OPRE 6201 Final for Fall 2003

This is an open textbook and open lecture notes exam. You may use a calculator although leaving quantities in fractional forms is perfectly acceptable and preferable. Cellular communication devices (phones, laptops, palm pilots, etc.) **cannot** be used during this exam. **Do not forget** to define any variables you introduce. Unless otherwise stated, x_i always denotes the ith decision variable. Good luck!

NAME (please print): ______________________________

Question	Out of	Points
1	20	
2	20	
3	20	
4	20	
5	20	
	100	

1. Put **T** before a statement if you think that statement is true. Otherwise put **X**.

   1. (   ) A shortest path can visit a city twice if all the links have positive lengths. X
   2. (   ) In a network with 10 cities shortest path is found exactly when R contains all 10 cities. X
   3. (   ) At every iteration of the minimum spanning tree algorithm exactly one node is added to the set T. X
   4. (   ) Every LP with maximization objective and nonnegative variables has a feasible solution. X
   5. (   ) A basic variable must be positive. X
   6. (   ) In a balanced transportation problem, a demand constraint and a supply constraint are redundant. X
   7. (   ) We can always find a feasible solution for an Integer Program by rounding down the solution of the LP relaxation. X
   8. (   ) In a network with 10 nodes where the distance between any two nodes is 1, a shortest path has a length of 1. T
   9. (   ) Integer Programs help us to relax the divisibility assumption of LP. T
   10. (   ) If an integer program is feasible than its LP relaxation is also feasible. T

2. Suppose that you are moving to a new apartment and you have only two boxes to pack your stuff in. The first box carries 22 kg and the second carries 28 kg. The weight and the value of your belongings are:

Item	A	B	C	D	E	F	G	H
Weight (kg)	10	9	15	3	11	6	3	4
Value (dollar)	5	2	7	6	1	6	8	6

a) Provide a formulation to maximize the value of items you can carry with these two boxes.

Let $x_{j,1} = 1$ ($x_{j,2} = 1$) if we put item j into the first (second) box or 0 otherwise.

Max $5(x_{A,1} + x_{A,2}) + 2(x_{B,1} + x_{B,2}) + 7(x_{C,1} + x_{C,2}) + 6(x_{D,1} + x_{D,2}) + (x_{E,1} + x_{E,2}) + 6(x_{F,1} + x_{F,2}) + 8(x_{G,1} + x_{G,2}) + 6(x_{H,1} + x_{H,2})$
Subject to

$$10x_{A,1} + 9x_{B,1} + 15x_{C,1} + 3x_{D,1} + 11x_{E,1} + 6x_{F,1} + 3x_{G,1} + 4x_{H,1} \leq 22$$
$$10x_{A,2} + 9x_{B,2} + 15x_{C,2} + 3x_{D,2} + 11x_{E,2} + 6x_{F,2} + 3x_{G,2} + 4x_{H,2} \leq 28$$
$$x_{j,1} + x_{j,2} \leq 1 \text{ for } j = \{A, B, C, D, E, F, G, H\}.$$

The last constraint makes sure that one item is not put into two boxes at the same time.

b) Since the box capacity is limited, you can not fit all your items into boxes. Can you identify an item, which will always (in all optimal solutions) be put in one of the boxes? Justify your answer.

Item G has the minimum weight and the maximum value. We can prove that item G will always be put in one of the boxes by contradiction. Suppose that Item G is not in either box in a given solution. Then take out any item from box 1 (or box 2) and put item G instead. This increases the value of items inside the boxes without violating the weight constraints.

At the same time G has the highest value per weight. But this does not guarantee that it will be in the optimal solution. To illustrate consider the example below:

Item	1	2	3
Weight (kg)	6	5	5
Value (dollar)	7	5	5

Suppose that the total weight can at most be 10. Then the optimal solution is taking items 2 and 3 although 1 has the largest value/weight ratio. People who based their arguments only on ratios lose points here.

