bean

Additional 100s	Expected Marginal Benefit	Expected Marginal Cost	Expected Marginal Contribution
11th	5500×.49 = 2695	500×.51 = 255	2695-255 = 2440
12th	5500×.29 = 1595	500×.71 = 355	1595-355 = 1240
13th	5500×.18 = 990	500×.82 = 410	990-410 = 580
14th	5500×.08 = 440	500×.92 = 460	440-460 = -20
15th	5500×.04 = 220	500×.96 = 480	220-480 = -260
16th	5500×.02 = 110	500×.98 = 490	110-490 = -380
17th	5500×.01 = 55	500×.99 = 495	55-495 = -440

Optimal level of product availability

p = sale price; *s* = outlet or salvage price; *c* = purchase price

***CSL* = Probability that demand will be at or below reorder point**

At optimal order size,

Expected Marginal Benefit from raising order size =

=P(Demand is above stock)*(Profit from sales)=$(1-CSL^*)(p - c)$

Expected Marginal Cost =

=P(Demand is below stock)*(Loss from discounting)=$CSL^*(c - s)$.

$C_o = c-s$; C_u=p-c

$$(1-CSL^*)C_u = CSL^* \times C_o,$$

$$CSL^* = C_u / (C_u + C_o)$$

Order Quantity for a Single Order

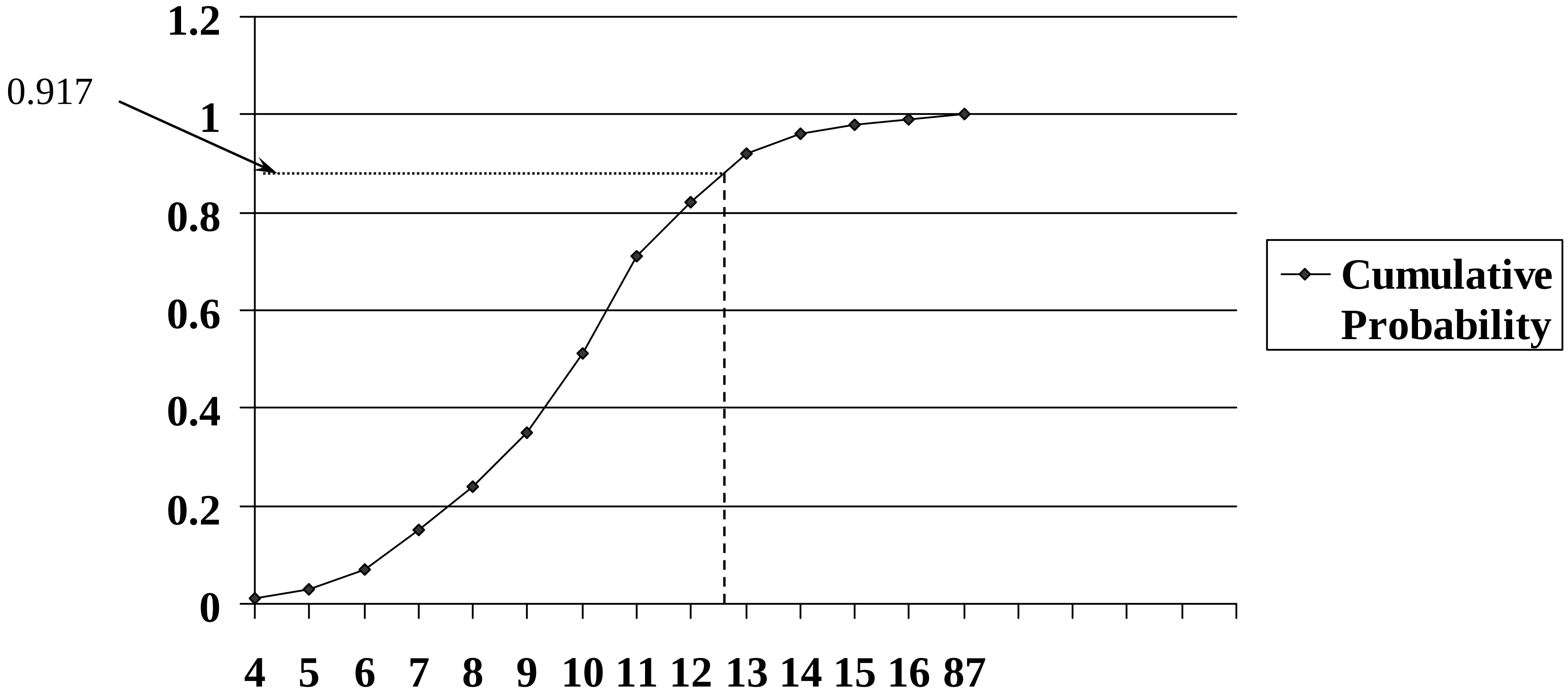
C_o = Cost of overstocking = $5

C_u = Cost of understocking = $55

Q^* = Optimal order size

$$CSL = P(Demand \le Q^*) \ge \frac{C_u}{C_u + C_o} = \frac{55}{55+5} = 0.917$$

