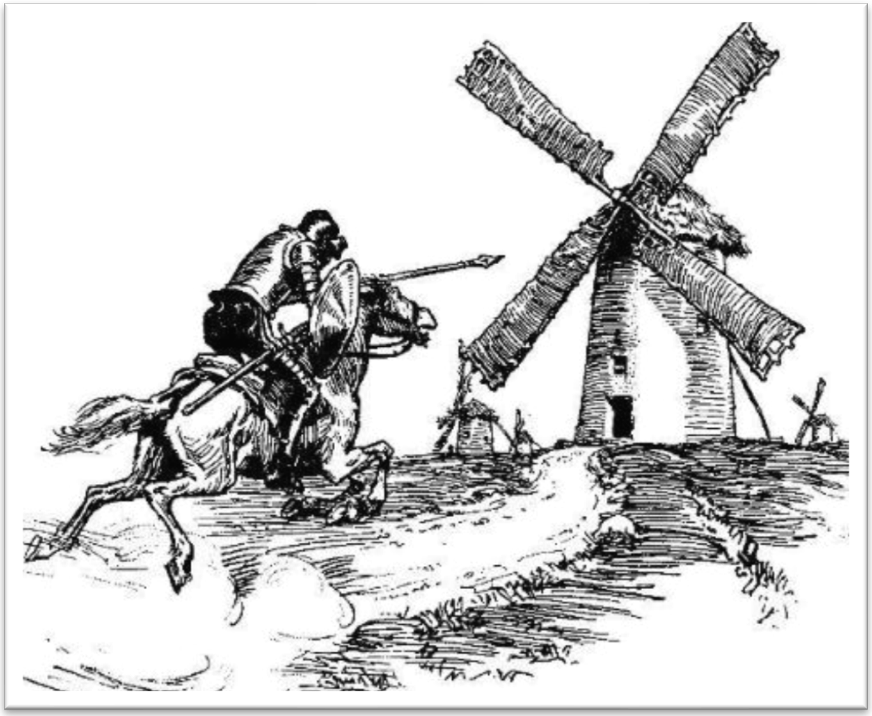
Wind

- Wind Turbine & Farms
- Physics: Generators
- Business
    - Power, Energy & Revenue
    - Cost & Profit
- Non-monetary issues

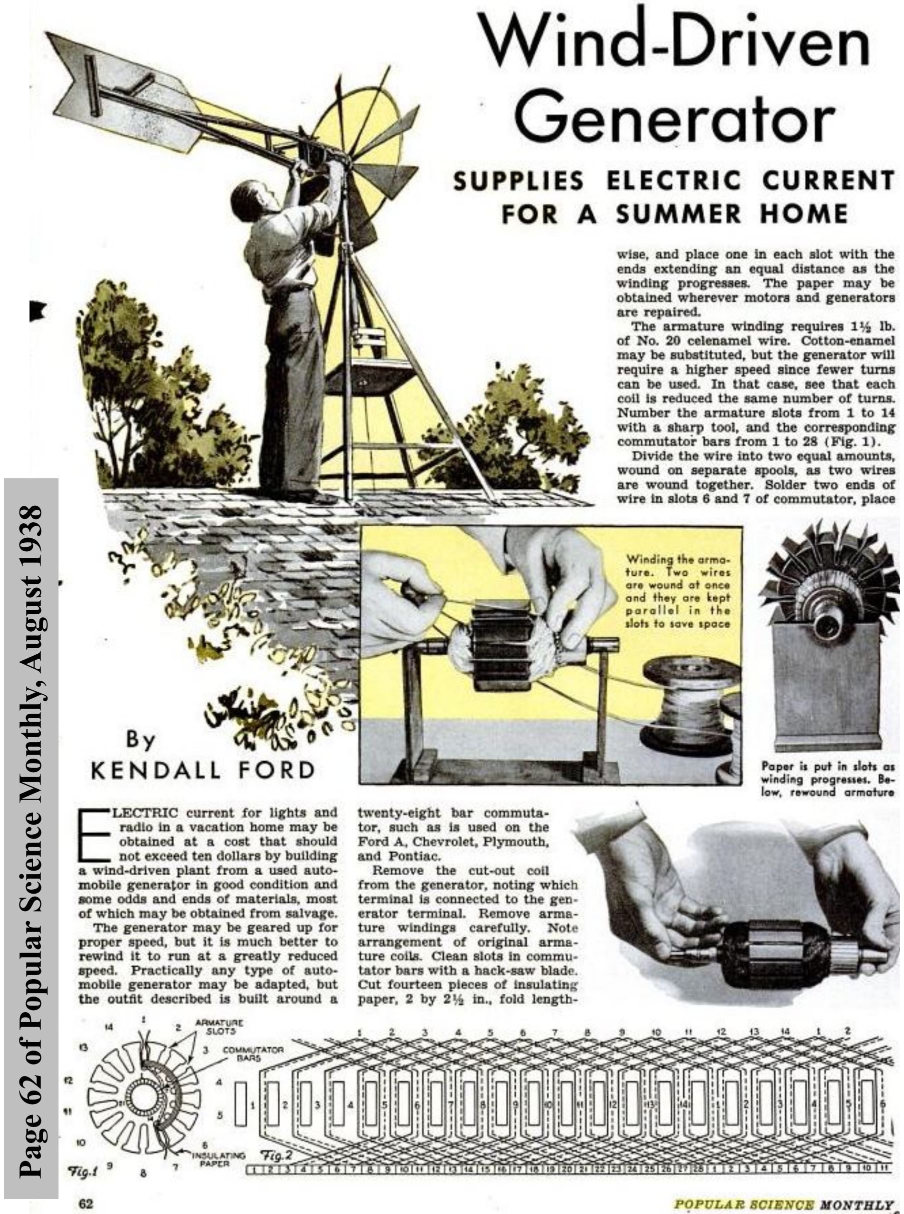
Updated in Spring 2019

Wind Power in Art & History



Tilting at windmills: Attacking invisible enemies.
Coined after 1605 novel Don Quixote by Cervantes.
Wind is invisible yet powerful.

Page 62 of Popular Science Monthly, August 1938



Wind-Driven Generator

SUPPLIES ELECTRIC CURRENT FOR A SUMMER HOME

By
KENDALL FORD

ELECTRIC current for lights and radio in a vacation home may be obtained at a cost that should not exceed ten dollars by building a wind-driven plant from a used automobile generator in good condition and some odds and ends of materials, most of which may be obtained from salvage.

The generator may be geared up for proper speed, but it is much better to rewind it to run at a greatly reduced speed. Practically any type of automobile generator may be adapted, but the outfit described is built around a twenty-eight bar commutator, such as is used on the Ford A, Chevrolet, Plymouth, and Pontiac.

Remove the cut-out coil from the generator, noting which terminal is connected to the generator terminal. Remove armature windings carefully. Note arrangement of original armature coils. Clean slots in commutator bars with a hack-saw blade. Cut fourteen pieces of insulating paper, 2 by 2½ in., fold lengthwise, and place one in each slot with the ends extending an equal distance as the winding progresses. The paper may be obtained wherever motors and generators are repaired.

The armature winding requires 1½ lb. of No. 20 celenamel wire. Cotton-enamel may be substituted, but the generator will require a higher speed since fewer turns can be used. In that case, see that each coil is reduced the same number of turns. Number the armature slots from 1 to 14 with a sharp tool, and the corresponding commutator bars from 1 to 28 (Fig. 1).

Divide the wire into two equal amounts, wound on separate spools, as two wires are wound together. Solder two ends of wire in slots 6 and 7 of commutator, place

Winding the armature. Two wires are wound at once and they are kept parallel in the slots to save space

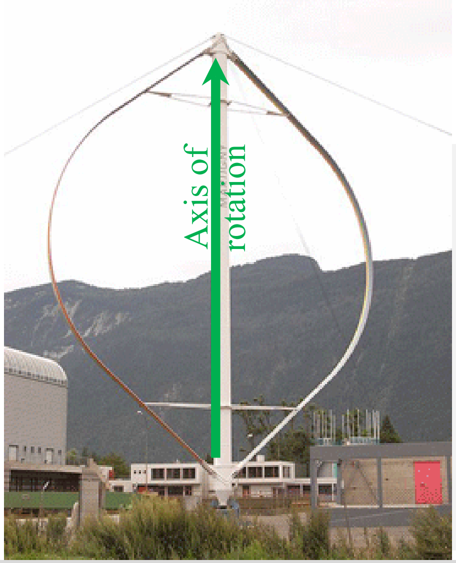
Paper is put in slots as winding progresses. Below, rewound armature

62 POPULAR SCIENCE MONTHLY

Wind Turbines

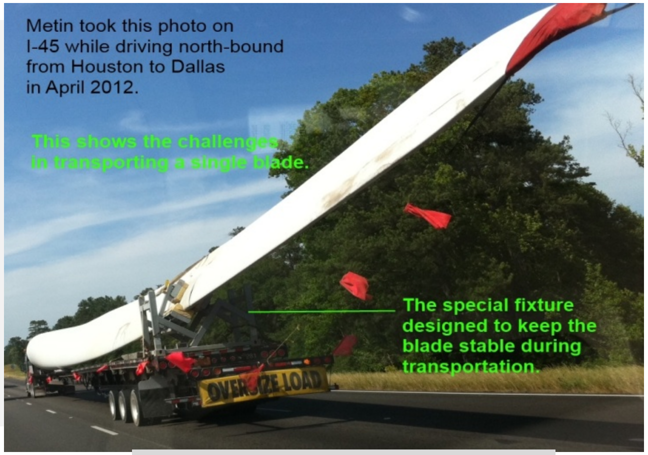
- Wind occurs from differences in air temperature and pressure - consequences of uneven solar heating of the earth.
- Wind has the kinetic energy, harvested with a wind turbine, the opposite of a fan.
- The kinetic is energy proportional to (∝)
  - The square of wind speed
  - The mass of wind through the blades ∝ with the wind speed.
- Wind power depends on the cube of the wind speed.
  - If wind speed ↑ by 2, the power ↑ by 8 in some range.
- Watch: Youtube video “What’s inside a wind turbine?”

Horizontal rotational axis (traditional) wind turbine.
70-100 metre tall; 80-100 metre of blade diameter.



Vertical rotational axis wind turbine.
30 meters tall; 15 meters wide.
These can operate at faster wind speeds.

- According to EIA, the largest wind farm is Horse Hollow Wind Energy Center in Taylor & Nolan counties of TX.
- Offshore wind farms can be built in North America & Europe.
  - Europe has 150,000 square kilometers of water that is shallower than 35 meters.
  - Texas shores are also feasible locations for wind farm development.



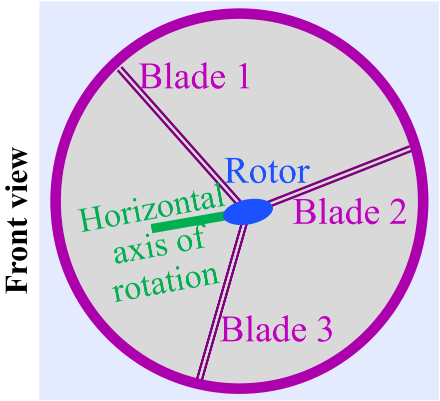
Metin took this photo on I-45 while driving north-bound from Houston to Dallas in April 2012.

This shows the challenges in transporting a single blade.

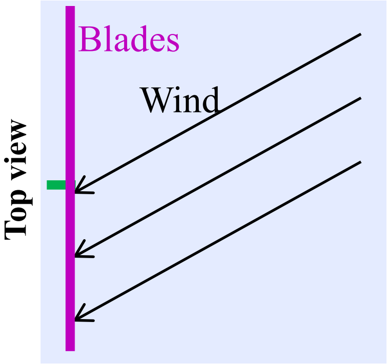
The special fixture designed to keep the blade stable during transportation.

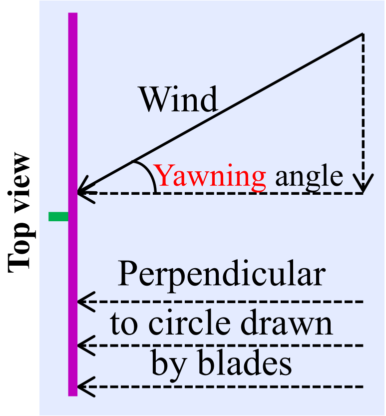
Blades are composites or carbon fiber enforced plastics

Yawning Angle & Turbine Components



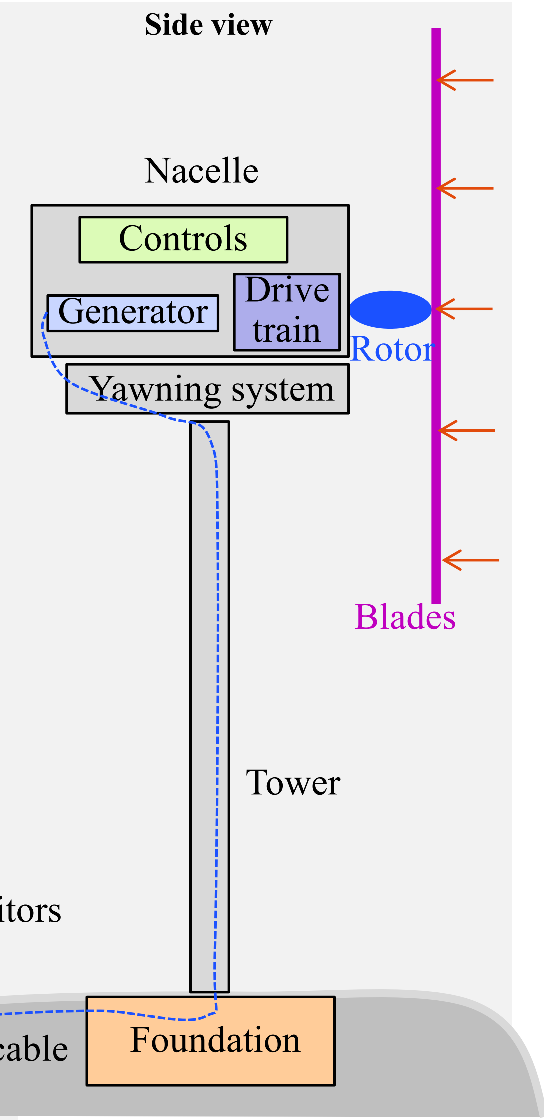
- Wind velocity perpendicular to blades
  = Velocity of wind times * Cosine (yawning angle)
- To max power, max perpendicular wind
  = Max cosine of the yawning angle,
- Turn the rotor for zero yawning angle.