3. Consider the transportation tableau:

		Sinks 1	Sinks 2	Sinks 3	
Sources	1	$\boxed{3}$	$\boxed{2}$ 45	$\boxed{3}$	45
	2	$\boxed{1}$	$\boxed{5}$	$\boxed{0}$ 35	35
	3	$\boxed{5}$ 30	$\boxed{4}$ 5	$\boxed{1}$ 25	60
		30	50	60	

a) Obtain a bfs using the least cost method. Write your solution into the table above.

b) Find the reduced cost of **all** cells, use the bfs of a).

$\bar{c}_{1,1} = 3 - 2 + 4 - 5 = 0$, $\bar{c}_{1,2} = 0$, $\bar{c}_{1,3} = 3 - 1 + 4 - 2 = 4$
$\bar{c}_{2,1} = 1 - 0 + 1 - 5 = -3$, $\bar{c}_{2,2} = 5 - 0 + 1 - 4 = 2$, $\bar{c}_{2,3} = 0$
$\bar{c}_{3,1} = 0$, $\bar{c}_{3,2} = 0$, $\bar{c}_{3,3} = 0$

c) Do a pivot of transportation simplex starting with the bfs of a). Write the result into the table below:

(2,1) is entering cell at the value of $\min\{30, 35\} = 30$ forcing (3,1) to exit the basis. :

		Sinks 1	Sinks 2	Sinks 3	
Sources	1	$\boxed{3}$	$\boxed{2}$ 45	$\boxed{3}$	45
	2	$\boxed{1}$ 30	$\boxed{5}$	$\boxed{0}$ 5	35
	3	$\boxed{5}$	$\boxed{4}$ 5	$\boxed{1}$ 55	60
		30	50	60	

d) Is the bfs after the pivot optimal or multiple optimal, why?

$\bar{c}_{1,1} = 3 - 2 + 4 - 1 + 0 - 1 = 3$, $\bar{c}_{1,2} = 0$, $\bar{c}_{1,3} = 3 - 1 + 4 - 2 = 4$
$\bar{c}_{2,1} = 0$, $\bar{c}_{2,2} = 5 - 0 + 1 - 4 = 2$, $\bar{c}_{2,3} = 0$
$\bar{c}_{3,1} = 5 - 1 + 0 - 1 = 3$, $\bar{c}_{3,2} = 0$, $\bar{c}_{3,3} = 0$

Since all reduced costs are nonnegative, the bfs is optimal. No nonbasic variable has zero reduced cost so we have a single optimal bfs.

4. Consider the following LP: Max $4x_1 + 2x_3 - 2x_4$

Subject to

$$x_1 - 2x_3 + x_4 \leq 4$$
$$2x_1 + x_2 + x_3 - x_4 = 2$$
$$x_1, x_2, x_3 \geq 0, x_4 \text{ urs}$$

a) Introduce any additional variables necessary and put this LP into a standard tableau.

We need to introduce the slack s_1 into the first constraint. x_2 can work as slack in the second constraint. To handle urs x_4 let $x_4 = x_4^+ - x_4^-$. Then

	z	x_1	x_2	x_3	x_4^+	x_4^-	s_1	RHS
R_0 :	1	-4	0	-2	2	-2	0	0
R_1 :	0	1	0	-2	1	-1	1	4
R_2 :	0	2	1	1	-1	1	0	2

b) List the basic variables in your tableau and their values:

$s_1 = 4$, $x_2 = 2$.

c) Do a simplex iteration and obtain a new bfs if the current tableau is not optimal.

x_1 enters into the basis and x_2 leaves:

	z	x_1	x_2	x_3	x_4^+	x_4^-	s_1	RHS
R_0 :	1	0	2	0	0	0	0	4
R_1 :	0	0	-1/2	-5/2	3/2	-3/2	1	3
R_2 :	0	1	1/2	1/2	-1/2	1/2	0	1

d) Read the values of the current basic variables and the objective value from the current tableau. Is the current bfs optimal or are there multiple optimal solutions, why?

$x_1 = 1$, $s_1 = 3$ and $z = 4$. The LP has multiple optimal solutions because all R_0 entries are nonnegative and the nonbasic variable x_4^+ has a coefficient of zero.

