Newsvendor Model

Chapter 11

These slides are based in part on slides that come with Cachon & Terwiesch book *Matching Supply with Demand* http://cachon-terwiesch.net/3e/. If you want to use these in your course, you may have to adopt the book as a textbook or obtain permission from the authors Cachon & Terwiesch.

Learning Goals

- Determine the optimal level of product availability
  - Demand forecasting
  - Profit maximization
- Service measures such as a fill rate

Motivation

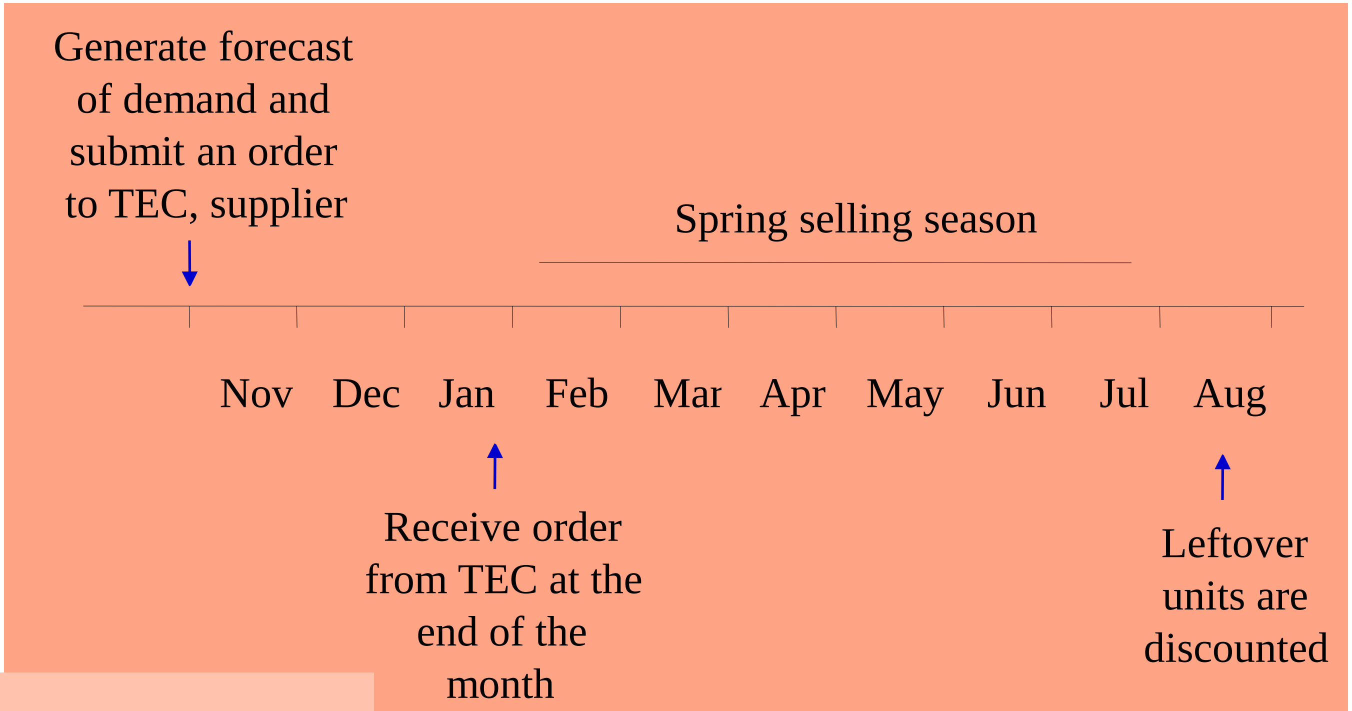
- **Determining optimal levels (purchase orders)**
  - **Single order (purchase) in a season**
  - **Short lifecycle items**
    - **1 month: Printed Calendars, Rediform**
    - **6 months: Seasonal Camera, Panasonic**
    - **18 months, Cell phone, Nokia**
    - **Motivating Newspaper Article for toy manufacturer Mattel**

> Mattel [*who introduced Barbie in 1959 and run a stock out for several years then on*] was hurt last year by inventory cutbacks at Toys "R" Us, and officials are also eager to avoid a repeat of the 1998 Thanksgiving weekend. Mattel had expected to ship a lot of merchandise after the weekend, but retailers, **wary of excess inventory**, stopped ordering from Mattel. That led the company to report a **$500 million sales shortfall in the last weeks of the year** ... For the crucial holiday selling season this year, Mattel said it will require retailers to place their **full orders before Thanksgiving**. And, for the first time, the company will no longer take reorders in December, Ms. Barad said. This will enable Mattel to tailor production more closely to demand and avoid building inventory for orders that don't come.
>
> - Wall Street Journal, Feb. 18, 1999

- For tax (in accounting), option pricing (in finance) and revenue management applications see newsvendorEx.pdf, basestcokEx.pdf and revenueEx.pdf.

O'Neill's Hammer 3/2 wetsuit

Hammer 3/2 timeline and economics



Economics:

Each suit sells for p = \$180

TEC charges c = \$110/suit

Discounted suits sell for v = \$90

The “too much/too little problem”:

- Order too much and inventory is left over at the end of the season
- Order too little and sales are lost.
- Marketing’s forecast for sales is 3200 units.