Optimal Order Quantity



Optimal Order Quantity = 13('00)

Revisit L.L. Bean as a Newsvendor Problem

- Total cost by ordering Q units:
  - C(Q) = overstocking cost + understocking cost

$$C(Q) = C_o \int_0^Q (Q - x) f(x) dx + C_u \int_Q^\infty (x - Q) f(x) dx$$

$$\frac{dC(Q)}{dQ} = C_o F(Q) - C_u (1 - F(Q)) = 0$$

Marginal cost of raising Q* - Marginal cost of decreasing Q* = 0

$$F(Q^*) = \frac{C_u}{C_o + C_u}$$

Ordering Women's Designer Boots Under Capacity Constraints

	Autumn	Leaves	Ruffle
Retail price	$150	$200	$250
Purchase price	$75	$90	$110
Salvage price	$40	$50	$90
Mean Demand	1000	500	250
Stand. deviation of demand	250	175	125

Available Store Capacity = 1,500.

Assuming No Capacity Constraints

	Autumn	Leaves	Ruffle
p_i-c_i	150-75=$75	200-90=$110	250-110=$140
c_i-s_i	75-40=$35	90 - 50 = $40	110-90 = $20
Critical Fractile	75/110 = 0.68	110/150= 0.73	140/160=0.875
z_i	0.47	0.61	1.15
Q_i	1118	607	394

Storage capacity is not sufficient to keep all models!

Algorithm for Ordering Under Capacity Constraints

{Initialization}
ForAll products, Q_i **:= 0**. **Remaining_capacity:=Total_capacity**.