5. PlaToy Company produces toys in Plano and Richardson. It has developed three new barbie dolls — Betsy, Vicky and Wendy — for possible inclusion in its product line for Xmas season. Setting up the production facilities to begin production would cost the same amount at both Plano and Richardson factories, see set up costs in dollars below. For administrative reasons, the same factory would be used for all new doll production: All dolls are produced at either Plano or Richardson Factories.

Regardless where they are produced, the contribution to margin for all the dolls are the same as given below. Also production rates at factories (units per hour) are also given below. Plano and Richardson Factories, respectively, have 300 hours and 200 hours of production time available before Xmas.

	Set up cost	Contribution to margin	Plano Factory Production rate	Richardson Factory Production rate
Betsy	50000	5	50	40
Vicky	60000	6	80	60
Wendy	40000	7	60	70

a) It is known that practically all barbies produced until Xmas can be sold. But these dolls will not be produced after Xmas. The problem is to determine how many units (if any) of each new toy should be produced before Xmas and at which factory to maximize the total profit. Formulate an MILP.

Let $y_{BP} = 1$ if Betsy is produced at Plano, 0 otherwise. Define binary variables $y_{BR}, y_{VP}, y_{VR}, y_{WP}, y_{WR}$ similarly. Let x_{BP} be the number Betsy produced at Plano. Define variables $x_{BR}, x_{VP}, x_{VR}, x_{WP}, x_{WR}$ similarly.

Maximize $5(x_{BP}+x_{BR})+6(x_{VP}+x_{VR})+7(x_{WP}+x_{WR})-50000(y_{BP}+y_{BR})-60000(y_{VP}+x_{VR})-40000(y_{WP}+y_{WR})$
ST:
$y_{BP}+y_{BR} \leq 1$; $y_{BP}+y_{VR} \leq 1$; $y_{BP}+y_{WR} \leq 1$.
$y_{VP}+y_{VR} \leq 1$; $y_{VP}+y_{BR} \leq 1$; $y_{VP}+y_{WR} \leq 1$.
$y_{WP}+y_{WR} \leq 1$; $y_{WP}+y_{BR} \leq 1$; $y_{WP}+y_{VR} \leq 1$.

These constraints say "the same factory would be used for all new doll production":
$y_{BP}=1 \Rightarrow y_{BR}=0, y_{VR}=0, y_{WR}=0$; $y_{BR}=1 \Rightarrow y_{BP}=0, y_{VP}=0, y_{WP}=0$;
$y_{VR}=1 \Rightarrow y_{BP}=0, y_{VP}=0, y_{WP}=0$; $y_{WR}=1 \Rightarrow y_{BP}=0, y_{VP}=0, y_{WP}=0$;
$y_{VP}=1 \Rightarrow y_{VR}=0, y_{BR}=0, y_{WR}=0$; $y_{WP}=1 \Rightarrow y_{WR}=0, y_{BR}=0, y_{VR}=0$.

$x_{BP}/50+x_{VP}/80+x_{WP}/60 \leq 300$; $x_{BR}/40+x_{VR}/60+x_{WR}/70 \leq 200$.

These constraints say that no more than available hours used for production.

$x_{BP} \leq 300(50)y_{BP}$; $x_{VP} \leq 300(80)y_{VP}$; $x_{WP} \leq 300(60)y_{WP}$;
$x_{BR} \leq 200(40)y_{BR}$; $x_{VR} \leq 200(60)y_{VR}$; $x_{WR} \leq 200(70)y_{WR}$;

These constraints say that production can take place if the factory is set up.

b) Suppose that different dolls can be produced at different factories but each doll is produced at only one factory. However, there is an additional \$10000 administrative cost incurred at a factory if any one of the new dolls are produced at the factory. Modify your MILP.

Let z_P be 1 if any one of the new dolls is produced at Plano. Similarly define z_R.

Subtract $10000(z_P+z_R)$ from the objective function.

Add constraints $y_{BP}+y_{VP}+y_{WP} \leq 3z_P$; $y_{BR}+y_{VR}+y_{WR} \leq 3z_R$.

Remove the first nine constraints from the formuolation in a) except for $y_{BP}+y_{BR} \leq 1$; $y_{VP}+y_{VR} \leq 1$; $y_{WP}+y_{WR} \leq 1$.