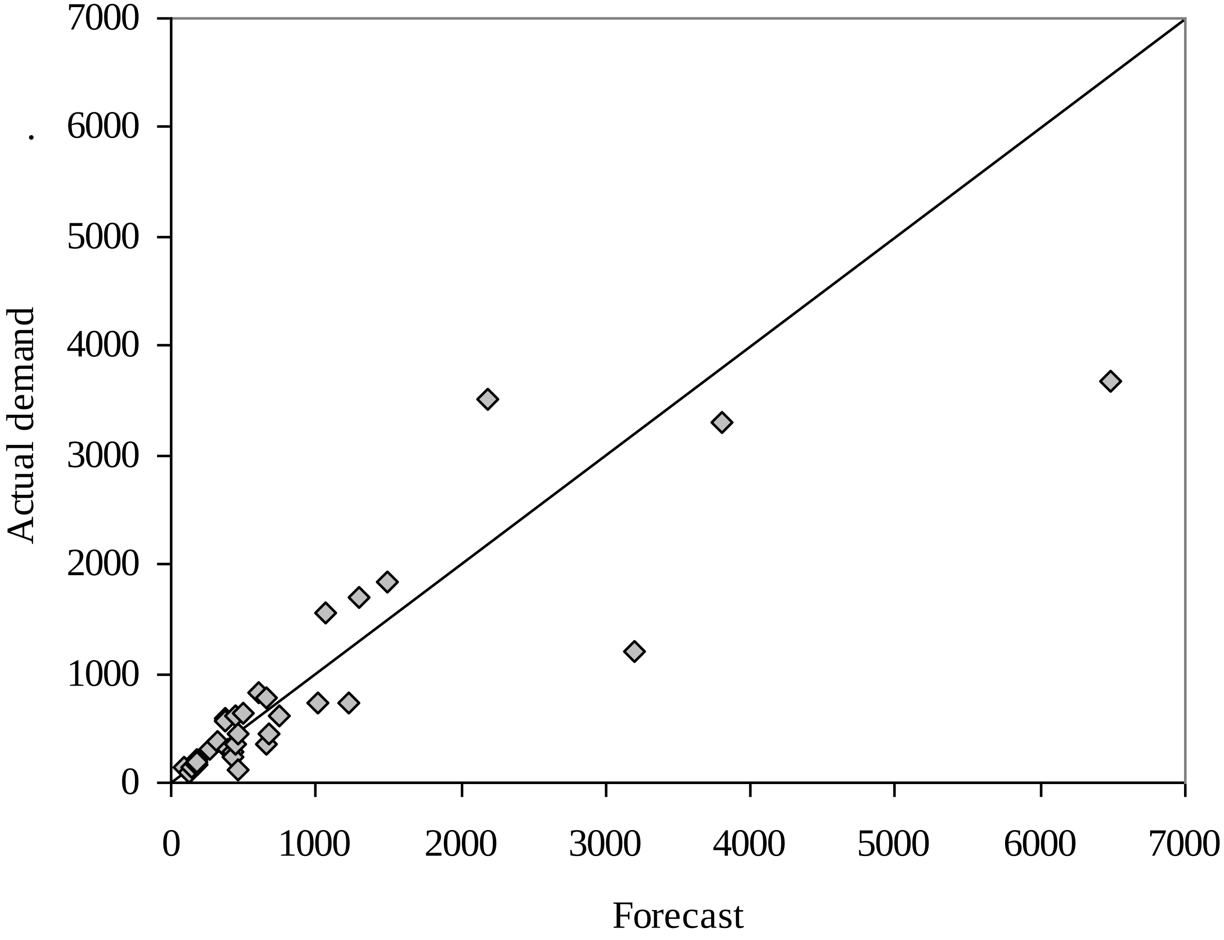
Newsvendor model implementation steps

1. Gather economic inputs:
   a) selling price,
   b) production/procurement cost,
   c) salvage value of inventory
2. Generate a demand model to represent demand
   a) Use empirical demand distribution
   b) Choose a standard distribution function: the normal distribution and the Poisson distribution – for discrete items
3. Choose an aim:
   a) maximize the objective of expected profit
   b) satisfy a fill rate constraint.
4. Choose a quantity to order.

The Newsvendor Model:

Develop a Forecast

Historical forecast performance at O'Neill



Forecasts and actual demand for surf wet-suits from the previous season

How do we know the actual when the actual demand > forecast demand

Are the number of stockout units (= unmet demand=demand-stock) observable, i.e., known to the store manager?

- Yes, if the store manager issues rain checks to customers.
- No, if the stockout demand disappears silently.
  - A vicious cycle

    Underestimate the demand → Stock less than necessary.

    Stocking less than the demand → Stockouts and lower sales.

    Lower sales → Underestimate the demand.
  - Demand filtering: Demand known exactly only when below the stock.
  - Shall we order more than optimal to learn about demand?
- Yes and no, if some customers complain about a stockout; see next page.

Observing a Portion of Unmet Demand

- Unmet demand are reported by partners (sales associates)
- Reported lost sales are based on customer complaints

Not everybody complains of a stock out,
Not every sales associate records complaints,
Not every complaint is reported,
Only a portion of complaints are observed by IM

??

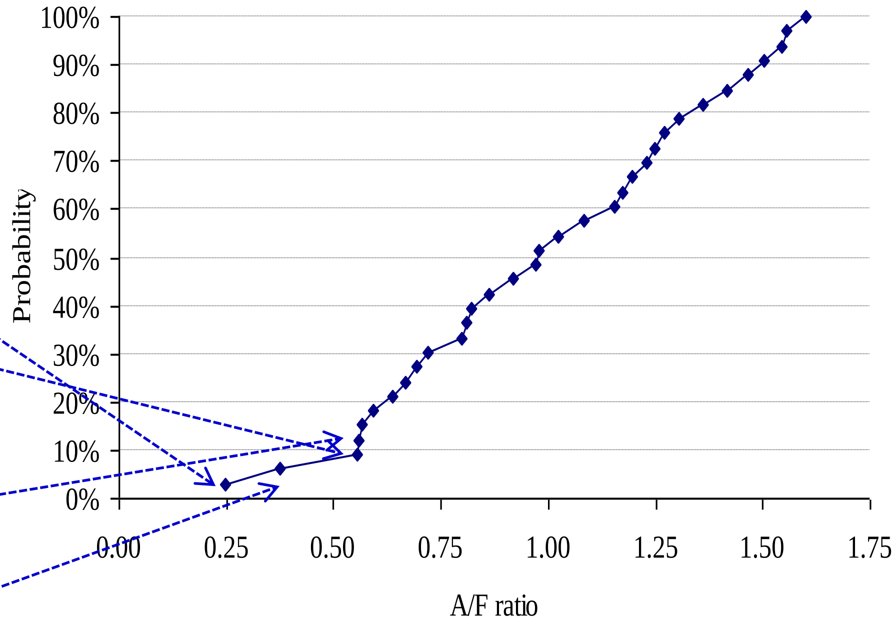
Empirical distribution of forecast accuracy
Order by A/F ratio