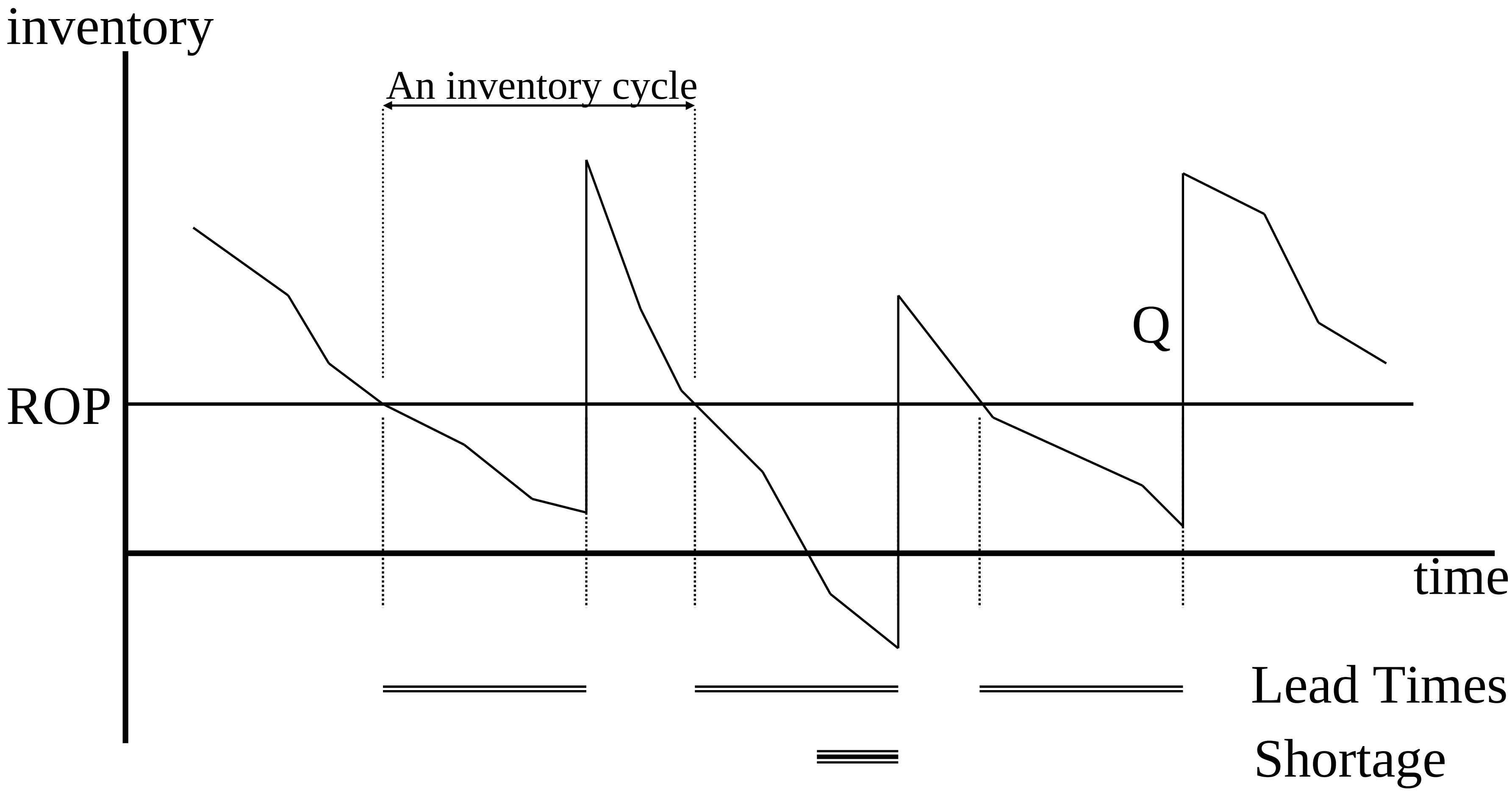
{Iterative step}
While **Remaining_capacity** > 0 do

ForAll products,
Compute the **marginal contribution** of increasing Q_i by 1
If all **marginal contributions** <=0, STOP
{Order sizes are already sufficiently large for all products}
else **Find** the product with the largest **marginal contribution**, call it **j**
{Priority given to the most profitable product}
$Q_j := Q_j + 1$ and **Remaining_capacity=Remaining_capacity-1**
{Order more of the most profitable product}

Marginal Contribution=(p-c)P(D>Q)-(c-s)P(D<Q)

	Order Quantity			Marginal Contribution		
Remaining_Capacity	Autumn	Leaves	Ruffle	Autumn	Leaves	Ruffle
1500	0	0	0	74.997	109.679	136.360
1490	0	0	10	74.997	109.679	135.611
1360	0	0	140	74.997	109.679	109.691
1350	0	0	150	74.997	109.679	106.103
1340	0	10	150	74.997	109.617	106.103
1330	0	20	150	74.997	109.543	106.103
1320	0	30	150	74.997	109.457	106.103
1310	0	40	150	74.997	109.357	106.103
890	0	380	230	74.997	73.033	70.170
880	10	380	230	74.996	73.033	70.170
870	20	380	230	74.995	73.033	70.170
290	580	400	230	69.887	67.422	70.170
280	580	400	240	69.887	67.422	65.101
1	788	446	265	53.196	53.176	52.359
0	789	446	265	53.073	53.176	52.359