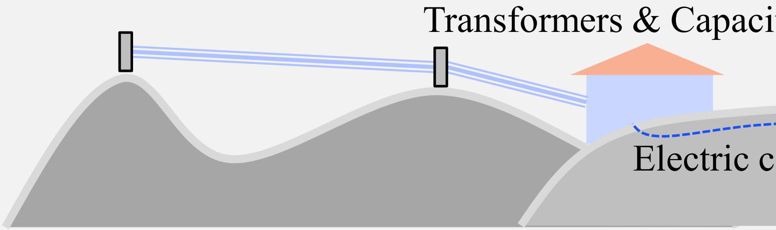




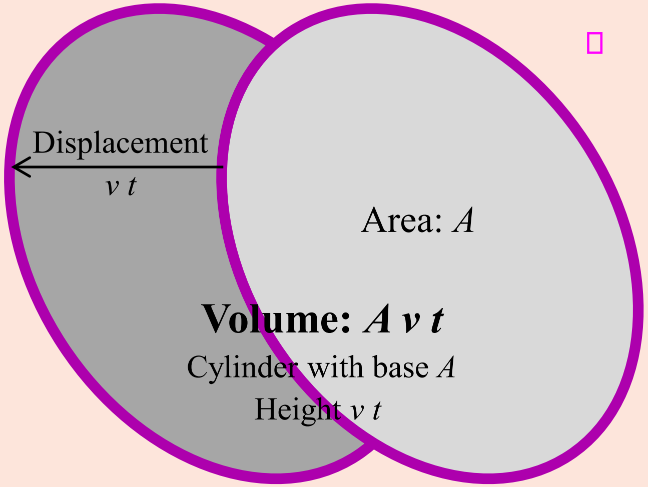
WLOG, wind is perpendicular to the blades

- Foundation: Flat and strong
- Tower: Steel or concrete, 3∗blade radius
- Yawning System: Rotates the nacelle to catch the wind, has vertical rotational axis and is connected to wind direction sensors on the nacelle
- Nacelle
  - Drive Train: Delivers the power to generator but adjusts the frequency of rotation via gears
  - Controls: Sensors, Electromechanical systems, Circuits, Switches, Motors, Computers
  - Generator: Transformation kinetic → electric energy
- Balancing & Stabilizing the electric output via transformers, inverters & capacitors



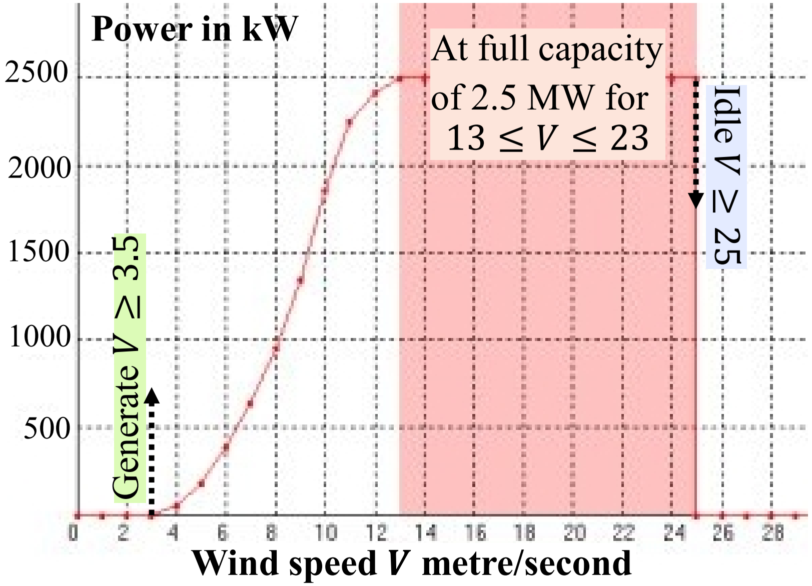


Power in Theory and in Practice



- Wind speed v; Area swept by blades A.
  - Volume of air displaced by the wind in t time:
    $A\ v\ t$
  - Mass of the air displaced by the wind in t time:
    (Density of air) $A\ v\ t$
  - Kinetic energy of the air displaced by the wind in t time:
    (Density of air) $A\ t\ v\ v^2 / 2$
  - Ideally wind power input
    (Density of air) $A\ v^3 / 2$

- For a specific General Electric turbine on the right, with cut-in speed 3.5 m/s & cut-out speed 25 m/s.
  - Over 3.5-10 m/s range, power (in kW) increases with the wind speed as shown on the right. This increase seems to be faster than a cubic.
  - Over 10-13 m/s range, power increases but slowly in the wind speed. This is to protect the blades and the tower.
  - Winds faster than 13 m/s does not increase power which is maxed out at 2500 kW.
- 3600 kW GE wind turbines are operational in Spain. These have blade radius of 52 meters.



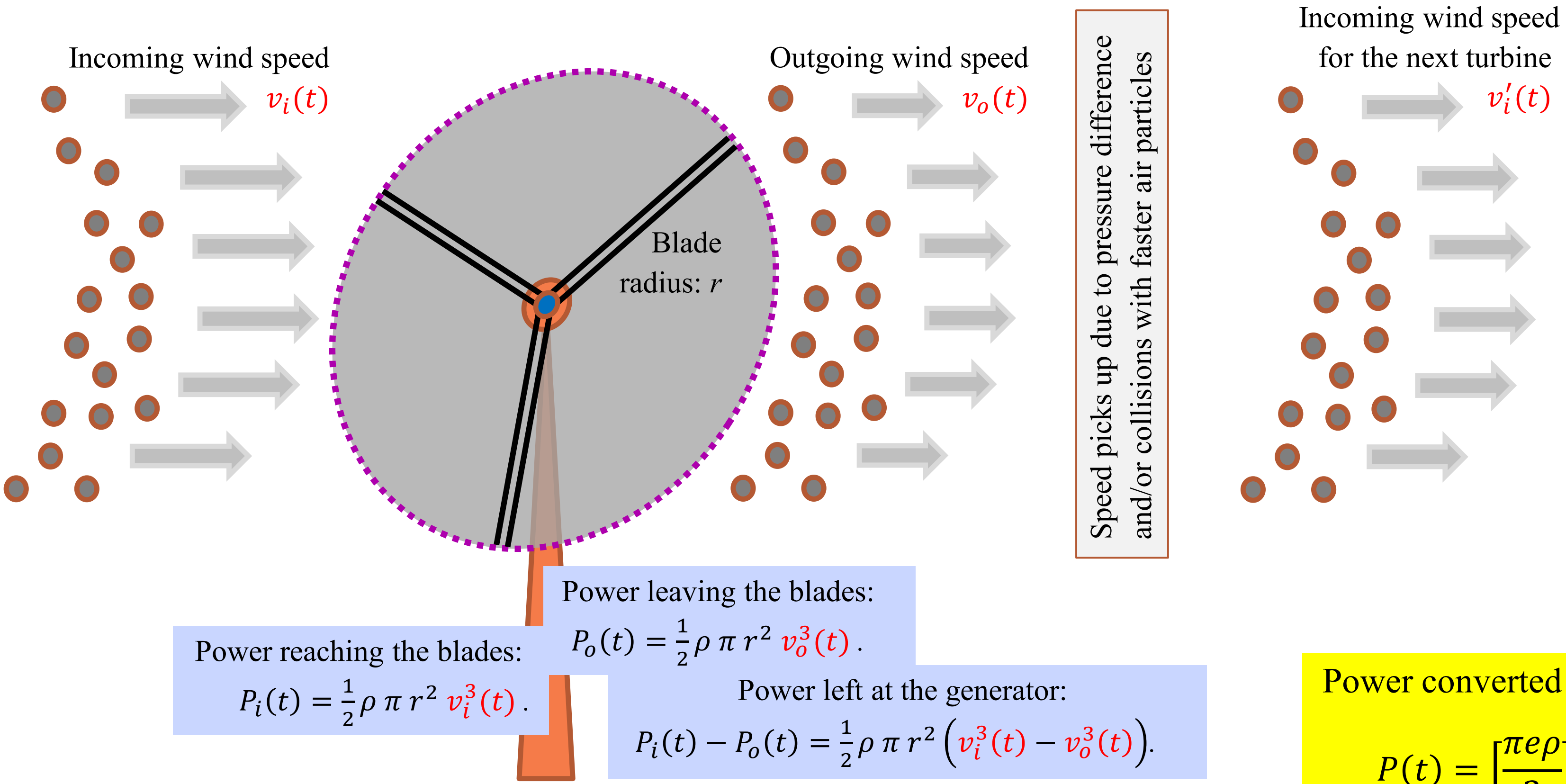
Wind Power in Detail

- Constant: $\pi = 3.14$
- Location dependent parameter ρ: air density in kilogram/cubicmetre
- Blade dependent parameters
  - r: blade radius in metre
  - e: kinetic energy → electricity conversion efficiency, ≈ 40%
- Variable $v_i(t)$: speed of incoming wind in metre/second

Betz upper limit for capturing kinetic energy in a turbine: $\frac{16}{27}$ with $\frac{v_o}{v_i} = \frac{1}{3}$, next page

Wind turbines convert $\frac{3}{4}$ to $\frac{4}{5}$ of the limit to electrical energy

$44.4\% = \frac{3}{4}\frac{16}{27}$ to $47.4\% = \frac{4}{5}\frac{16}{27}$



Speed picks up due to pressure difference and/or collisions with faster air particles

Power reaching the blades:

$$P_i(t) = \frac{1}{2}\rho\,\pi\,r^2\,v_i^3(t)\,.$$

Power leaving the blades:

$$P_o(t) = \frac{1}{2}\rho\,\pi\,r^2\,v_o^3(t)\,.$$

Power left at the generator:

$$P_i(t) - P_o(t) = \frac{1}{2}\rho\,\pi\,r^2\left(v_i^3(t) - v_o^3(t)\right).$$

Power converted to electricity :

$$P(t) = \left[\frac{\pi e \rho}{2}\right] r^2\,v_i^3(t)$$

Betz Limit for Capturing Wind Power

- Assume stationarity in time, drop the time index t.
- Recall the power reaching and leaving the blades that collide with a mass of Δm particles

Power reaching the blades: $P_i = \frac{1}{2}\rho A\, v_i\, v_i^2$ $-$ Power leaving the blades: $P_o = \frac{1}{2}\rho A\, v_o v_o^2$ $=$ Power left at the generator: $P_i - P_o = \frac{1}{2}\Delta m\left(v_i^2 - v_o^2\right) = \frac{1}{2}\rho A\left(\frac{v_i + v_o}{2}\right)\left(v_i^2 - v_o^2\right)$

- $\Delta m = \text{mass per time} = \text{density} * \text{area} * \text{velocity} = \rho A\left(\frac{v_i + v_o}{2}\right)$
- Consider the wind speed ratio v_o/v_i
  - Due to air particles losing momentum $0 \le \frac{v_o}{v_i} \le 1$. For example $\frac{v_o}{v_i} = 0.4$ means that the air particles lose 60% of their speed
  - Let $x = v_o/v_i$
- Using new variable x, consider the ratio of power left to incoming power

$$\frac{\text{Power left at the generator: } P_i - P_o = \frac{1}{2}\frac{1}{2}\rho A(v_i + v_o)\left(v_i^2 - v_o^2\right)}{\text{Power reaching the blades: } P_i = \frac{1}{2}\rho A\, v_i v_i^2} = \frac{1}{2}(1 + x)(1 - x^2)$$

Albert Betz, 1885-1968
Originally for water flow

- Maximum efficiency possible is

$$\max_{0 \le x \le 1} \frac{1}{2}(1 + x)(1 - x^2)$$

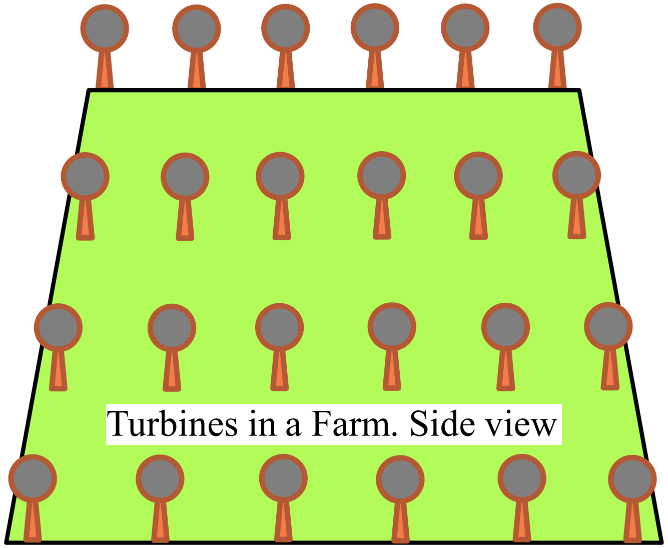
- To find the maximum of this concave function, set the derivative equal to zero