Product description	Forecast	Actual demand	Error*	A/F Ratio**
JR ZEN FL 3/2	90	140	-50	1.56
EPIC 5/3 W/HD	120	83	37	0.69
JR ZEN 3/2	140	143	-3	1.02
WMS ZEN-ZIP 4/3	170	163	7	0.96
HEATWAVE 3/2	170	212	-42	1.25
JR EPIC 3/2	180	175	5	0.97
WMS ZEN 3/2	180	195	-15	1.08
ZEN-ZIP 5/4/3 W/HOOD	270	317	-47	1.17
WMS EPIC 5/3 W/HD	320	369	-49	1.15
EVO 3/2	380	587	-207	1.54
JR EPIC 4/3	380	571	-191	1.50
WMS EPIC 2MM FULL	390	311	79	0.80
HEATWAVE 4/3	430	274	156	0.64
ZEN 4/3	430	239	191	0.56
EVO 4/3	440	623	-183	1.42
ZEN FL 3/2	450	365	85	0.81
HEAT 4/3	460	450	10	0.98
ZEN-ZIP 2MM FULL	470	116	354	0.25
HEAT 3/2	500	635	-135	1.27
WMS EPIC 3/2	610	830	-220	1.36
WMS ELITE 3/2	650	364	286	0.56
ZEN-ZIP 3/2	660	788	-128	1.19
ZEN 2MM S/S FULL	680	453	227	0.67
EPIC 2MM S/S FULL	740	607	133	0.82
EPIC 4/3	1020	732	288	0.72
WMS EPIC 4/3	1060	1552	-492	1.46
JR HAMMER 3/2	1220	721	499	0.59
HAMMER 3/2	1300	1696	-396	1.30
HAMMER S/S FULL	1490	1832	-342	1.23
EPIC 3/2	2190	3504	-1314	1.60
ZEN 3/2	3190	1195	1995	0.37
ZEN-ZIP 4/3	3810	3289	521	0.86
WMS HAMMER 3/2 FULL	6490	3673	2817	0.57

* Error = Forecast - Actual demand

** A/F Ratio = Actual demand divided by Forecast

33 products, so increment probability by 3%.



Empirical distribution function for the historical A/F ratios.

Normal distribution tutorial

- All normal distributions are specified by 2 parameters, mean = μ and st_dev = σ.
- Each normal distribution is related to the standard normal that has mean = 0 and st_dev = 1.
- For example:
  - Let Q be the order quantity, and (μ, σ) the parameters of the normal demand forecast.
  - *Prob*{demand is Q or lower} = *Prob*{the outcome of a standard normal is z or lower}, where

$$z = \frac{Q-\mu}{\sigma} \quad \text{or} \quad Q = \mu + z \times \sigma$$

  - The above are two ways to write the same equation, the first allows you to calculate z from Q and the second lets you calculate Q from z.
  - Look up *Prob*{the outcome of a standard normal is z or lower} in the

Standard Normal Distribution Function Table.

Using historical A/F ratios to choose a Normal distribution for the demand forecast

- Start with an initial forecast generated from hunches, guesses, etc.
  - O'Neill's initial forecast for the Hammer 3/2 = 3200 units.
- Evaluate the A/F ratios of the historical data:

$$A/F\ ratio = \frac{Actual\ demand}{Forecast}$$

- Set the mean of the normal distribution to

$$Expected\ actual\ demand = Expected\ A/F\ ratio \times Forecast$$

- Set the standard deviation of the normal distribution to

$$\begin{aligned} &Standard\ deviation\ of\ actual\ demand = \\ &\quad Standard\ deviation\ of\ A/F\ ratios \times Forecast \end{aligned}$$

O'Neill's Hammer 3/2 normal distribution forecast

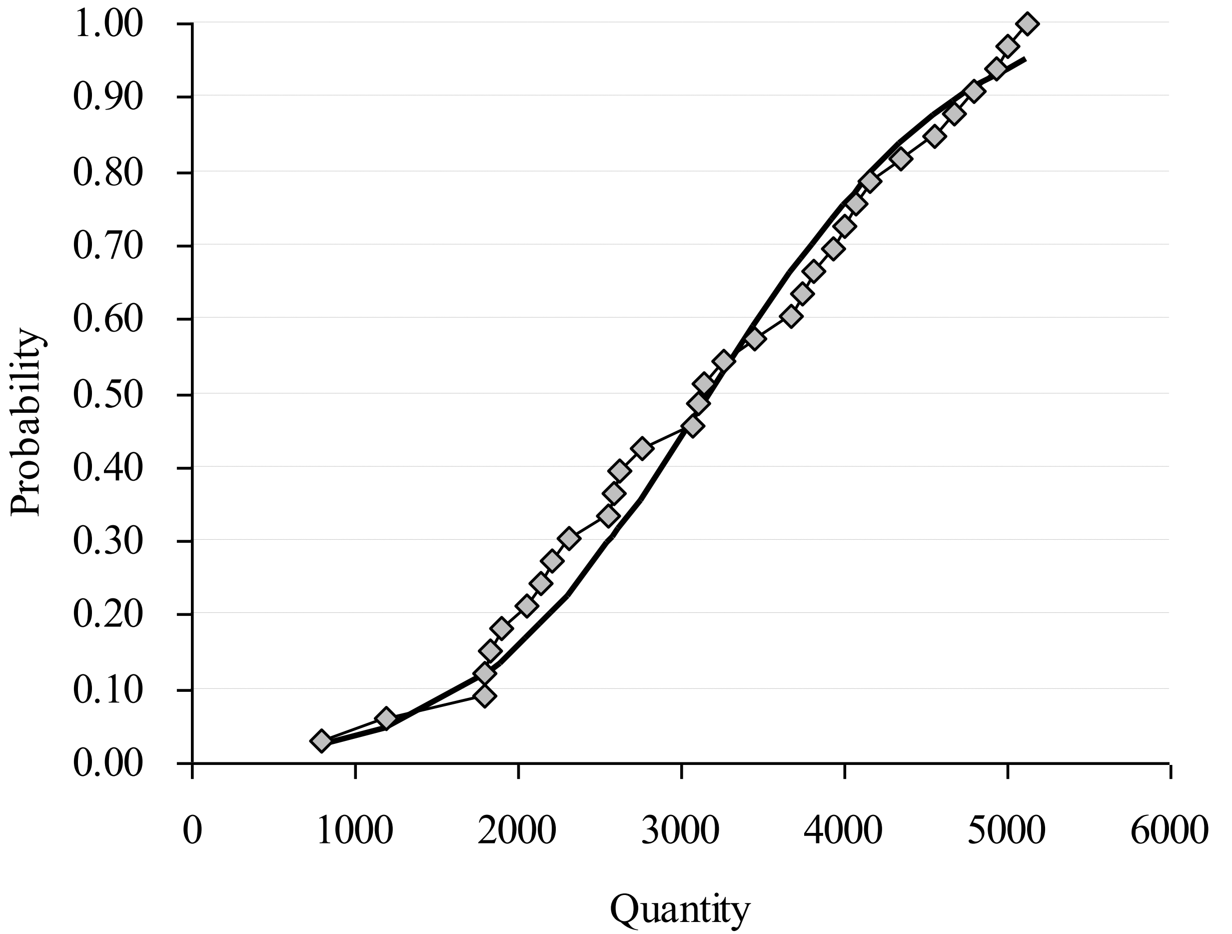
Product description	Forecast	Actual demand	Error	A/F Ratio
JR ZEN FL 3/2	90	140	-50	1.5556
EPIC 5/3 W/HD	120	83	37	0.6917
JR ZEN 3/2	140	143	-3	1.0214
WMS ZEN-ZIP 4/3	170	156	14	0.9176
• • •	• • •	• • •	• • •	• • •
ZEN 3/2	3190	1195	1995	0.3746
ZEN-ZIP 4/3	3810	3289	521	0.8633
WMS HAMMER 3/2 FULL	6490	3673	2817	0.5659
Average				0.9975
Standard deviation				0.3690