Optimal Safety Inventory and Order Levels: (ROP,Q) ordering model



A Cost minimization approach as opposed to the last chapter's service based approach

- Fixed ordering cost = S R / Q
- Holding cost = h C (Q/2+ss)

  where ss = ROP – L R

- Backordering cost (based on per unit backordered)

$$\frac{R}{Q} C_u \int_{ROP}^{\infty} (x - ROP)\, f(x) dx$$

- Total cost=C(Q,r)=

$$S\frac{R}{Q} + hC\{\frac{Q}{2} + ROP - LR\} + \frac{R}{Q}\int_{ROP}^{\infty} C_u (x - ROP)\, f(x) dx$$

Optimal Q (for high service level) and ROP

- Q*=Optimal lot size
- ROP*=Optimal reorder point

$$Q^* = \sqrt{\frac{2SR}{hC}} \qquad CSL^* = F(ROP^*) = 1 - \frac{hCQ}{RC_u}$$

- A cost / benefit analysis:
- $(1\text{-}CSL)C_u$= per cycle benefit of increasing ROP by 1
- HQ^*/R= per cycle cost of increasing ROP by 1
  - » Q^*/R is the duration of 1 inventory cycle
- $(1\text{-}CSL^*)C_u = HQ^*/R$

Optimal Safety Inventory Levels

R = 100 gallons/week; σ_R= 20; H = \$0.6/gal./year

L = 2 weeks; Q = 400; ROP = 300.

What is the imputed cost of backordering?

$$\boldsymbol{CSL = 1\text{-}HQ^{*}/C_{u}R}$$

$$CSL = F(ROP, RL, \sqrt{L}\sigma_R) = 0.9998$$

$$C_u = \frac{HQ}{(1-CSL)R} = \$230.8 \text{ per gallon per week}$$

Levers for Increasing Supply Chain Profitability

- Increase salvage value or
  - Obermeyer sells winter clothing in south America during the summer.
- Decrease the margin lost from a stock out
  - Car part suppliers, McMaster-Carr and Grainger, are competitors but they buy from each other to satisfy the customer demand during a stock out.
- Improve forecasting to lower uncertainty
- Quick response by decreasing replenishment lead time which leads to a larger number of orders per season
- Postponement of product differentiation
- Tailored (dual) sourcing

Impact of Improving Forecasts

Demand: Normally distributed with a mean of R = 350 and standard deviation of σ_R = 100

Purchase price = \$100 , Retail price = \$250

Disposal value = \$85 , Holding cost for season = \$5

How many units should be ordered as σ_R changes?

Understocking cost=\$150, Overstocking cost=\$20

Impact of Improving Forecasts

σ_R	***Q****	***Expected Overstock***	***Expected Understock***	***Expected Profit***
150	526	186.7	8.6	$47,469
120	491	149.3	6.9	$48,476
90	456	112.0	5.2	$49,482
60	420	74.7	3.5	$50,488
30	385	37.3	1.7	$51,494
0	350	0	0	$52,500

Quick Response: Multiple Orders per Season

- **Ordering shawls at a department store**
  - **Selling season = 14 weeks**
  - **Cost per shawl = $40**
  - **Sale price = $150**
  - **Disposal price = $30**
  - **Holding cost = $2 per week**
- **Expected weekly demand = 20**
- **StDev of weekly demand = 15**

Ordering Twice as Opposed to Once

- The second order can be used to correct the demand supply mismatch in the first order
  - At the time of placing the second order, take out the on-hand inventory from the demand the second order is supposed to satisfy. This is a simple correction idea.
- Between the time first and second orders are placed, more information becomes available to demand forecasters. The second order is typically made against less uncertainty than the first order is.