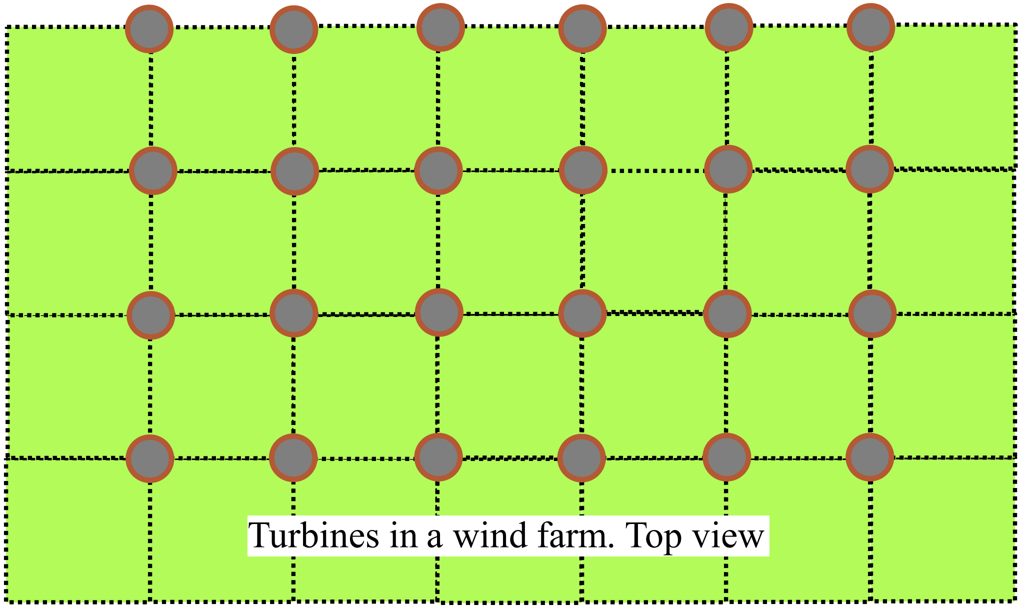
$$0 = \frac{d}{dx}\frac{1}{2}(1 - 2x - 3x^2) = -\frac{1}{2}(x - 1)(3x - 1)$$

- Then the maximum captured is at $x = 1/3$ and is $\frac{1}{2}\left(1 + \frac{1}{3}\right)\left(1 - \frac{1}{3^2}\right) = \frac{4}{9}\frac{4}{3} = \frac{16}{27}$

Betz upper limit: $\frac{16}{27}$

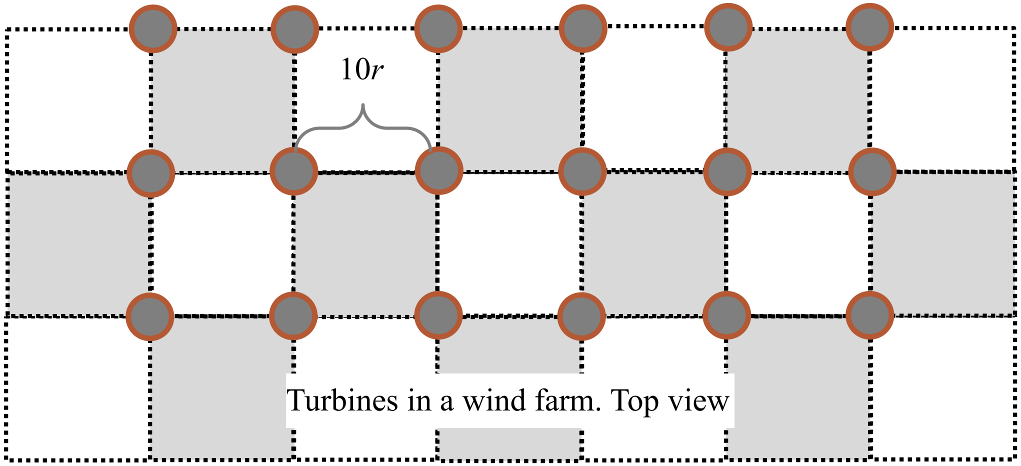
Wind Farm


Turbines in a Farm. Side view


Turbines in a wind farm. Top view

- A farm has multiple turbines
  - To the extent possible, turbines are located in a grid pattern
  - Turbines are sufficiently apart
- A farm requires some land
    - Offshore is different

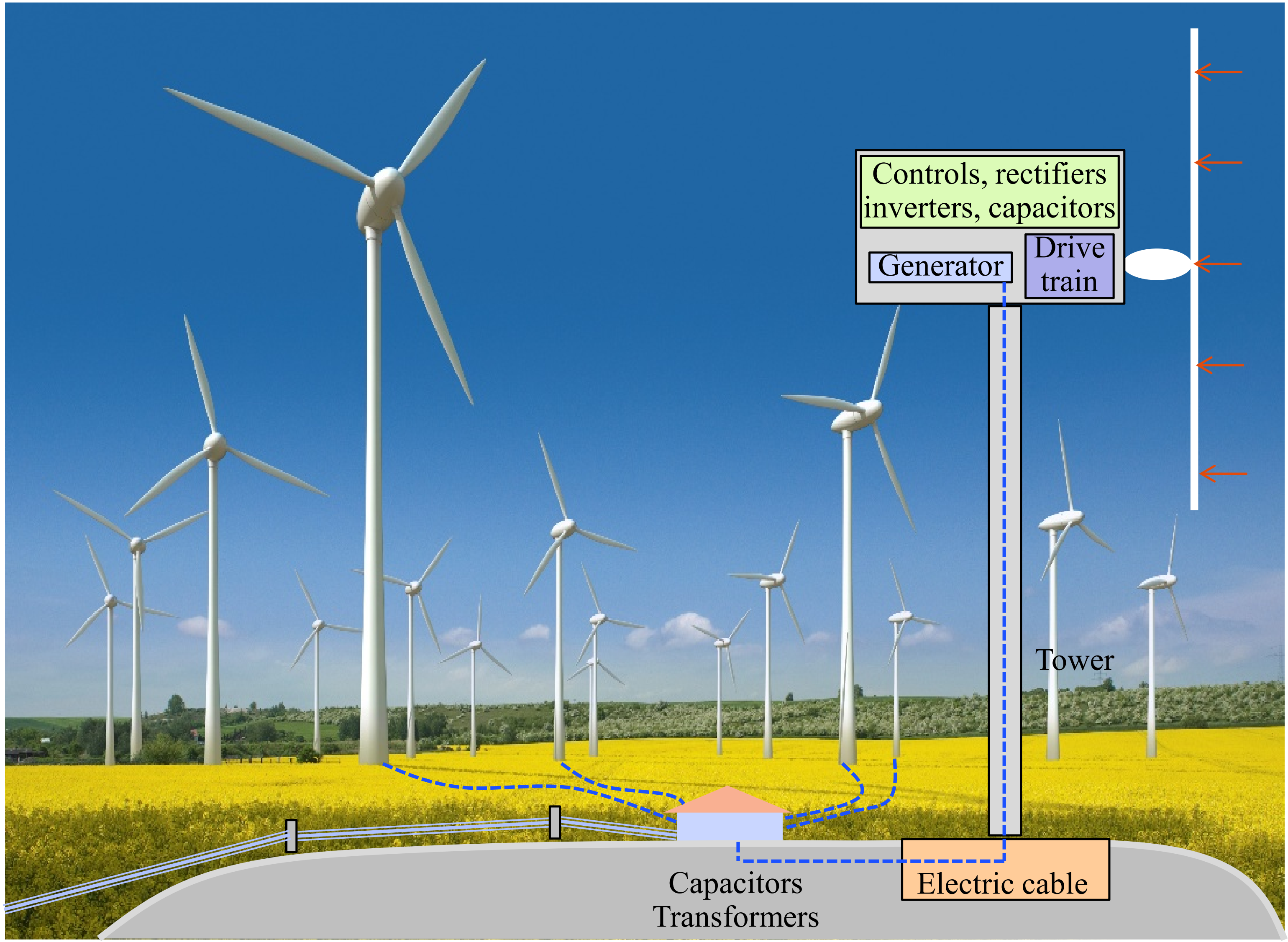
Wind Farm Area & Power Density



Turbines in a wind farm. Top view

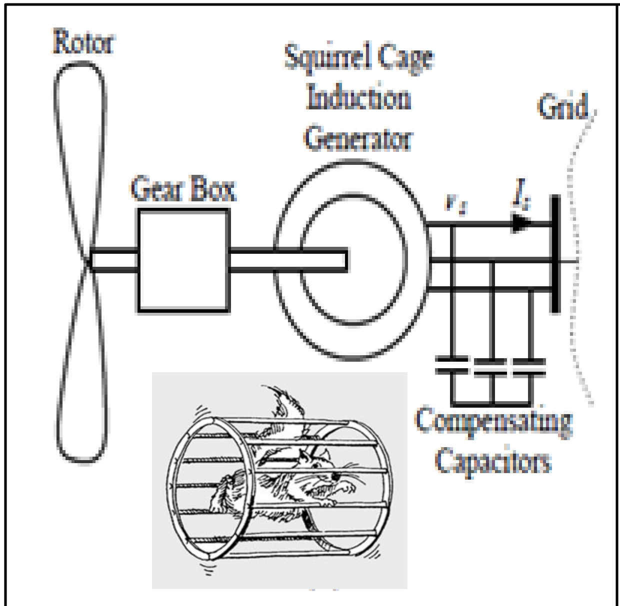
- Turbines must be located sufficiently apart so that the wind picks up the speed before the next turbine
  - The distance between turbines can be about 10 r (=blade radius)
- Ex: A wind farm is to locate turbines 10 r apart. The radius of turbines have not been determined yet but their efficiency is $e = 38\%$. The air density in the area is $\rho = 1.225 \text{ kg/m}^3$. What is the power density Watt per square metre when the wind blows at 12 m/s?
  - Power of $P(t)$ is obtained from the area $10r \times 10r$ of a square above.
  - $P(t) = \left[\frac{\pi e \rho}{2}\right] r^2 \, v_i^3(t) = \left[\frac{(3.14)(0.38)(1.225)}{2}\right] 12^3 \, r^2 = 1262.883 \, r^2$
  - $Area = 100 \, r^2$
  - Power density $= \frac{1262.883 \, r^2}{100 \, r^2} = 12.63 \text{ Watt/m}^2$
- Wind power density of 12.63 Watt/m^2 $\ll$ 1000 Watt/m^2 of Solar power density, borrowed from solar slides

Electric Systems in Detail

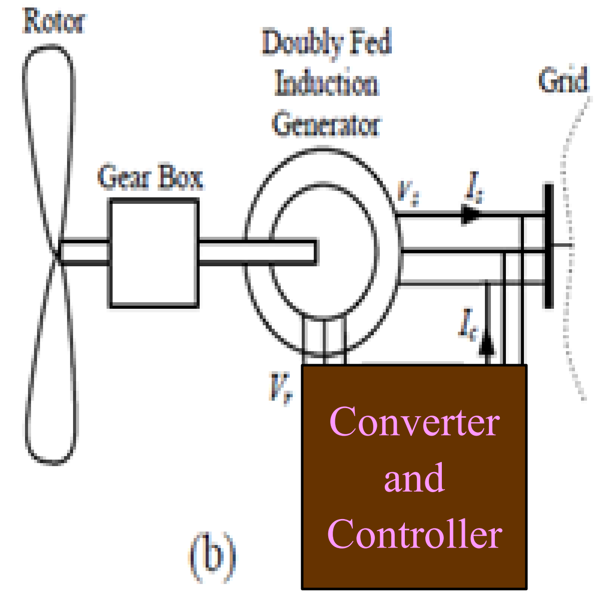


Generator Types

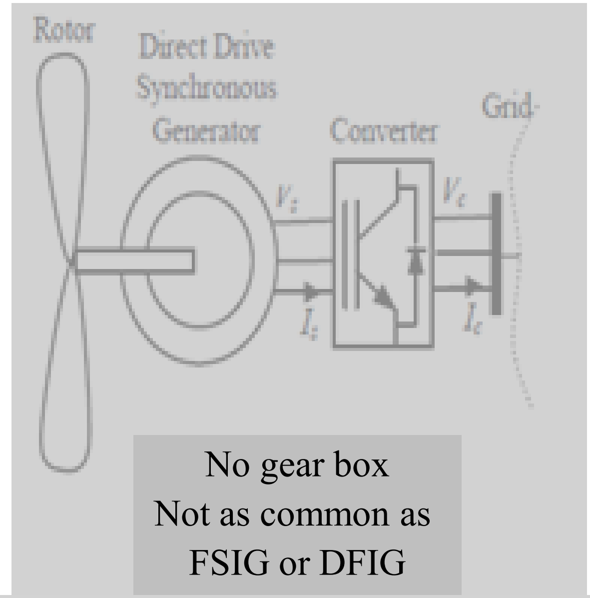
- Induction generator
  - Gear box brings up (sometimes down) the rotational speed of the shaft at a desired levels
  - The wire on the shaft (conducting bars) creates a magnetic field around it.
  - The magnetic field rotates with the wire (bar)
  - Rotating magnetic field induces current on the wires of the squirrel cage
- Initiation at low wind speed:
  - Induction motor: If low frequency of rotation, pull electric energy from the grid/storage to rotate the shaft at higher speeds



Fixed Speed Induction Generator
FSIG



Doubly Fed Induction Generator
DFIG



No gear box
Not as common as
FSIG or DFIG

Direct Drive Synchronous Generator
(DFIG)

- Speed Maintenance: Once the shaft is rotating at a desired speed
  - Maintain the shaft’s speed with wind’s speed in FSIG
  - Boost up the shaft’s speed by feeding electric energy from the grid/capacitors to run at an ideal speed in DFIG

Fixed Speed Induction vs. Doubly Fed Induction

Fixed Speed Induction Generator (FSIG)

Advantages:

- Simple and cheap
- It can improve both voltage and transient stability

Disadvantages:

- Lack of control possibilities for active/reactive power.
- Gearbox breakdown due to large mechanical loads.
- Large fluctuations in output power.