$$\text{Expected actual demand} = 0.9975 \times 3200 = 3192$$

$$\text{Standard deviation of actual demand} = 0.369 \times 3200 = 1181$$

- Choose a normal distribution with mean 3192 and st_dev 1181 to represent demand for the Hammer 3/2 during the Spring season.
- Why not a mean of 3200?

Fitting Demand Distributions: Empirical vs normal demand distribution



Empirical distribution function (diamonds) and normal distribution function with mean 3192 and standard deviation 1181 (solid line)

An Example of Empirical Demand: Demand for Candy (in the Office Candy Jar)

- An OPRE 6302 instructor believes that passing out candies (candies, chocolate, cookies) in a late evening class builds morale and spirit.
- This belief is shared by office workers as well. For example, secretaries keep office candy jars, which are irresistible:

  "… 4-week study involved the chocolate candy consumption of 40 adult secretaries. The study utilized a 2x2 within-subject design where candy proximity was crossed with visibility. Proximity was manipulated by placing the chocolates on the desk of the participant or 2 m from the desk. Visibility was manipulated by placing the chocolates in covered bowls that were either clear or opaque. Chocolates were replenished each evening. "

  People ate an average of 2.2 more candies each day when they were visible, and 1.8 candies more when they were proximately placed on their desk vs 2 m away." They ate 3.1 candies/day when candies were in an opaque container.
- Candy demand is fueled by the proximity and visibility.
- What fuels the candy demand in the OPRE 6302 class?

- What undercuts the demand? Hint: The aforementioned study is titled "The office candy dish: proximity's influence on estimated and actual consumption" and published in *International Journal of Obesity* (2006) 30: 871–875.

The Newsvendor Model:

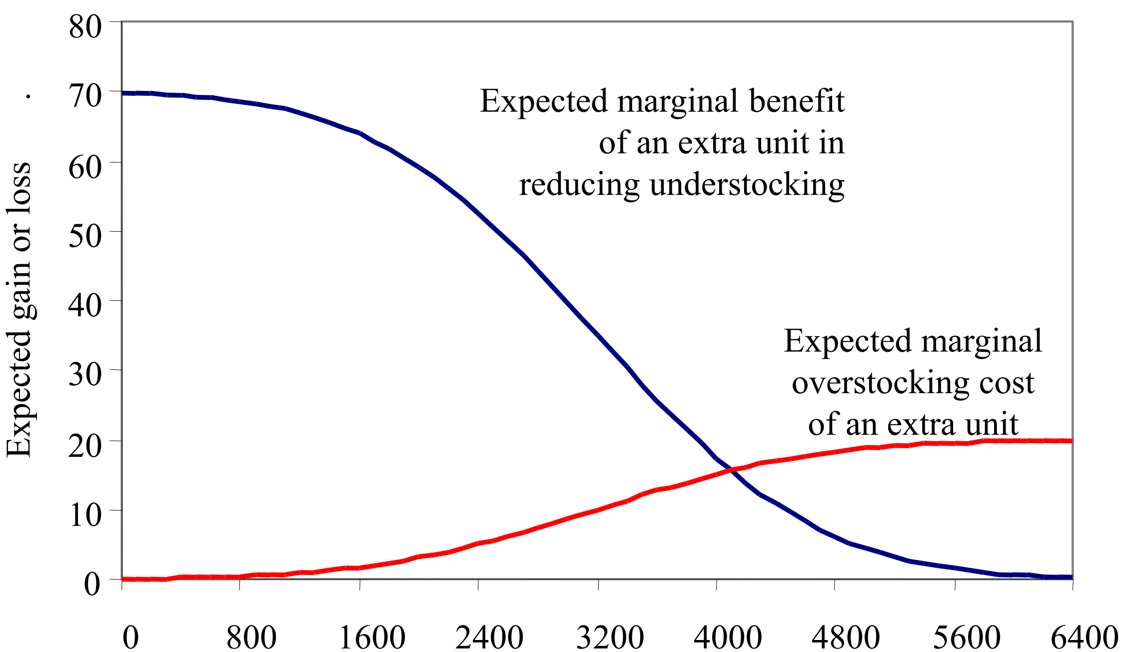
The order quantity that maximizes expected profit