Impact of Quick Response
Correcting the mismatch with second order

Single Order				*Two Orders in Season*				
Service Level	Order Size	Ending Invent.	Expect. Profit	Initial Order	**OUL** for 2nd Order	Ending Invent.	Average Total Order	Expect. Profit
0.96	378	97	$23,624	209	209	69	349	$26,590
0.94	367	86	$24,034	201	201	60	342	$27,085
0.91	355	73	$24,617	193	193	52	332	$27,154
0.87	343	66	$24,386	184	184	43	319	$26,944
0.81	329	55	$24,609	174	174	36	313	$27,413
0.75	317	41	$25,205	166	166	32	302	$26,916

OUL: Ideal **O**rder **U**p to **L**evel of inventory at the beginning of a cycle

Forecasts Improve for the Second Order Uncertainty reduction from SD=15 to 3

Single Order				***Two Orders in Season***				
Service Level	Order Size	Ending Invent.	Expect. Profit	Initial Order	OUL for 2^{nd} Order	Average Total Order	Ending Invent.	Expect. Profit
0.96	378	96	$23,707	209	153	292	19	$27,007
0.94	367	84	$24,303	201	152	293	18	$27,371
0.91	355	76	$24,154	193	150	288	17	$26,946
0.87	343	63	$24,807	184	148	288	14	$27,583
0.81	329	52	$24,998	174	146	283	14	$27,162
0.75	317	44	$24,887	166	145	282	14	$27,268

With two orders retailer buys less, supplier sells less.

Why should supplier reduce its replenishment lead time?

Postponement is a cheaper way of providing product variety

- Dell delivers customized PC in a few days
- Electronic products are customized according to their distribution channels
- Toyota is promising to build cars to customer specifications and deliver them in a few days
- Increased product variety makes forecasts for individual products inaccurate
  - Lee and Billington (1994) reports 400% forecast errors for high technology products
  - Demand supply mismatch is a problem
    - » Huge end of the season inventory write-offs. Johnson and Anderson (2000) estimates the cost of inventory holding in PC business 50% per year.
- Not providing product flexibility leads to market loss.
  - An American tool manufacturer failed to provide product variety and lost market share to a Japanese competitor. Details in McCutcheon et. al. (1994).
- Postponement: Delaying the commitment of the work-in-process inventory to a particular product. A.k.a. end of line configuration, late point differentiation, delayed product differentiation.

Postponement

- Postponement is delaying customization step as much as possible
- Need:
  - Indistinguishable products before customization
  - Customization step is high value added
  - Unpredictable demand
  - Flexible SC to allow for any choice of customization step
  - Negatively correlated products

Forms of Postponement by Zinn and Bowersox (1988)

- Labeling postponement: Standard product is labeled differently based on the realized demand.
  - HP printer division places labels in appropriate language on to printers after the demand is observed.
- Packaging postponement: Packaging performed at the distribution center.
  - In electronics manufacturing, semi-finished goods are transported from SE Asia to North America and Europe where they are localized according to local language and power supply
- Assembly and manufacturing postponement: Assembly or manufacturing is done after observing the demand.
  - McDonalds assembles meal menus after customer order.

Examples of Postponement

- HP DeskJet Printers
  - Printers localized with power supply module, power cord terminators, manuals
- IBM RS/6000 Assembly
  - 50-75 end products differentiated by 10 features or components. Assembly used to start from scratch after customer order. Takes too long.
  - Instead IBM stocks semi finished RS/6000 called vanilla boxes. Vanilla boxes are customized according to customer specification.
- Xilinx Integrated Circuits
  - Semi-finished products, called dies, are held in the inventory. For easily/fast customizable products, customization starts from dies and no finished goods inventory is held. For more complicated products finished goods inventory is held and is supplied from the dies inventory.
  - New programmable logic devices which can be customized by the customer using a specific software.
- Motorola cell phones
  - DC has cell phones, phone service provider logos and service provider literature. The product is customized for different service providers after demand is materialized.