Doubly Fed Induction Generator (DFIG)

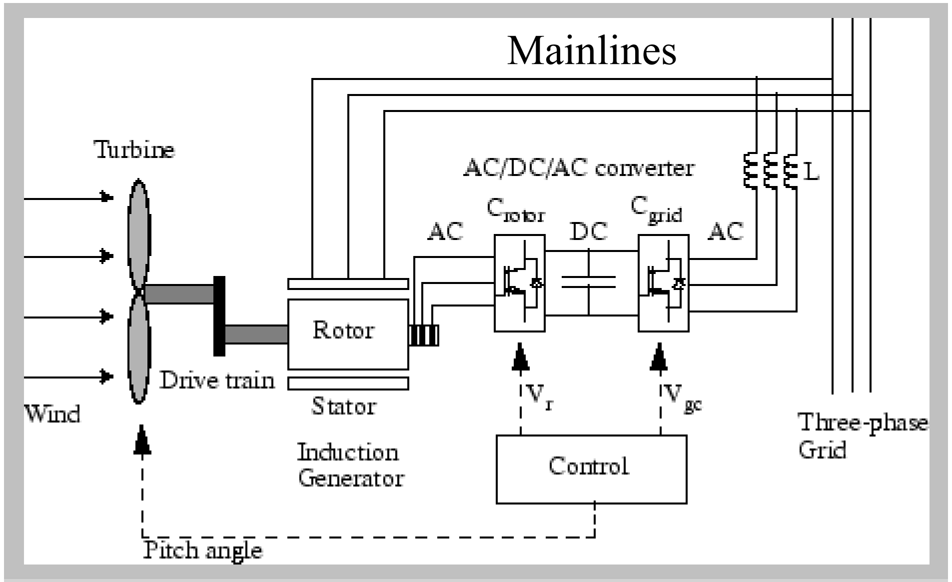
Advantages:

- Its controllability of both active & reactive power;
- Large rotor inertia smoothes the variations of wind speed and as a result it has fewer fluctuations in output power;
- The ability to ride through fault by its uninterruptable operation & it can successfully ride through grid faults.

Disadvantages:

- Low efficiency
- Fragile gearbox
- Hard to fix: High maintenance cost:12.9% of the cost.

Doubly Fed Induction Generator: Converter & Controller



DFIG uses a wound rotor induction generator, both rotor and stator are connected. Rotor has wound coils of cable and rotates between stationary Stators that generate the magnetic field.

Unlike fixed speed induction generator (FSIG), capacitor in doubly fed induction generator (DFIG) is connected

- parallel to the mainline,
- through a direct current.

Capacitor does not have to (dis-)charge continuously. So, it lasts longer.

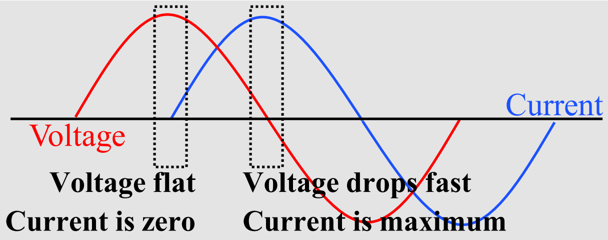
Unlike FSIG, DFIG has a controller that detects wind speed and rotates blades (possibly each individually) to face the wind or to avoid the wind. This is called pitch control. Typically, pitch is controlled by electromechanical motors rather than hydraulic motors.

When the wind is fast, first rotate blades to avoid some of it. If it is too fast, use extra energy to charge the capacitor rather than feeding that into the grid. Grid must be fed with stable voltage and frequency.

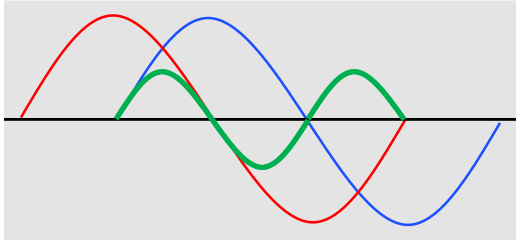
If the capacitor cannot absorb the surge in the wind speed or is aged, it heats up and burns. This can start up a fire in the turbine.

Dealing with Challenges of Alternating Current

Why capacitors? Current through a capacitor is proportional to the change in the voltage. When the voltage drops at a constant rate, the capacitor generates (discharges) a current equal to that constant. This compensates for voltage drops (wind speed) drops.



Power=Voltage*Current



Power is made more stable with a capacitor

Rotating systems generate alternating current: $\text{Voltage}(t) = \text{Max Voltage} * \sin(2\pi f t + \phi)$

t: time in seconds;

f: frequency in 1/seconds; Grid's alternating current has the frequency of $f = 60$ Hertz $= 60$ rotations/second. Max voltage in 1/60 seconds.

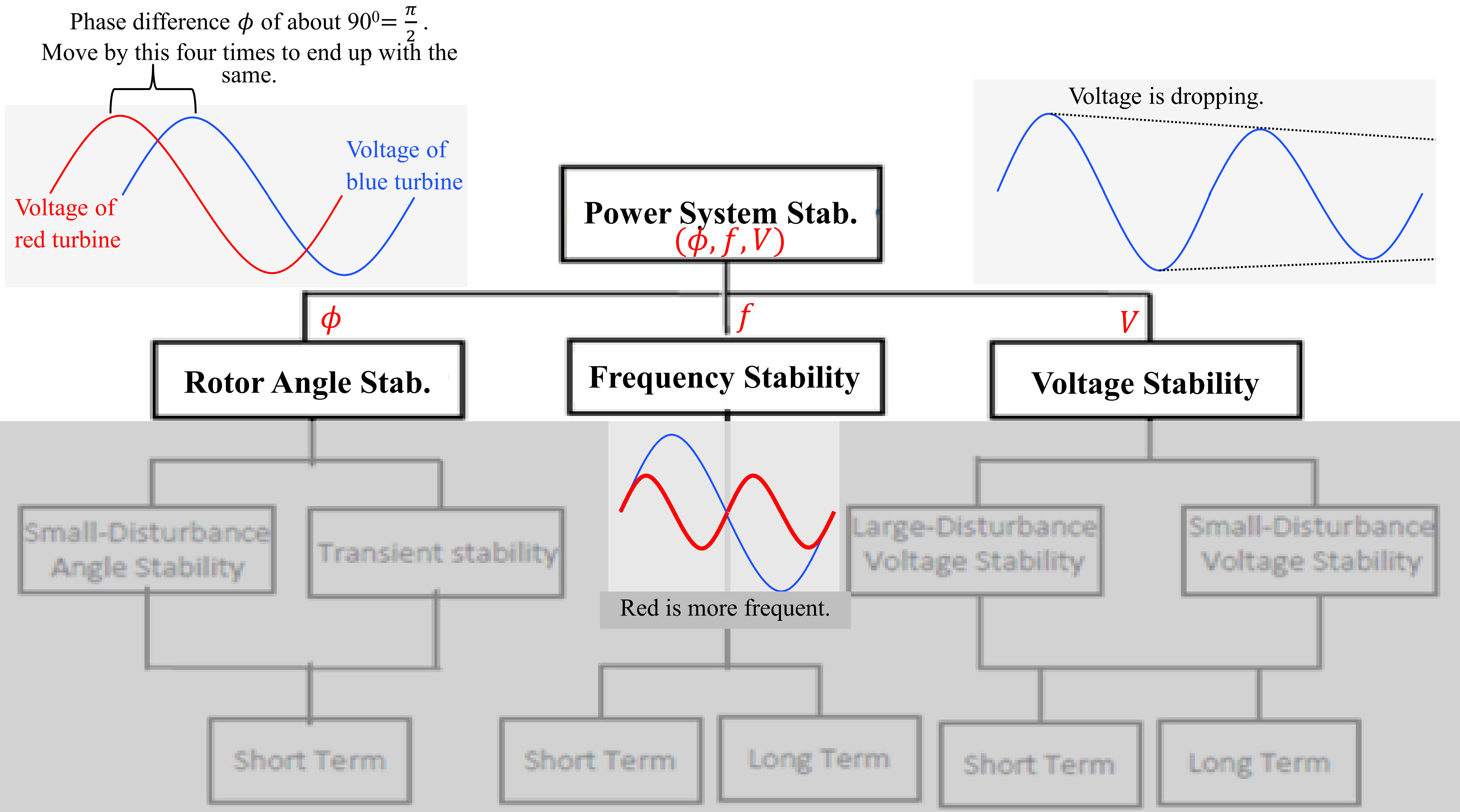
ϕ: phase difference in radians, e.g., $\frac{\pi}{2}$ between the red and blue voltage curves above.

Why rectifiers? Rectifiers convert Alternating current to Direct current, e.g., to store energy in batteries

Why inverters? Inverters convert Direct current to Alternating current, e.g., to feed back to the induction generator to increase shaft's speed.

Why 3 cables? AC has 3 phases: Phases are 1/(3*60) second apart, e.g., $\phi = \frac{2\pi}{3}$ for two closest currents. Every 1/180 second one reaches max voltage. Summing up the phase difference between the first and second, second and third, and third and first we find $\frac{2\pi}{3} + \frac{2\pi}{3} + \frac{2\pi}{3} = 2\pi$, note $\sin(2\pi f t + 2\pi) = \sin(2\pi f t)$, no phase difference between the first and itself.

Stability Classification at a Glance



Phase difference ϕ of about $90^0 = \frac{\pi}{2}$. Move by this four times to end up with the same.

Voltage of red turbine

Voltage of blue turbine

Voltage is dropping.

Power System Stab. (ϕ, f, V)

ϕ **Rotor Angle Stab.**

f **Frequency Stability**

V **Voltage Stability**

Small-Disturbance Angle Stability

Transient stability

Short Term

Red is more frequent.

Short Term

Long Term

Large-Disturbance Voltage Stability

Small-Disturbance Voltage Stability

Short Term

Long Term

Profit from a Wind Farm

Constants
Number pi
Air density at the site
Blade radius
Conversion efficiency

Variables
Wind speed
Wholesale price

Revenue = Energy ∗ Price

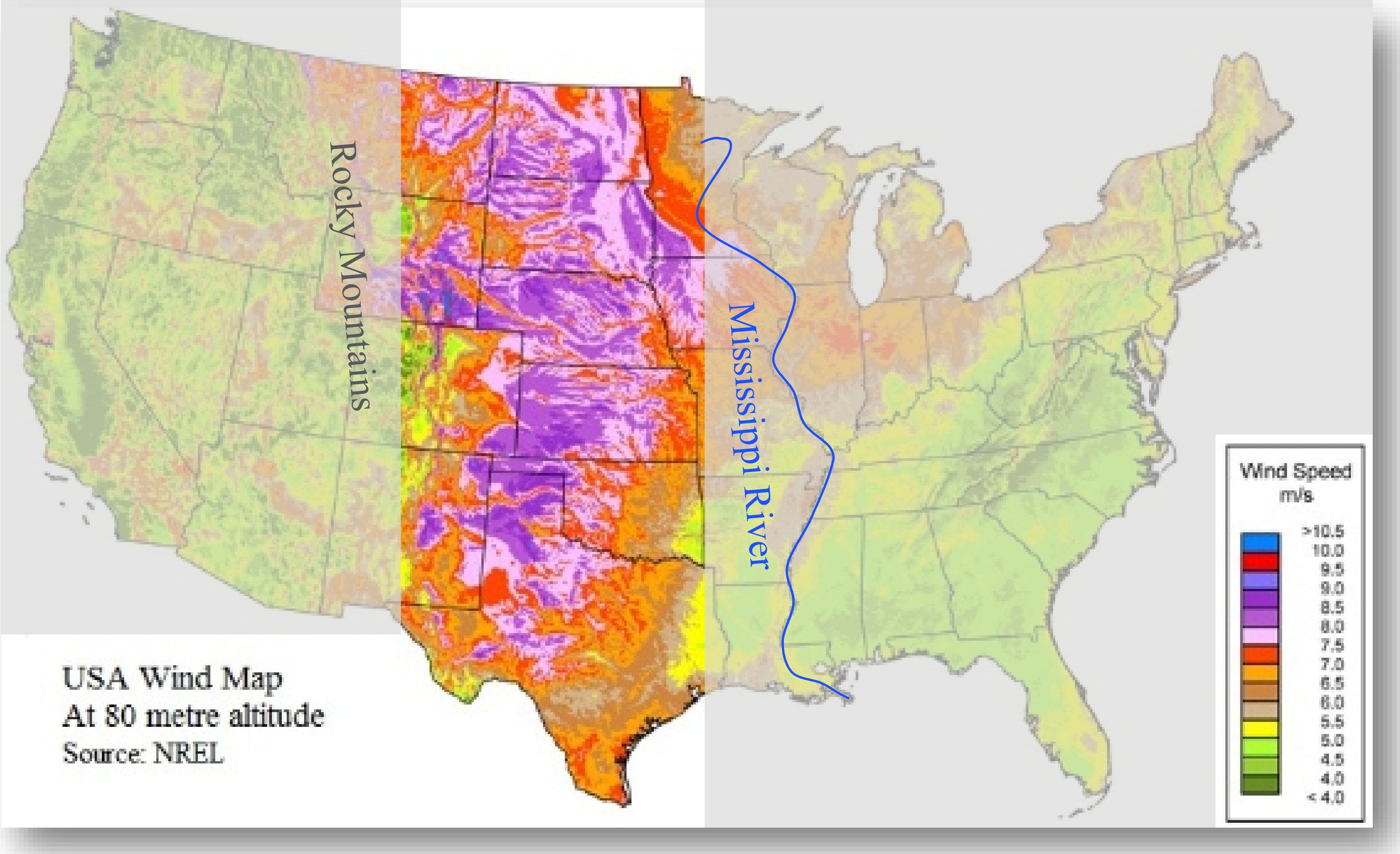
Cost of capital
Turbine cost
Foundation cost
Road cost
Line cost
Lifetime of farm