“Too much” and “too little” costs

- C_o = overage cost
  - The cost of ordering one more unit than what you would have ordered had you known demand.
  - In other words, suppose you had left over inventory (i.e., you over ordered). C_o is the increase in profit you would have enjoyed had you ordered one fewer unit.
  - For the Hammer $C_o = Cost - Salvage\ value = c - v = 110 - 90 = 20$
- C_u = underage cost
  - The cost of ordering one fewer unit than what you would have ordered had you known demand.
  - In other words, suppose you had lost sales (i.e., you under ordered). C_u is the increase in profit you would have enjoyed had you ordered one more unit.
  - For the Hammer $C_u = Price - Cost = p - c = 180 - 110 = 70$

Balancing the risk and benefit of ordering a unit

- Ordering one more unit increases the chance of overage
  - Probability of overage $F(Q) = Prob\{Demand \le Q\}$
  - Expected loss on the Q^{th} unit = C_o x $F(Q)$ = "Marginal cost of overstocking"
- The benefit of ordering one more unit is the reduction in the chance of underage
  - Probability of underage $1-F(Q)$
  - Expected benefit on the Q^{th} unit = C_u x $(1-F(Q))$ = "Marginal benefit of understocking"



As more units are ordered,

- the expected marginal benefit from ordering 1 more unit decreases
- while the expected marginal cost of ordering 1 more unit increases.

Expected profit maximizing order quantity

- To minimize the expected total cost of underage and overage, order Q units so that the expected marginal cost with the Q^{th} unit equals the expected marginal benefit with the Q^{th} unit:

$$C_o \times F(Q) = C_u \times (1 - F(Q))$$

- Rearrange terms in the above equation → $F(Q) = \frac{C_u}{C_o + C_u}$

- The ratio $C_u \,/\, (C_o + C_u)$ is called the *critical ratio*.
- Hence, to minimize the expected total cost of underage and overage, choose Q such that we do not have lost sales (i.e., demand is Q or lower) with a probability that equals to the critical ratio

Expected cost minimizing order quantity with the empirical distribution function

- Inputs: Empirical distribution function table; p = 180; c = 110; v = 90; C_u = 180-110 = 70; C_o = 110-90 =20
- Evaluate the critical ratio:

$$\frac{C_u}{C_o + C_u} = \frac{70}{20+70} = 0.7778$$

- Look up 0.7778 in the empirical distribution function graph
- Or, look up 0.7778 among the ratios:
  - If the critical ratio falls between two values in the table, choose the one that leads to the *greater* order quantity

Product description	Forecast	Actual demand	A/F Ratio	Rank	Percentile
⋮	⋮	⋮	⋮	⋮	⋮
HEATWAVE 3/2	170	212	1.25	24	72.7%
HEAT 3/2	500	635	1.27	25	75.8%
HAMMER 3/2	1300	1696	1.30	26	78.8%
⋮	⋮	⋮	⋮	⋮	⋮

  - Convert A/F ratio into the order quantity

$$Q = Forecast * A/F = 3200 * 1.3 = 4160.$$

Expected cost minimizing order quantity with the normal distribution

- Inputs: p = 180; c = 110; v = 90; C_u = 180-110 = 70; C_o = 110-90 =20; critical ratio = 0.7778; mean = μ = 3192; standard deviation = σ = 1181
- Look up critical ratio in the Standard Normal Distribution Function Table:

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389

  – If the critical ratio falls between two values in the table, choose the greater z-statistic
  – Choose z = 0.77
- Convert the z-statistic into an order quantity:

$$Q = \mu + z \times \sigma$$
$$= 3192 + 0.77 \times 1181 = 4101$$

- Or, Q = norminv(0.778,3192,1181) =3192+1181norminv(0.778,0,1) = 4096

Another Example: Apparel Industry
How many L.L. Bean Parkas to order?

Demand data / distribution

Demand D_i	Probability of demand being this size	Cumulative Probability of demand being this size or less, $F(.)$	Probability of demand greater than this size, $1-F(.)$
4	.01	.01	.99
5	.02	.03	.97
6	.04	.07	.93
7	.08	.15	.85
8	.09	.24	.76
9	.11	.35	.65
10	.16	.51	.49
11	.20	.71	.29
12	.11	.82	.18
13	.10	.92	.08
14	.04	.96	.04
15	.02	.98	.02
16	.01	.99	.01
17	.01	1.00	.00

Expected demand is 1,026 parkas,
order 1026 parkas regardless of costs?

Cost/Profit data

Cost per parka = c = $45
Sale price per parka = p = $100
Discount price per parka = $50
Holding and transportation cost = $10
Salvage value per parka = v = 50-10=$40

Profit from selling parka = p-c = 100-45 = $55
Cost of overstocking = c-v = 45-40 = $5

Had the costs and demand been symmetric, we would have ordered the average demand.

Cost of understocking=$55
Cost of overstocking=$5
Costs are almost always antisymmetric.

Demand is sometimes antisymmetric

Optimal Order Q*

p = sale price; *v* = outlet or salvage price; *c* = purchase price

***F*(*Q*)=*CSL* = In-stock probability = Cycle Service Level**

= Probability that demand will be at or below reorder point

Raising the order size if the order size is already optimal

Expected Marginal Benefit =

=P(Demand is above stock)*(Profit from sales)=(1-CSL)(p - c)

Expected Marginal Cost =

=P(Demand is below stock)*(Loss from discounting)=CSL(c - v)