Postponement

- Saves Inventory holding cost by reducing safety stock
  - Inventory pooling
  - Resolution of uncertainty
- Saves Obsolescence cost
- Increases Sales
- Stretches the Supply Chain
  - Suppliers
  - Production facilities, redesigns for component commonality
  - Warehouses

Value of Postponement: Benetton case

- **For each color, 20 weeks in advance forecasts**
  - **Mean demand= 1,000; Standard Deviation= 500**
- **For each garment**
  - **Sale price = $50**
  - **Salvage value = $10**
  - **Production cost using option 1 (long lead time) = $20**
    - » **Dye the thread and then knit the garment**
  - **Production cost using option 2 (short lead time) = $22**
    - » **Knit the garment and then dye the garment**
- What is the value of postponement?
  - Expected profit increases from $94,576 to $98,092

Postponement Downside

- By postponing all three garment types, production cost of each product goes up
- When this increase is substantial or a single product's demand dominates all other's (causing limited uncertainty reduction via aggregation), a partial postponement scheme is preferable to full postponement.

Partial Postponement: Dominating Demand

- **Color with dominant demand: Mean = 3,100, SD = 800**
- **Other three colors: Mean = 300, SD = 200**

- Expected profit without postponement = $102,205
- Expected profit with postponement = $99,872

- Are these cases comparable?
  - Total expected demand is the same=4000
  - Total variance originally = 4*250,000=1,000,000
  - Total variance now=800*800+3(200*200)=640,000+120,00=760,000
- Dominating demand yields less profit even with less total variance.

Partial Postponement: Benetton case

- For each product a part of the demand is aggregated, the rest is not
- Produce Q_1 units for each color using Option 1 and Q_A units (aggregate) using Option 2, results from simulation:

Q_1 for each	Q_A	Profit
1337	0	$94,576
0	4524	$98,092
1100	550	$99,180
1000	850	$100,312
800	1550	$104,603

Tailored (Dual) Sourcing

- Tailored sourcing does not mean buying from two arbitrary sources. These two sources must be complementary:
  - Primary source: Low cost, long lead time supplier
    - » Cost = $245, Lead time = 9 weeks
  - Complementary source: High cost, short lead time supplier
    - » Cost = $250, Lead time = 1 week

- An example CWP (Crafted With Pride) of apparel industry bringing out competitive advantages of buying from domestic suppliers vs international suppliers.
- Another example is Benetton's practice of using international suppliers as primary and domestic (Italian) suppliers as complementary sources.

Tailored Sourcing: Multiple Sourcing Sites

Characteristic	*Complementary Site*	*Primary Site*
Manufacturing Cost	High	Low
Flexibility (Volume/Mix)	High	Low
Responsiveness	High	Low
Engineering Support	High	Low

Dual Sourcing Strategies from the Semiconductor Industry

Strategy	*Complementary Site*	*Primary Site*
Volume based dual sourcing	Fluctuation	Stable demand
Product based dual sourcing	Unpredictable products, Small batch	Predictable, large batch products
Model based dual sourcing	Newer products	Older stable products

Tailored Sourcing Strategies for Benetton

Fraction of demand from primary supplier	Annual Profit
0%	$37,250
50%	$51,613
60%	$53,027
100%	$48,875

Learning Objectives

- **Optimal order quantities are obtained by trading off cost of lost sales and cost of excess stock**
- **Levers for improving profitability**
  - **Increase salvage value and decrease cost of stockout**
  - **Improved forecasting**
  - **Quick response with multiple orders**
  - **Postponement**
  - **Tailored sourcing**