Cost of Operations & Maintenence

Cost of Land
For buying or leasing

- Annual Profit = Annual Revenue – Annual Cost

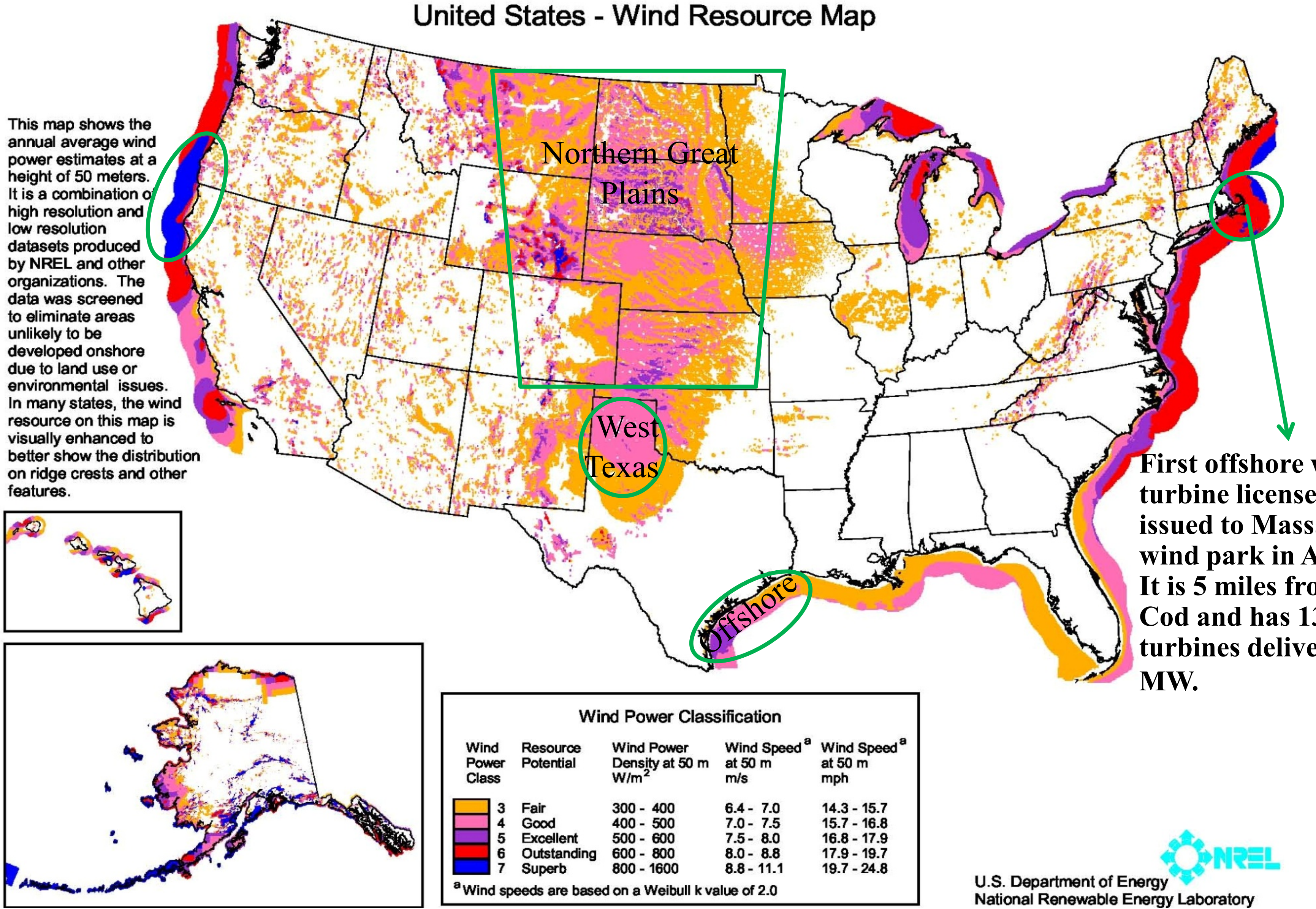
Abundance of Wind in the Absence of Transmission Lines



USA Wind Map
At 80 metre altitude
Source: NREL

- Wind is strong in the middle (between Rockies and Mississippi) of USA
  - Relatively cheap land to build farms there
  - At capacity transmission lines to bring energy to consumers
    - SPP (Southwest Power Pool), Dallas Conference, Sep 2016.

Wind Speed Matters



United States - Wind Resource Map

This map shows the annual average wind power estimates at a height of 50 meters. It is a combination of high resolution and low resolution datasets produced by NREL and other organizations. The data was screened to eliminate areas unlikely to be developed onshore due to land use or environmental issues. In many states, the wind resource on this map is visually enhanced to better show the distribution on ridge crests and other features.

Wind Power Classification

Wind Power Class		Resource Potential	Wind Power Density at 50 m W/m^2	Wind Speed[a] at 50 m m/s	Wind Speed[a] at 50 m mph
🟧	3	Fair	300 - 400	6.4 - 7.0	14.3 - 15.7
🟪	4	Good	400 - 500	7.0 - 7.5	15.7 - 16.8
🟪	5	Excellent	500 - 600	7.5 - 8.0	16.8 - 17.9
🟥	6	Outstanding	600 - 800	8.0 - 8.8	17.9 - 19.7
🟦	7	Superb	800 - 1600	8.8 - 11.1	19.7 - 24.8

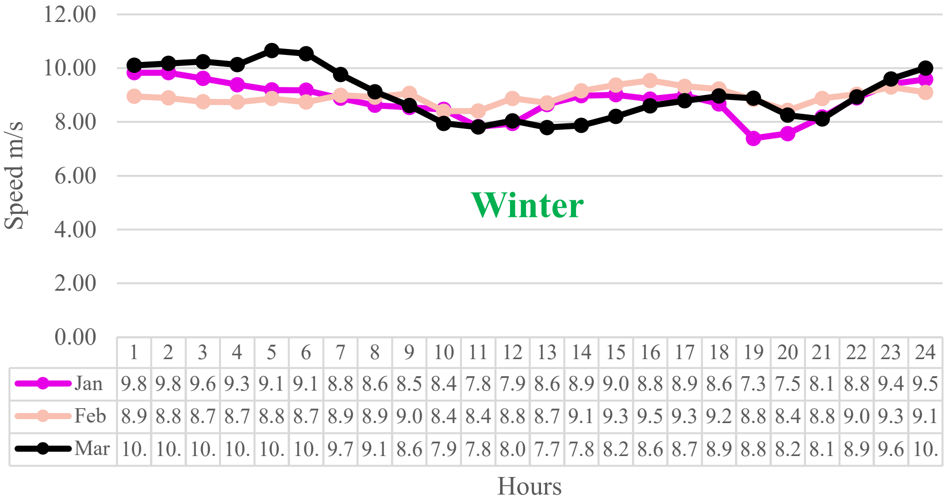
[a] Wind speeds are based on a Weibull k value of 2.0

U.S. Department of Energy
National Renewable Energy Laboratory

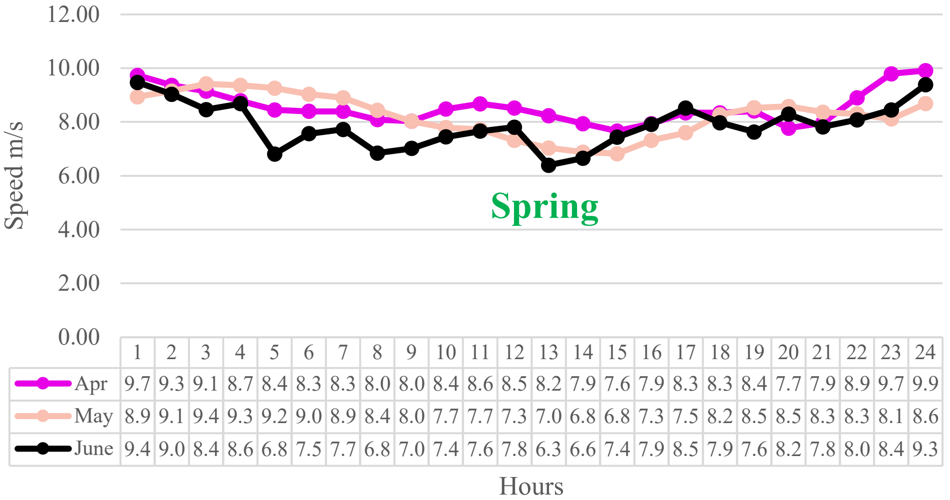
First offshore wind turbine license in US issued to Massachusetts wind park in April 2011. It is 5 miles from Cape Cod and has 130 turbines delivering 420 MW.

Random Wind Speed: Empirical Analysis

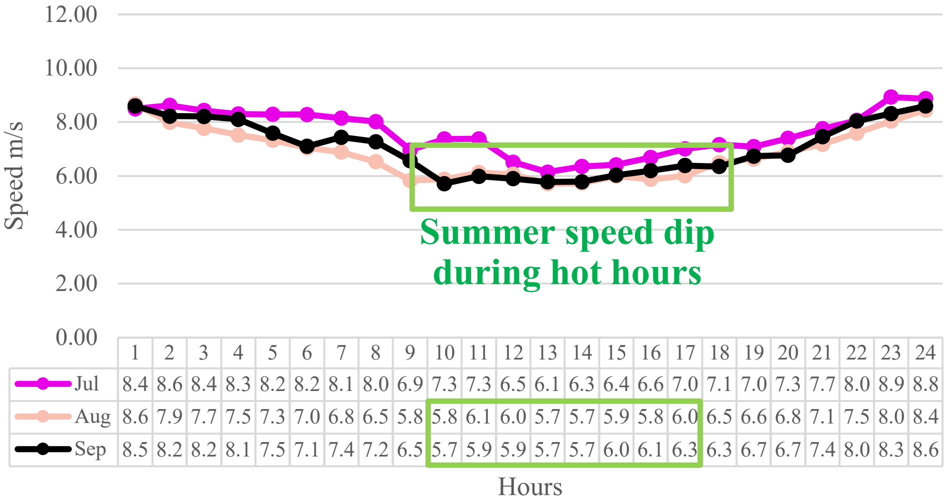
- Wind does not blow always. Wind turbine outputs are generally ≈40% of their capacity.
  - Every watt from wind means one fewer watt from fossil fuels. Energy storage is very important.
- Ex: West Texas wind speed profiles from White Deer in 2012. Each point is a monthly average, so graphs are smoother.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Jan	9.8	9.8	9.6	9.3	9.1	9.1	8.8	8.6	8.5	8.4	7.8	7.9	8.6	8.9	9.0	8.8	8.9	8.6	7.3	7.5	8.1	8.8	9.4	9.5
Feb	8.9	8.8	8.7	8.7	8.8	8.7	8.9	8.9	9.0	8.4	8.4	8.8	8.7	9.1	9.3	9.5	9.3	9.2	8.8	8.4	8.8	9.0	9.3	9.1
Mar	10.	10.	10.	10.	10.	10.	9.7	9.1	8.6	7.9	7.8	8.0	7.7	7.8	8.2	8.6	8.7	8.9	8.8	8.2	8.1	8.9	9.6	10.

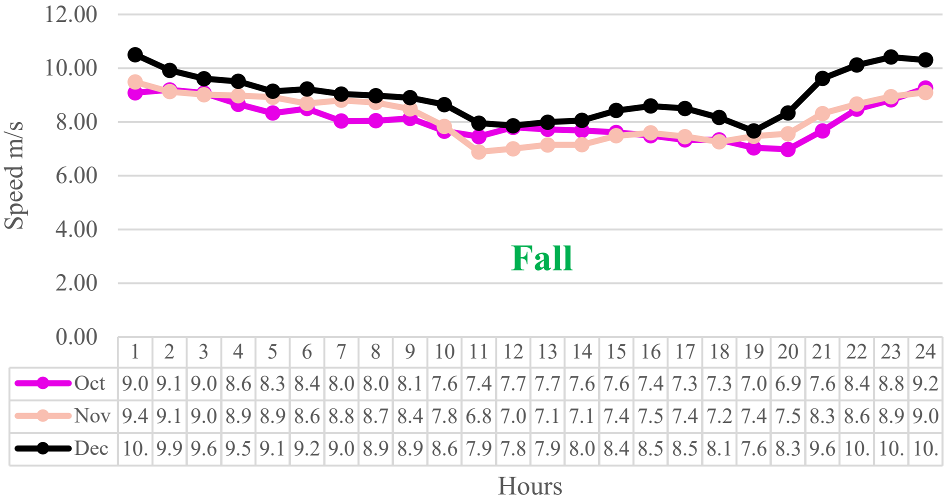


	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Apr	9.7	9.3	9.1	8.7	8.4	8.3	8.3	8.0	8.0	8.4	8.6	8.5	8.2	7.9	7.6	7.9	8.3	8.3	8.4	7.7	7.9	8.9	9.7	9.9
May	8.9	9.1	9.4	9.3	9.2	9.0	8.9	8.4	8.0	7.7	7.7	7.3	7.0	6.8	6.8	7.3	7.5	8.2	8.5	8.5	8.3	8.3	8.1	8.6
June	9.4	9.0	8.4	8.6	6.8	7.5	7.7	6.8	7.0	7.4	7.6	7.8	6.3	6.6	7.4	7.9	8.5	7.9	7.6	8.2	7.8	8.0	8.4	9.3



Summer speed dip during hot hours

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Jul	8.4	8.6	8.4	8.3	8.2	8.2	8.1	8.0	6.9	7.3	7.3	6.5	6.1	6.3	6.4	6.6	7.0	7.1	7.0	7.3	7.7	8.0	8.9	8.8
Aug	8.6	7.9	7.7	7.5	7.3	7.0	6.8	6.5	5.8	5.8	6.1	6.0	5.7	5.7	5.9	5.8	6.0	6.5	6.6	6.8	7.1	7.5	8.0	8.4
Sep	8.5	8.2	8.2	8.1	7.5	7.1	7.4	7.2	6.5	5.7	5.9	5.9	5.7	5.7	6.0	6.1	6.3	6.3	6.7	6.7	7.4	8.0	8.3	8.6



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Oct	9.0	9.1	9.0	8.6	8.3	8.4	8.0	8.0	8.1	7.6	7.4	7.7	7.7	7.6	7.6	7.4	7.3	7.3	7.0	6.9	7.6	8.4	8.8	9.2
Nov	9.4	9.1	9.0	8.9	8.9	8.6	8.8	8.7	8.4	7.8	6.8	7.0	7.1	7.1	7.4	7.5	7.4	7.2	7.4	7.5	8.3	8.6	8.9	9.0
Dec	10.	9.9	9.6	9.5	9.1	9.2	9.0	8.9	8.9	8.6	7.9	7.8	7.9	8.0	8.4	8.5	8.5	8.1	7.6	8.3	9.6	10.	10.	10.