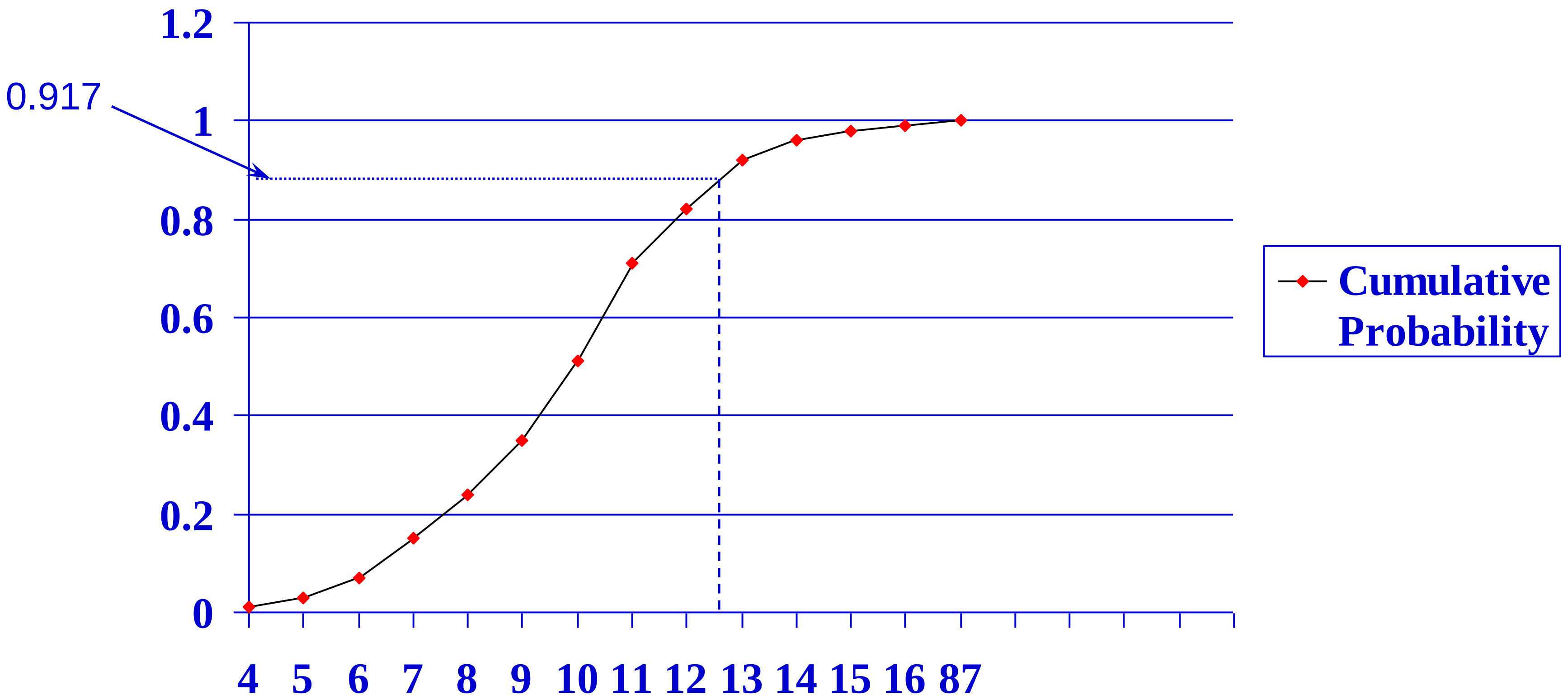
Define C_o= c-v=overstocking cost; C_u=p-c=understocking cost

$$\mathbf{(1\text{-}CSL)C_u = CSL\ C_o}$$

$$\mathbf{CSL = C_u / (C_u + C_o)}$$

$$\mathrm{CSL} = F(Q^*) = \mathrm{P}(\mathrm{Demand} \le Q^*) \ge \frac{C_u}{C_u + C_o} = \frac{55}{55+5} = 0.917$$

Optimal Order Quantity



Optimal Order Quantity = 13('00)

Marginal Profits at L.L. Bean

Approximate additional (marginal) expected profit from ordering 1('00) extra parkas if 10('00) are already ordered

=(55.P(D>1000) - 5.P(D≤1000)) 100=(55.(0.49) - 5.(0.51)) 100 =2440

Approximate additional (marginal) expected profit from ordering 1('00) extra parkas if 11('00) are already ordered

=(55.P(D>1100) - 5.P(D≤1100)) 100=(55.(0.29) - 5.(0.71)) 100 =1240

Additional 100s	Expected Marginal Benefit	Expected Marginal Cost	Expected Marginal Contribution
10→11	5500×.49 = 2695	500×.51 = 255	2695-255 = 2440
11→12	5500×.29 = 1595	500×.71 = 355	1595-355 = 1240
12→13	**5500×.18 = 990**	**500×.82 = 410**	**990-410 = 580**
13→14	5500×.08 = 440	500×.92 = 460	440-460 = -20
14→15	5500×.04 = 220	500×.96 = 480	220-480 = -260
15→16	5500×.02 = 110	500×.98 = 490	110-490 = -380
16→17	5500×.01 = 55	500×.99 = 495	55-495 = -440

Revisit Newsvendor Problem with Calculus

- Total cost by ordering Q units:

  C(Q) = overstocking cost + understocking cost

$$C(Q) = C_o \int_0^Q (Q-x) f(x)dx + C_u \int_Q^\infty (x-Q) f(x)dx$$

$$\frac{dC(Q)}{dQ} = C_o F(Q) - C_u (1 - F(Q)) = F(Q)(C_o + C_u) - C_u = 0$$

Marginal cost of raising Q* - Marginal cost of decreasing Q* = 0

$$F(Q^*) = P(D \le Q^*) = \frac{C_u}{C_o + C_u}$$

Safety Stock

Inventory held in addition to the expected demand is called the safety stock

The expected demand is 1026 parkas but

we order 1300 parkas.

So the safety stock is 1300-1026=274 parkas.

The Newsvendor Model:

Performance measures

Newsvendor model performance measures

- For any order quantity we would like to evaluate the following performance measures:
  - *Expected lost sales*
    - » The average number of demand units that exceed the order quantity
  - *Expected sales*
    - » The average number of units sold.
  - *Expected left over inventory*
    - » The average number of inventory units that exceed the demand
  - *Expected profit*
  - *Fill rate*
    - » The fraction of demand that is satisfied immediately from the stock (no backorder)
  - *In-stock probability*
    - » Probability all demand is satisfied
  - *Stockout probability*
    - » Probability that some demand is lost (unmet)

Expected (lost sales=shortage)