Weibull for Wind Speed Modelling

- Recall wind power with constant wind speed v is $(\pi e\rho r^2/2)v^3$
- Wind power with random wind speed V is a derived random variable $(\pi e\rho r^2/2)V^3$

- Expected wind power $(\pi e\rho r^2/2)\mathrm{E}(V^3)$, where $\mathrm{E}(V^3)$ is the 3rd moment of the wind speed variable
- For any nonnegative random variable V, $\mathrm{E}(V^3) \ge \left(\mathrm{E}(V)\right)^3$:
  - Third moment is larger than the cube of the average
  - Inserting the cube of average into power formula underestimates the expected wind power
  - Tricking the partner in a transaction: If you are buying, underestimate power with $\left(\mathrm{E}(V)\right)^3$.

- Weibull: An appropriate distribution for wind speed (Carta et al., 2009)
  - Parameters α, β
  - Weibull has a nonnegative range as the speed does: More appropriate than Normal
  - Weibull reduces to exponential distribution with $\alpha = 1$: More general than Exponential
  - Cumulative distribution: $F(\mathrm{v}) = \mathrm{P}(V \le v) = 1 - \exp\left(-\left(\frac{v}{\beta}\right)^{\alpha}\right)$ for $v \ge 0$.
- In preparation for Weibull moments $\Gamma(n) = \int_0^\infty x^n \exp(-x)dx$ = gamma(n) in Excel

Third Moment of Wind Speed for Wind Energy Computation

- Moments of Weibull: $\mathrm{E}(V) = \beta\Gamma\left(1 + \frac{1}{\alpha}\right)$; $\mathrm{E}(V^2) = \beta^2\Gamma\left(1 + \frac{2}{\alpha}\right)$; $\mathrm{E}(V^3) = \beta^3\Gamma\left(1 + \frac{3}{\alpha}\right)$.

- Ex: What are the first third moments of the wind speed distributed by $(\alpha = 4, \beta = 6)$?

  - $\mathrm{E}(V) = \beta\Gamma\left(1 + \frac{1}{\alpha}\right) = 6\,gamma\left(1 + \frac{1}{4}\right) = 5.44\ \mathrm{m/s}$; $\mathrm{E}(V^2) = 6^2\mathrm{gamma}\left(1 + \frac{2}{4}\right) = 31.90\ \mathrm{m}^2/\mathrm{s}^2$;
  - $\mathrm{E}(V^3) = \beta^3\Gamma\left(1 + \frac{3}{\alpha}\right) = 6^3 gamma\left(1 + \frac{3}{4}\right) = 198.52\ \mathrm{m}^3/\mathrm{s}^3$.
  - Compare $\mathrm{E}(V^3)$ and $\left(\mathrm{E}(V)\right)^3$

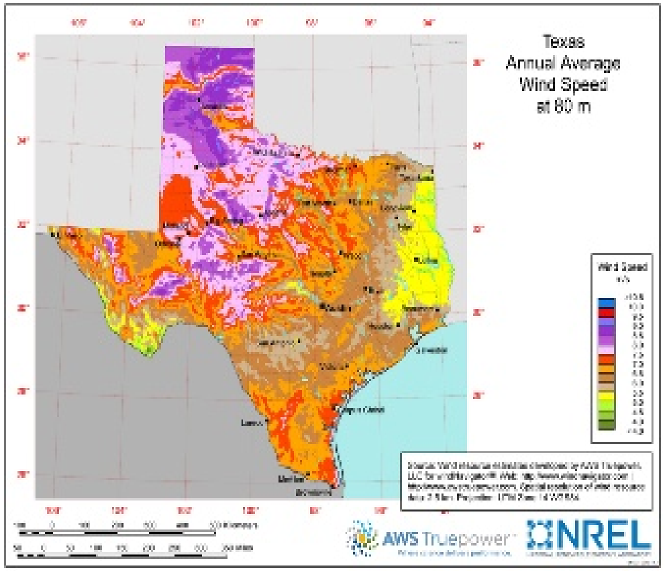
- Ex: A turbine has e =42%, r =25 and is subject to Weibull distributed winds with parameters $\alpha = 1.3$, $\beta = 6.2$ the next year. What is the expected amount of energy generated over a year?

  - For an arbitrary hour, the expected amount of energy generated is $\left(\frac{\pi e \rho r^2}{2}\right)\beta^3\Gamma\left(1 + \frac{3}{\alpha}\right) = \frac{(3.14)(0.42)(1.225)25^2}{2} 6.2^3 gamma\left(1 + \frac{3}{1.3}\right) = 504.85 * 238.33 * 2.70 = 325{,}457\ \mathrm{Wh}$

  - Over a year, the energy generated is $365 * 24 * (0.3255) = 2{,}851$ MWh.
  - Recall 1 MegaWatt-hour = 1,000 KiloWatt-hour = 1,000,000 Watt-hour

Estimation of Wind Speed Distribution Method of Moments with White Deer Data

- Method of Moments Estimation
  - Given sample mean μ_E and variance S_E of the wind speed
  - Consider theoretical mean and variance:
    - » $\mathrm{E}(V;\alpha,\beta)=\beta\Gamma\left(1+\frac{1}{\alpha}\right)$; $\mathrm{E}(V^2;\alpha,\beta)=\beta^2\Gamma\left(1+\frac{2}{\alpha}\right)-\left(\mathrm{E}(V)\right)^2$
  - $\mathrm{E}(V;\alpha,\beta)=\mu_E$ and $\mathrm{E}(V^2;\alpha,\beta)=S_E$ to solve for α,β

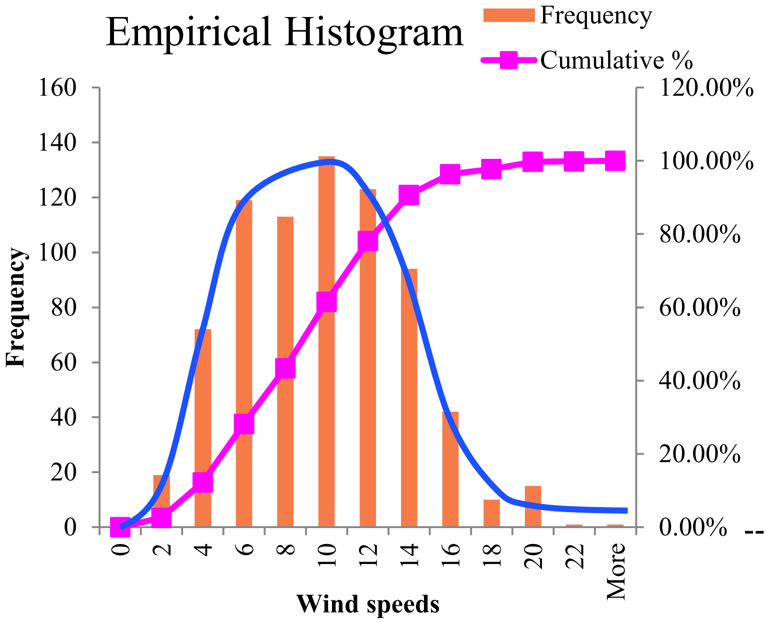


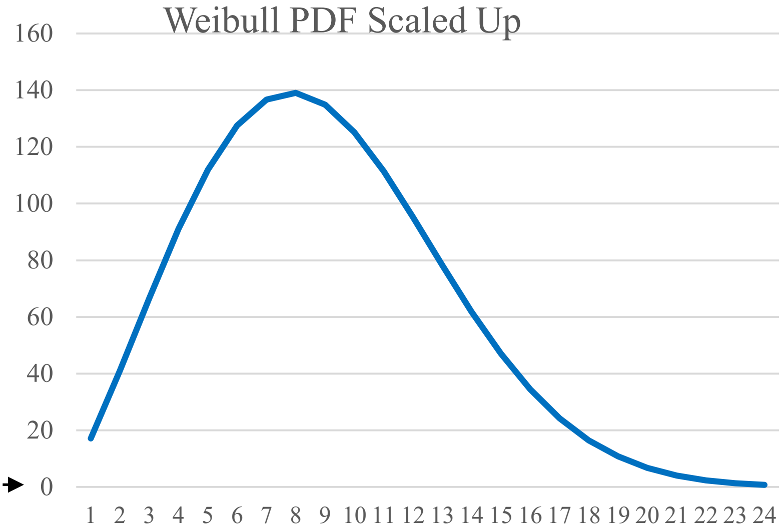
- White Deer is in Texas Panhandle, Amarillo ↔ Oklahoma City
- Consider hourly wind speeds in January, $N=31*24=744$

- Ex: Method of Moments Estimation. Blue are known, red are estimated.
  - Sample mean $\mu_E=8.81$ & variance $S_E=16.24$
  - $\mathrm{E}(V;\alpha,\beta)=\mu_E=8.81$ & $\mathrm{E}(V^2;\alpha,\beta)=S_E=16.24$ yields $\alpha=2.3203,\ \beta=9.9437$

Estimation of Wind Speed Distribution CDF Fitting with White Deer Data

- Weibull CDF fitting
  - For N observations, let V_n be the nth order statistics and set empirical CDF as
    $$F_E(v) = \frac{n}{N} \quad \text{for} \quad v = V_n \quad \text{and} \quad n = 1 \dots N$$
  - The theoretical CDF is $F(v; \alpha, \beta)$ as given above
  - Find α, β by minimizing the sum of squared errors empirical and theoretical CDFs
    $$\text{Min}_{\alpha,\beta} \sum_{n=1}^{N} \left(F_E(V_n) - F(V_n; \alpha, \beta)\right)^2$$





- Weibull CDF fitting
  - Solving for $\text{Min}_{\alpha,\beta} \sum_{n=1}^{N} \left(F_E(V_n) - F(V_n; \alpha, \beta)\right)^2$ yields $\alpha = 2.29, \beta = 10.08$

Estimation of Third Moment Empirically with Data

- Method of Moments Estimators: $\alpha = 2.3203,\ \beta = 9.9437$
- Weibull CDF Fitting Estimators: $\alpha = 2.2863,\ \beta = 10.081$

- Third moment of Weibull $\mathrm{E}(V^3) = \beta^3\Gamma\left(1 + \frac{3}{\alpha}\right)$.
- Empirically with data: $\frac{1}{\mathrm{N}}\sum_{n=1}^{N} v_n^3$, where $N = 744$ for January

$\mathrm{E}(V^3)$ **by Method of Moments**	$\mathrm{E}(V^3)$ **by Weibull CDF Fitting**	$\mathrm{E}(V^3)$ **by Empirically with Data**
1142.3	1204.0	1131.3
101%	106%	100%

All methods of estimation are close; differing ≤ 6%

Annual Revenue from a Wind Farm

- Let the hour index in a year be $t = 1 \ldots 8760 = 365 * 24$
- Wholesale price random variable be $W(t)$ in hour t and the revenue is $\left(\frac{\pi e \rho r^2}{2}\right) \mathrm{E}(V^3(t) W(t))$
- Suppose that the wind speed and wholesale price are independent, then the expected revenue is

$$R(t) = \left(\frac{\pi e \rho r^2}{2}\right) \mathrm{E}(V^3(t) W(t)) = \left(\frac{\pi e \rho r^2}{2}\right) \mathrm{E}(V^3(t))\ \mathrm{E}\big(W(t)\big)$$

The annual revenue is tedious but is $\sum_{t=1}^{8760} R(t)$.

- Further suppose that wind speed and wholesale price are stationary, so $V = V(t)$ and $W = W(t)$ for every hour. $V = V(t)$ is an approximation whereas $W = W(t)$ can be from a contract.

$$R = \left(\frac{\pi e \rho r^2}{2}\right) \mathrm{E}(V^3)\ \mathrm{E}(W) = \text{Constant in time} * 3^{\text{rd}} \text{ moment of speed} * 1^{\text{st}} \text{ moment of price}$$

The annual revenue is tedious but is $8760\, R = 8760 \left(\frac{\pi e \rho r^2}{2}\right) \mathrm{E}(V^3)\ \mathrm{E}(W)$.

- Possible MERIT projects: Check independence and/or stationarity.

- Ex: A turbine has *e*=40%, *r*=28 m and is subject to Weibull distributed winds with parameters $\alpha = 1.8, \beta = 5.8$ for the next year. If the energy is sold at an average price of \$120 per MWh, what is the revenue made from the turbine?
  - The annual expected revenue is

$$8760\, R = 8760 \left(\frac{\pi e \rho r^2}{2}\right) \mathrm{E}(V^3)\ \mathrm{E}(W) = 8760 \left(\frac{(3.14)(0.4)(1.225)28^2}{2}\right) (5.8^3)\, gamma\left(1 + \frac{3}{1.8}\right) 120\ /\ 1{,}000{,}000$$

$$= 8760\ (603.13)\ (195.11)\ (1.50)\ 120\ /\ 1{,}000{,}000 = \$\ 186{,}000 \text{ per year}$$

- In general, expect to make a few hundred thousand dollars of annual revenue per turbine

Annual Cost of a Wind Farm

- Turbines cost much more than the foundation, roads, transmission lines
- A turbine can cost \$ 3-4 million and have a lifetime of 20 years; see appendix after summary.
  - Ex: What is the annualized cost x of a \$3 (= a) million turbine over 5 years at the annual interest rate of i=8%?
    - » $3 = x(1.08^{-1}) + x(1.08^{-2}) + x(1.08^{-3}) + x(1.08^{-4}) + x(1.08^{-5}) = x\frac{1-1.08^{-5}}{0.08} = x\frac{0.3194}{0.08} = 3.9927x$
    - » $x = a\frac{i}{1-(1+i)^{-n}} = 3\frac{0.08}{0.3194} = 0.751$ million or \$ 751,000
  - Ex: What is the cost of the same turbine over 20 years at i=8%?
    - » $x = a\frac{i}{1-(1+i)^{-n}} = 3\frac{0.08}{0.7854} = 3(0.102) = 0.306$ million or \$ 306,000
  - Ex: What is the cost of the same turbine over 20 years at i=10%?
    - » $x = a\frac{i}{1-(1+i)^{-n}} = 3\frac{0.10}{0.8514} = 3(0.117) = 0.352$ million or \$ 352,000

See spreadsheet capitalCost.xlsx on course website.

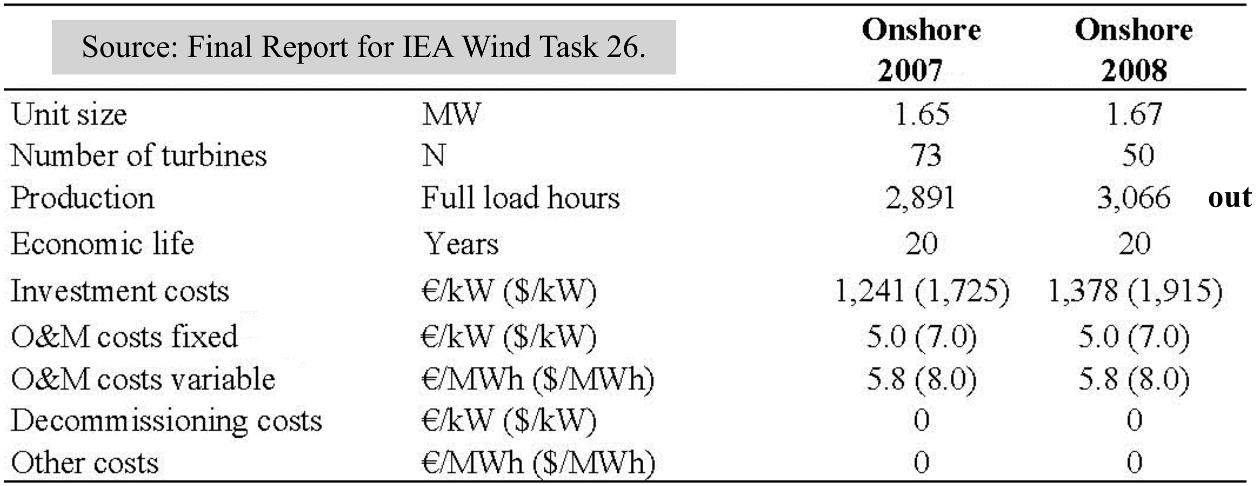
- Operation and Maintenance cost is \$ 8 per MWh.
  - Ex: A turbine has e=40%, r=28 m and is subject to Weibull distributed winds with parameters $\alpha = 1.8, \beta = 5.8$ for the next year. What is the annual O & M cost of the turbine?

$8760\left(\frac{\pi e \rho r^2}{2}\right) \mathrm{E}(V^3)\ 8/1{,}000{,}000$ = 8760 (603.13) (195.11) (1.50) 8 / 1,000,000 = \$12,408 per year

- Land costs vary
  - Ex: A turbine with r=28 m is installed on a 200 × 200 square metre land leased at \$ 0.1 per square metre per year. What is the annual land leasing cost?
    - » It is 0.1*200*200= \$ 4,000 per year

Capital costs dominate others by an order of magnitude.

Economics of Wind Power in USA



Source: Final Report for IEA Wind Task 26.

		Onshore 2007	Onshore 2008
Unit size	MW	1.65	1.67
Number of turbines	N	73	50
Production	Full load hours	2,891	3,066
Economic life	Years	20	20
Investment costs	€/kW ($/kW)	1,241 (1,725)	1,378 (1,915)
O&M costs fixed	€/kW ($/kW)	5.0 (7.0)	5.0 (7.0)
O&M costs variable	€/MWh ($/MWh)	5.8 (8.0)	5.8 (8.0)
Decommissioning costs	€/kW ($/kW)	0	0
Other costs	€/MWh ($/MWh)	0	0

out of 8760

2,000 kW turbine costs $3.8 M

Proportional to Capacity

Proportional to Power

- Unit size is the capacity of a single turbine.
- Number of turbines per wind farm is average among USA farms established in 2007 and 2008.
- Investment cost (turbine and tower) is $1,725 per kW in 2007. The same number is $2,000 per kW for nuclear reactors. Turbines are not significantly cheaper than nuclear power plants.
- What is the investment cost for an average wind farm in 2008? $157,030,000 = 1,670*50*1,915. This gives a capacity of only 83.5 MW while a nuclear power plant gives 1000-2000 MW. Smaller initial investment is easier to justify.
- O & M are operations and maintenance cost. It is based on power $/kWh and energy $/MWh. O & M cost was 1.5 cent/kWh = $15/MWh for nuclear power plants. O & M of wind turbines is cheaper.

Issues with the Wind Power

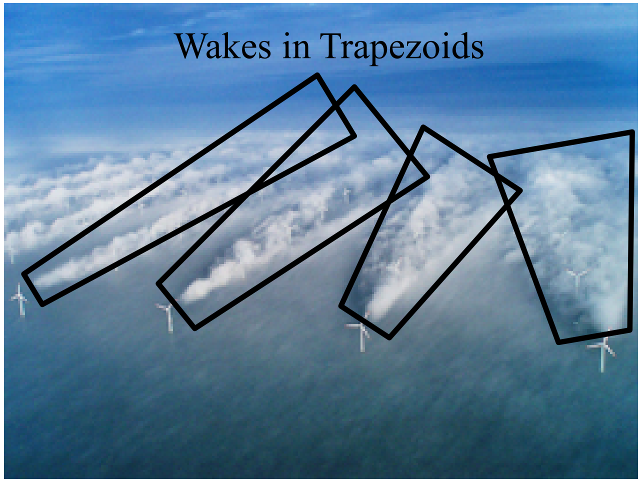
- Wind speed is random, so wind power is intermittent. Use storage in battery and other means.

- Geographical isolation. Turbines are further away from main transmission lines.
  - $5 Billion investment into transmission lines to transmit electricity from West Texas CREZ (Competitive Renewable Energy Zone) to East Texas.

- Maintenance/Repair: Turbines must be maintained and fail due to age and power surge. Wind farm operators have maintenance/repair contracts with the turbine companies. Attempts to maintain/repair by operators can annul the contract/warranty. Operator must wait for the certified crew to arrive at isolated locations and perform maintenance/repair 80 meters above the ground. This takes time and money.

- Wind power causes warming, but less than hydrocarbon resources
  - » Source: Miller and Keith. 2018. *Climatic Impacts of Wind Power*. Joule 2: 1-15.

- Wind Wake: Turbines reduce the speed of the wind and change wind patterns (sometimes creating turbulence behind them).
  - ``A decrease of the mean wind speed is found as the wind flows through the wind farms, leaving a velocity deficit of 8–9% on average” in the offshore Danish wind farms according to M.B. Christiansen, C.B. Hasager (2005).
  - Placing turbines back to back reduces efficiency. Location of turbines should be arranged to minimize wake effects. Every location on a hill is not feasible, the ground must be relatively flat and strong enough to hold a 100 metre tower.



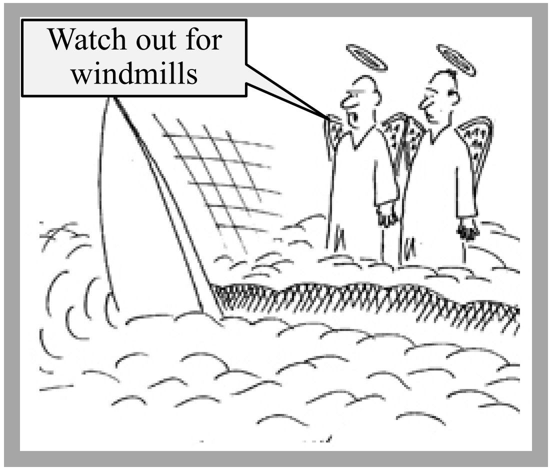
Wakes in Trapezoids

Issues with the Wind Power

- Sound and look of the wind turbines. People like them as long as they are not in their backyards (Not in my backyard, nimby, problem.
  - Swishing sound of blade is heard from several hundred feet of a wind turbine.
  - Even offshore turbines are criticized for blocking the open ocean vistas of boating enthusiasts.
- Blades of wind turbines hit and kill birds
  - Red-throated diver birds stop the second phase of London Array wind farm.
    - London Array wind farm is located in the south east of London where river Thames flows into the sea.
    - The first phase opened in July 2014, has 175 turbines with a total generating capacity of 630 MWatts.
    - Owners of the second phase: Dong Energy, Denmark; Eon, Germany; Masdar, Abu Dhabi.
    - Second phase is subjected to a hold for 3 years to assess the effect of the wind farm on red-throated diver birds that live in Thames Estuary.
    - The owners of the second phase surrendered the lease on the site.
  - UK installed significant wind powered generation capacity recently; 3.6 GW since 2000. There is a pipeline of projects up to 40 GW. Environmental issues and subsidy structure is creating uncertainty.

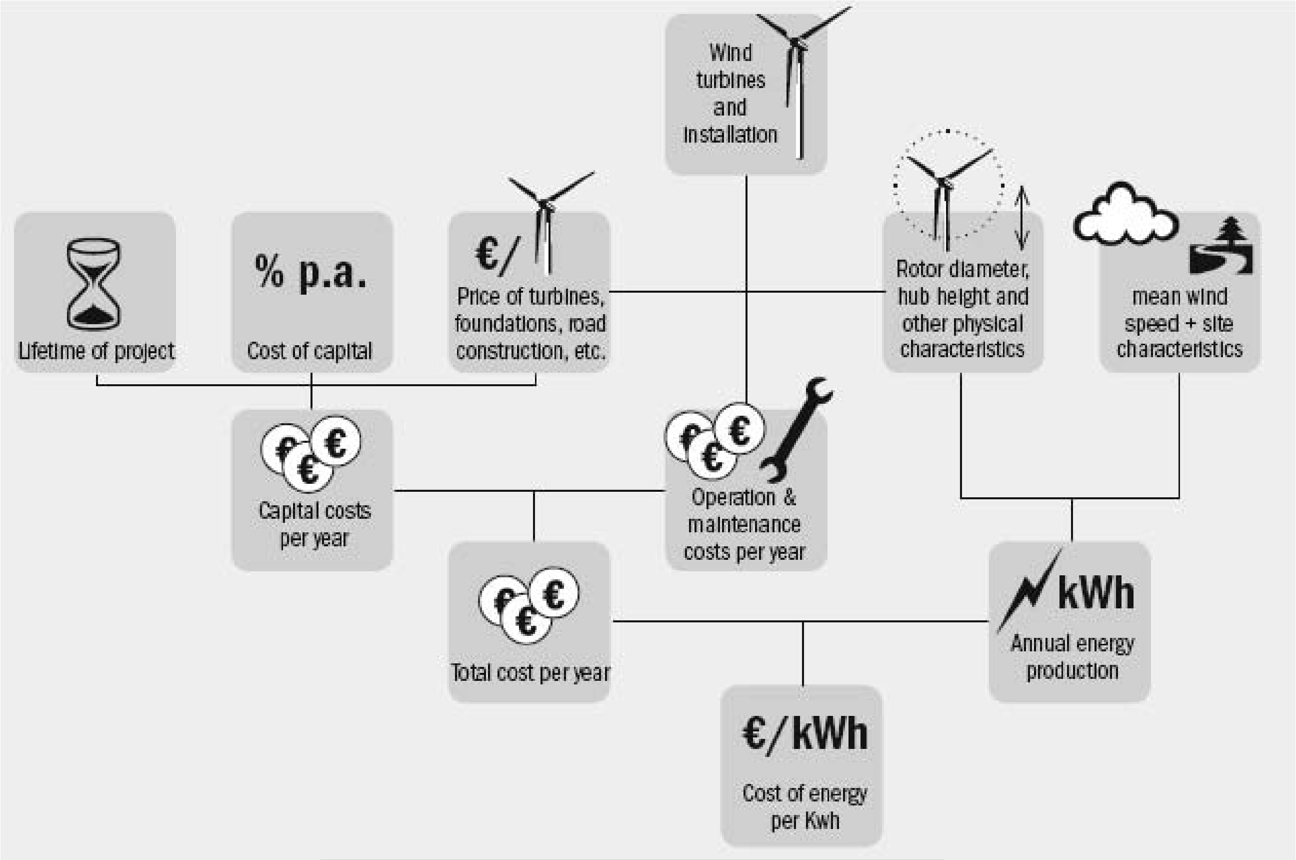
Source: "Red-throated Diver bird kills off second phase of wind farm". P. Clark. FT, Feb 19 2014.

Red-throated diver bird



- Wind power is subsidized with feed-in-tariff rates (in Europe), tax incentives (in USA) and quotas.
  - Tax incentives apply to the electricity produced within the first ten years of the turbine. Tax incentives in USA expire and must be renewed. They expired in 1999, 2001 and 2003. They were renewed in 2000, 2002, 2004. These created artificial swings in capacity installment.