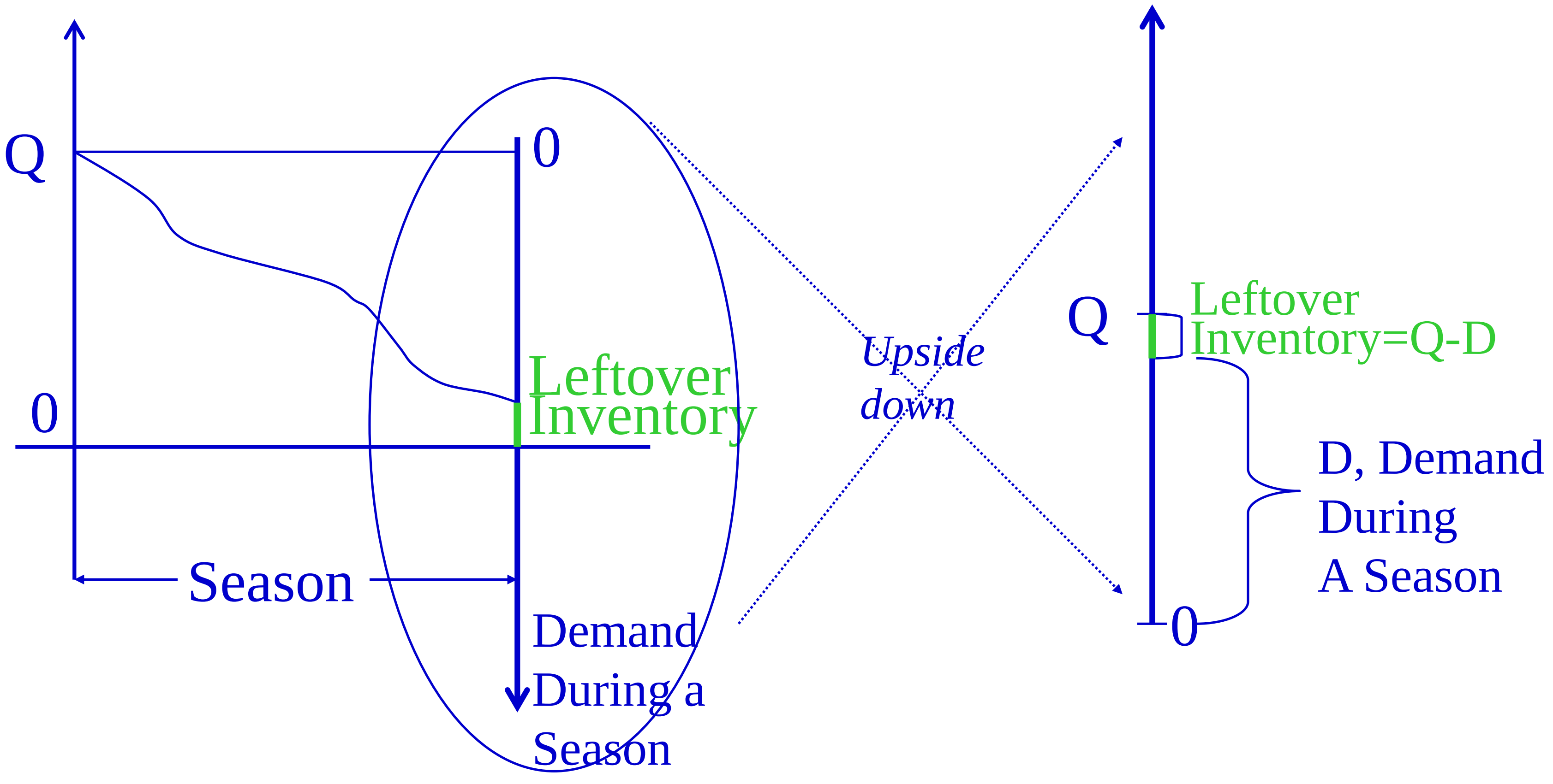
- **ESC is the expected shortage in a season (cycle)**
- **ESC is not a percentage, it is the number of units,** **also see next page**

$$\text{Shortage} = \begin{cases} \text{Demand - Q} & \text{if} \quad \text{Demand} \geq \text{Q} \\ 0 & \text{if} \quad \text{Demand} \leq \text{Q} \end{cases}$$

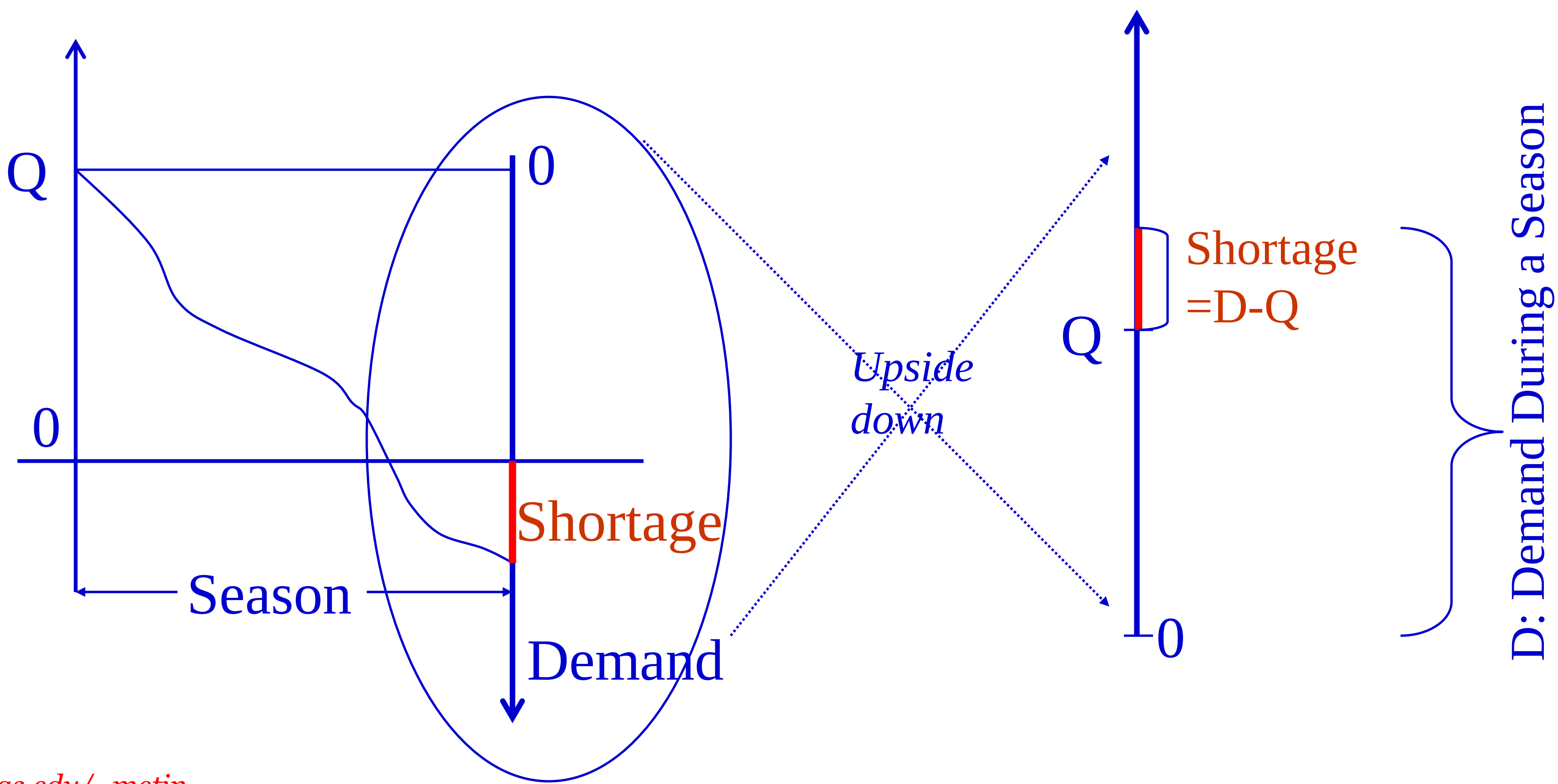
$$\text{ESC} = \text{E}(\max\{\text{Demand in a season} - \text{Q}, 0\})$$