Cost Components of Wind Power

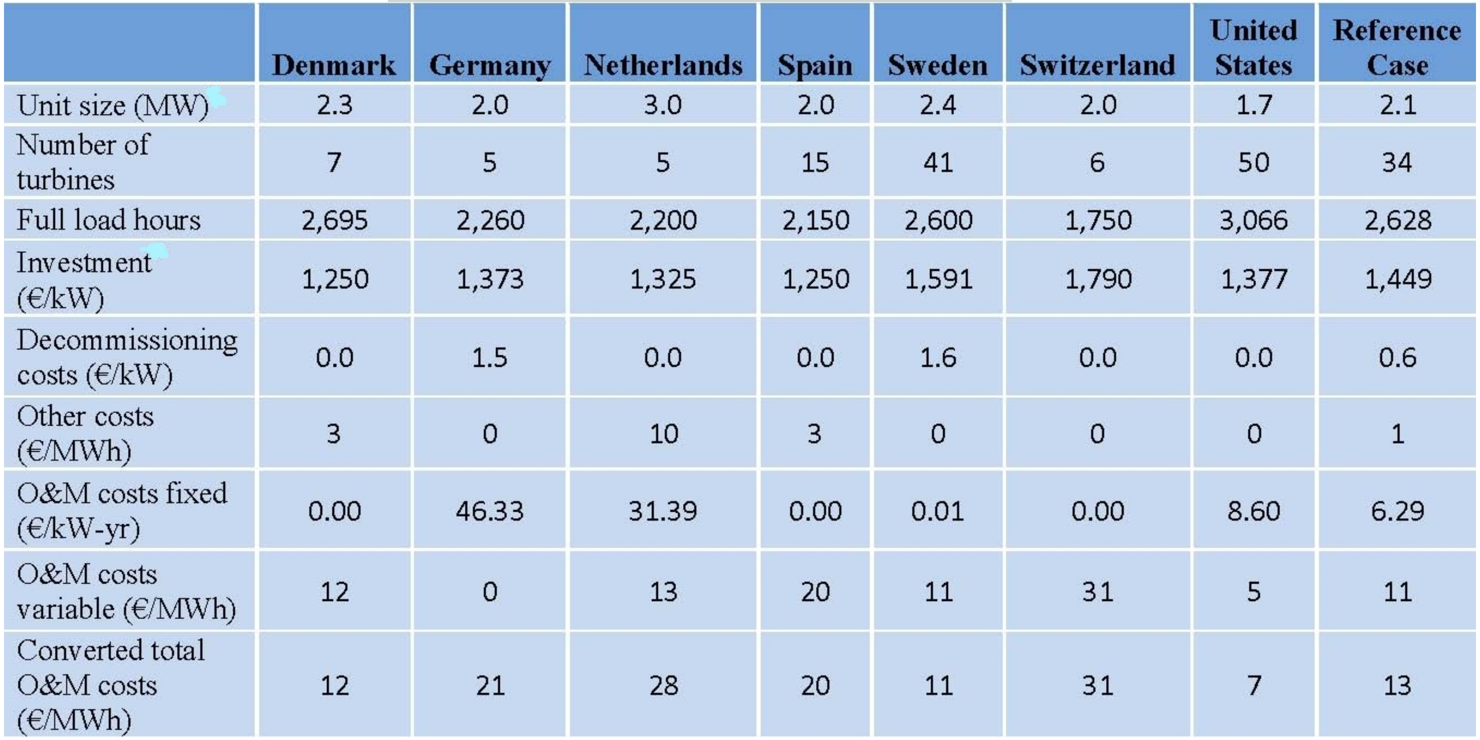


Source: Final Report for IEA Wind Task 26.

- With the data above, we can find the total annual cost of turbine, tower, construction, O & M. We can also find the total energy production. Dividing former by the latter, we obtain LCOE (Levelized Cost of Electricity) for wind.

Economics of Wind Power Globally

Source: Final Report for IEA Wind Task 26.



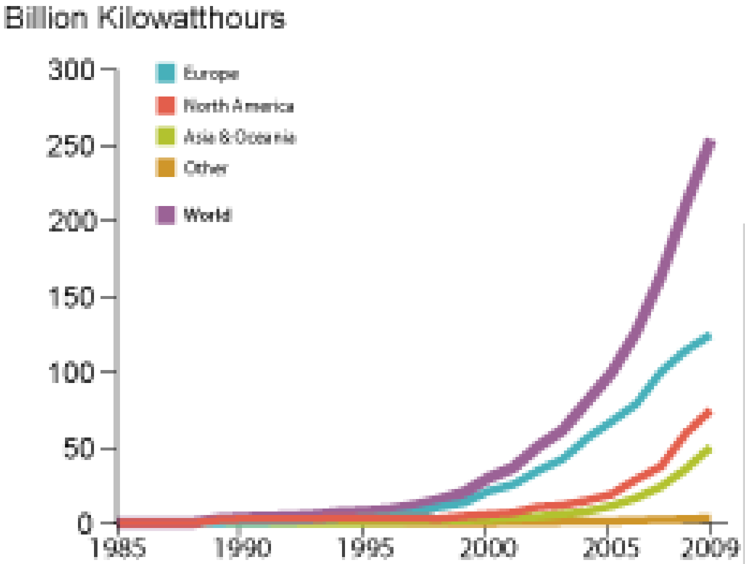
	Denmark	Germany	Netherlands	Spain	Sweden	Switzerland	United States	Reference Case
Unit size (MW)	2.3	2.0	3.0	2.0	2.4	2.0	1.7	2.1
Number of turbines	7	5	5	15	41	6	50	34
Full load hours	2,695	2,260	2,200	2,150	2,600	1,750	3,066	2,628
Investment (€/kW)	1,250	1,373	1,325	1,250	1,591	1,790	1,377	1,449
Decommissioning costs (€/kW)	0.0	1.5	0.0	0.0	1.6	0.0	0.0	0.6
Other costs (€/MWh)	3	0	10	3	0	0	0	1
O&M costs fixed (€/kW-yr)	0.00	46.33	31.39	0.00	0.01	0.00	8.60	6.29
O&M costs variable (€/MWh)	12	0	13	20	11	31	5	11
Converted total O&M costs (€/MWh)	12	21	28	20	11	31	7	13

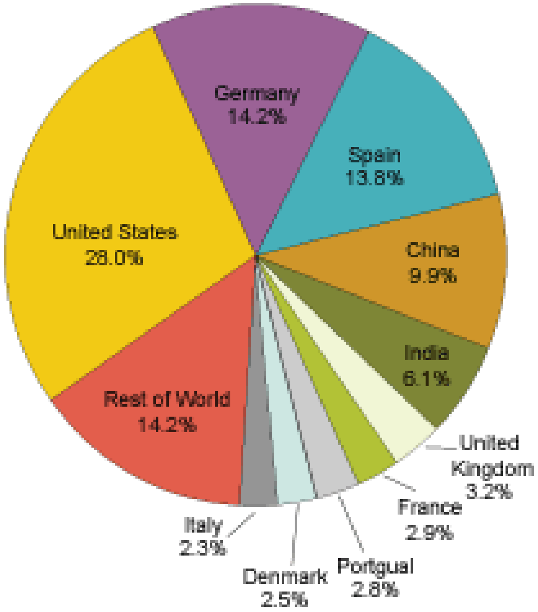
- US turbines are smaller but farms include more turbines with respect to reference case.
- US turbines are utilized most as indicated by full load hours.
- Investment costs are slightly less in the US.
- Accounting systems can complicate a direct comparison of O&M costs. Power or energy based? When they are converted for comparison, the US O&M costs are the lowest.

Wind Generated Electricity

- As of 2009, global wind generated electricity and various countries contributions are shown on the right.

- Denmark generates 20% of its electricity from wind. The country does not want to generate more from wind because of intermittency of wind.
- The same percentage for Portugal is 16%, Spain 13%, Ireland 11%, Germany 7%, USA 2%.
  - In March 2018, Portugal produced more renewable energy than its electric energy demand
  - Spanish utilities were required to pay a premium price for wind generated electricity. This feed-in-tariff encouraged the adoption of wind energy.
- Texas broke a wind record at 8:41 pm on March 7, 2012 by generating 7600 MW from wind which corresponded to 22% of the load 34,318 MW. On average, Texas generates 7-11% of its electricity from wind.



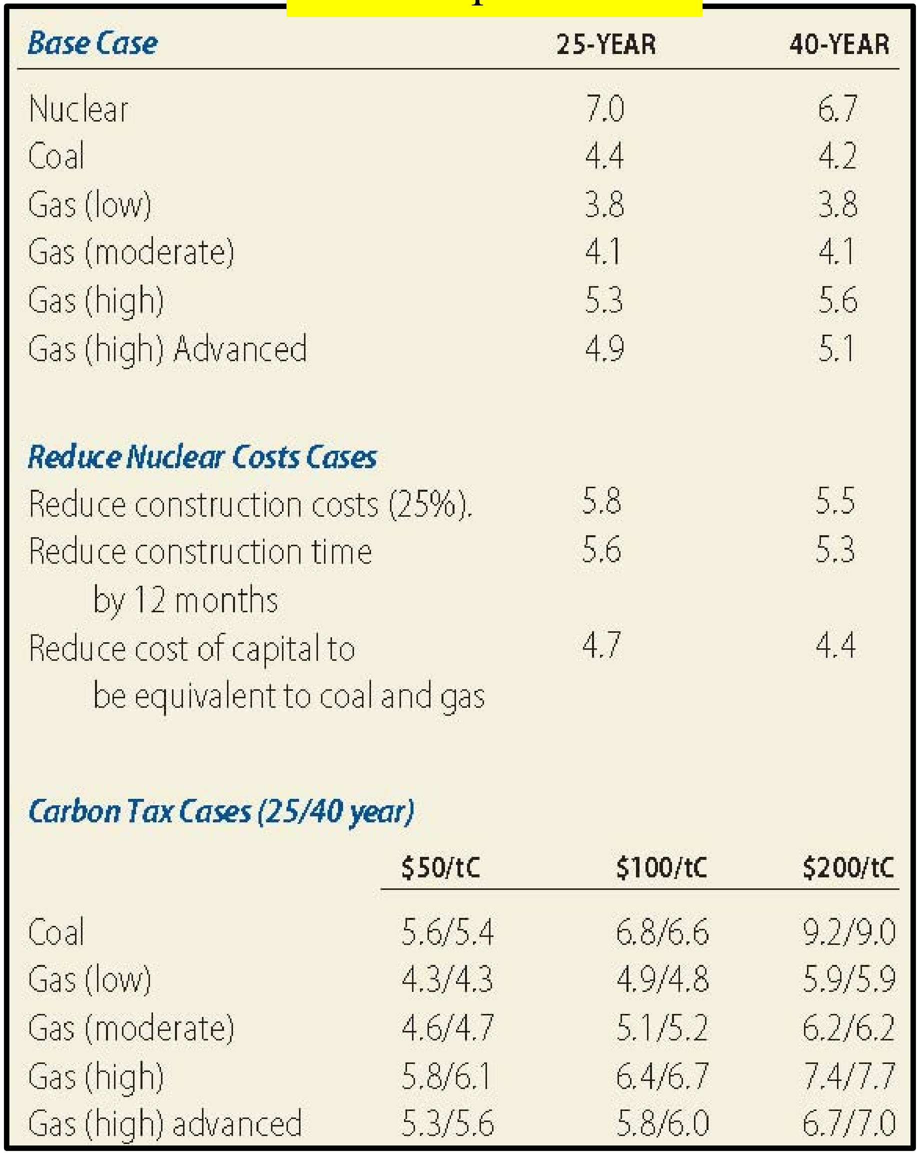


Source: www.eia.gov/energy_in_brief/wind_power.cfm

LCOE of Wind Power and Others

Country	LCOE Cents/kWh
Switzerland	16.7
Netherlands	13.1
Germany	11.8
Spain	11.5
Sweden	9.3
US	9.1
Denmark	8.5

Cents per kWh



Base Case	25-YEAR	40-YEAR
Nuclear	7.0	6.7
Coal	4.4	4.2
Gas (low)	3.8	3.8
Gas (moderate)	4.1	4.1
Gas (high)	5.3	5.6
Gas (high) Advanced	4.9	5.1
Reduce Nuclear Costs Cases		
Reduce construction costs (25%).	5.8	5.5
Reduce construction time by 12 months	5.6	5.3
Reduce cost of capital to be equivalent to coal and gas	4.7	4.4

Carbon Tax Cases (25/40 year)

	$50/tC	$100/tC	$200/tC
Coal	5.6/5.4	6.8/6.6	9.2/9.0
Gas (low)	4.3/4.3	4.9/4.8	5.9/5.9
Gas (moderate)	4.6/4.7	5.1/5.2	6.2/6.2
Gas (high)	5.8/6.1	6.4/6.7	7.4/7.7
Gas (high) advanced	5.3/5.6	5.8/6.0	6.7/7.0

- In US, Wind energy without subsidies is not cheaper than nuclear, coal or gas. But it can be when carbon taxes are factored in.
- Wind can provide a good alternative in other countries and some specific states such as Texas.

Summary

- Wind Turbine & Farms
- Physics: Generators
- Business
- Non-monetary issues

Based on

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Appendix: Annualization of Cost

- What is net present value (NPV) of fixed *n* payments of *x* under interest rate *i*?

- The NPV is $a := x(1+i)^{-1} + x(1+i)^{-2} + x(1+i)^{-3} + \cdots + x(1+i)^{-n}$
- Then $(1+\mathrm{i})a = x + x(1+i)^{-1} + x(1+i)^{-2} + \cdots + x(1+i)^{-n+1}$
- Hence, by differencing side by side: $ai = x - x(1+i)^{-n}$ or

$$a = x\frac{1-(1+i)^{-n}}{i}$$

- Finally, annualized cost of debt a is

$$x = a\frac{i}{1-(1+i)^{-n}}$$

Appendix: Meaning of a Moment

- A MERIT 2019 student asked: “*What is the meaning of moment when we talk of the first, second, third moment of the wind speed*?
- Formal answer: The first moment is the expected value of the wind speed, the second moment is the expected value of the square of speed, the third moment is the expected value of the cube
  - In general, for random variable V and integer k, the kth moment is $\mathrm{E}(\mathrm{V}^k)$
  - We can talk of the zeroth moment: $\mathrm{E}(V^0) = \mathrm{E}(1) = 1$
  - The concept of moment exists also in analysis but is applied to functions
    - For function f and integer k, the kth moment is $\int x^k f(x)dx$
  - The concept of moment of inertia exists in studying rotational motion in Physics
    - Moment of inertia of an object is defined with respect to a rotational axis and is proportional to the mass of the object and the square of distance of the object from the rotational axis.
    - In this course, we never talk about moment of inertia
  - Momentum is another concept in Physics but has no relation to moment
  - Also, moment is a short period of time in daily speech
- There also is “One Moment in Time” sang by W. Houston and associated with Olympics: https://www.youtube.com/watch?v=96aAx0kxVSA.
  - The song is about how important / meaningful a single moment can be.

What seems to be out of place in this picture?

Wind Turbines on Offshore Oil Rigs!

- Picture is from an offshore natural gas rig operated by Teikoku-Esso-Kokusai group 45 kms from mainland Japan.

- The rig has 21 small wind turbines that power a small grid on the rig.
  - Each turbine provides 1 kW
  - Uses rechargeable VRLA (valve-regulated lead-acid) batteries

- Reliable system that operated under hurricanes (wind speeds of 50 meters per second = 180 kms per hour = 110 miles per hour).

- This is an integrated application:
      - » Renewable Energy to extract Nonrenewable Energy.

http://www.uslbo.com/images/EnergyGeneration_x_OIL_RIGs.pdf