$$\text{ESC} = \int_{x=Q}^{\infty} (x - Q) f(x) dx, \quad f \text{ is the probability density of demand.}$$

Inventory and Demand during a season
Leftover inventory



Inventory and Demand during a season Shortage



Expected shortage during a season

$$\text{Expected shortage} = E(\max\{D-Q, 0\})$$
$$= \sum_{d=Q+1}^{\infty} (D-Q)P(D=d)$$

- Ex:

$$Q=10,\ D=\begin{Bmatrix} d_1 = 9 \ \text{ with prob } p_1 = 1/4 \\ d_2 = 10 \ \text{ with prob } p_2 = 2/4 \\ d_3 = 11 \ \text{ with prob } p_3 = 1/4 \end{Bmatrix},\quad \text{Expected Shortage?}$$

$$\text{Expected shortage} = \sum_{\mathrm{i}=1}^{3} \max\{0, (d_i - Q)\} p_i = \sum_{d=11}^{11} (d-10)\} P(D=d)$$
$$= \max\{0, (9-10)\}\frac{1}{4} + \max\{0, (10-10)\}\frac{2}{4} + \max\{0, (11-10)\}\frac{1}{4} = \frac{1}{4}$$

Expected shortage during a season

$$\text{Expected shortage} = E(\max\{D-Q,0\})$$

$$= \int_{D=Q}^{\infty} (D-Q) f(D) dD \quad \text{where f is pdf of Demand.}$$

- Ex:

$Q=10,\ D=Uniform(6,12),\quad$ Expected Shortage?

$$\text{Expected shortage} = \int_{D=10}^{12} (D-10)\frac{1}{6}dD = \frac{1}{6}\left(\frac{D^2}{2}-10D\right)\Bigg|_{D=10}^{D=12} = \frac{1}{6}\left(\frac{12^2}{2}-10(12)\right)-\frac{1}{6}\left(\frac{10^2}{2}-10(10)\right)$$

$$=2/6$$

Expected lost sales of Hammer 3/2s with *Q* = 3500

Normal demand with mean 3192, standard deviation=1181

- Step 1: normalize the order quantity to find its *z*-statistic.

$$z = \frac{Q-\mu}{\sigma} = \frac{3500-3192}{1181} = 0.26$$

- Step 2: Look up in the Standard Normal Loss Function Table the expected lost sales for a standard normal distribution with that *z*-statistic: *L*(0.26)=0.2824 see textbook Appendix B Loss Function Table.
  - » or, in Excel L(z)=normdist(z,0,1,0)-z*(1-normdist(z,0,1,1)) see textbook Appendix D.

- Step 3: Evaluate lost sales for the actual normal distribution:

$$Expected\ \ lost\ sales\ \ = \ \sigma \times L(z) = 1181 \times 0.2824 = 334$$

Keep 334 units in mind, we shall repeatedly use it

The Newsvendor Model:

Cycle service level and fill rate

Type I service measure: Instock probability = CSL Cycle service level

Instock probability: percentage of seasons without a stock out

For example consider 10 seasons:

$$\text{Instock Probability} = \frac{1+1+0+1+1+1+0+1+0+1}{10}$$

Write 0 if a season has stockout, 1 otherwise

$$\text{Instock Probability} = 0.7$$

$$\text{Instock Probability} = 0.7 = \text{Probability that a single season has sufficient inventory}$$

$$[\text{Sufficient inventory}] = [\text{Demand during a season} \leq Q]$$

$$\boxed{\text{Instock Probability} = P(\text{Demand} \leq Q)}$$

Instock Probability with Normal Demand

N(μ,σ) denotes a normal demand with mean μ and standard deviation σ

$$
\begin{aligned}
&P\left(N(\mu,\sigma) \le Q\right) \\
&\quad = \text{Normdist}(\text{Q}, \mu, \sigma, 1) \\
&\quad = P\left(N(\mu,\sigma) - \mu \le Q - \mu\right) \quad \text{Taking out the mean} \\
&\quad = P\left(\frac{N(\mu,\sigma) - \mu}{\sigma} \le \frac{Q - \mu}{\sigma}\right) \quad \text{Dividing by the StDev} \\
&\quad = P\left(N(0,1) \le \frac{Q - \mu}{\sigma}\right) \quad \text{Obtaining standard normal distribution} \\
&\quad = \text{Normdist}\left(\frac{Q - \mu}{\sigma}, 0, 1, 1\right)
\end{aligned}
$$

Example: Finding Instock probability for given Q

μ = 2,500; σ= 500; Q = 3,000;

Instock probability if demand is Normal?

Instock probability = Normdist((3,000-2,500)/500,0,1,1)

Example: Finding Q for given Instock probability

μ = 2,500/week; σ= 500;

To achieve Instock Probability=0.95, what should Q be?

Q = Norminv(0.95, 2500, 500)

Type II Service measure
Fill rate

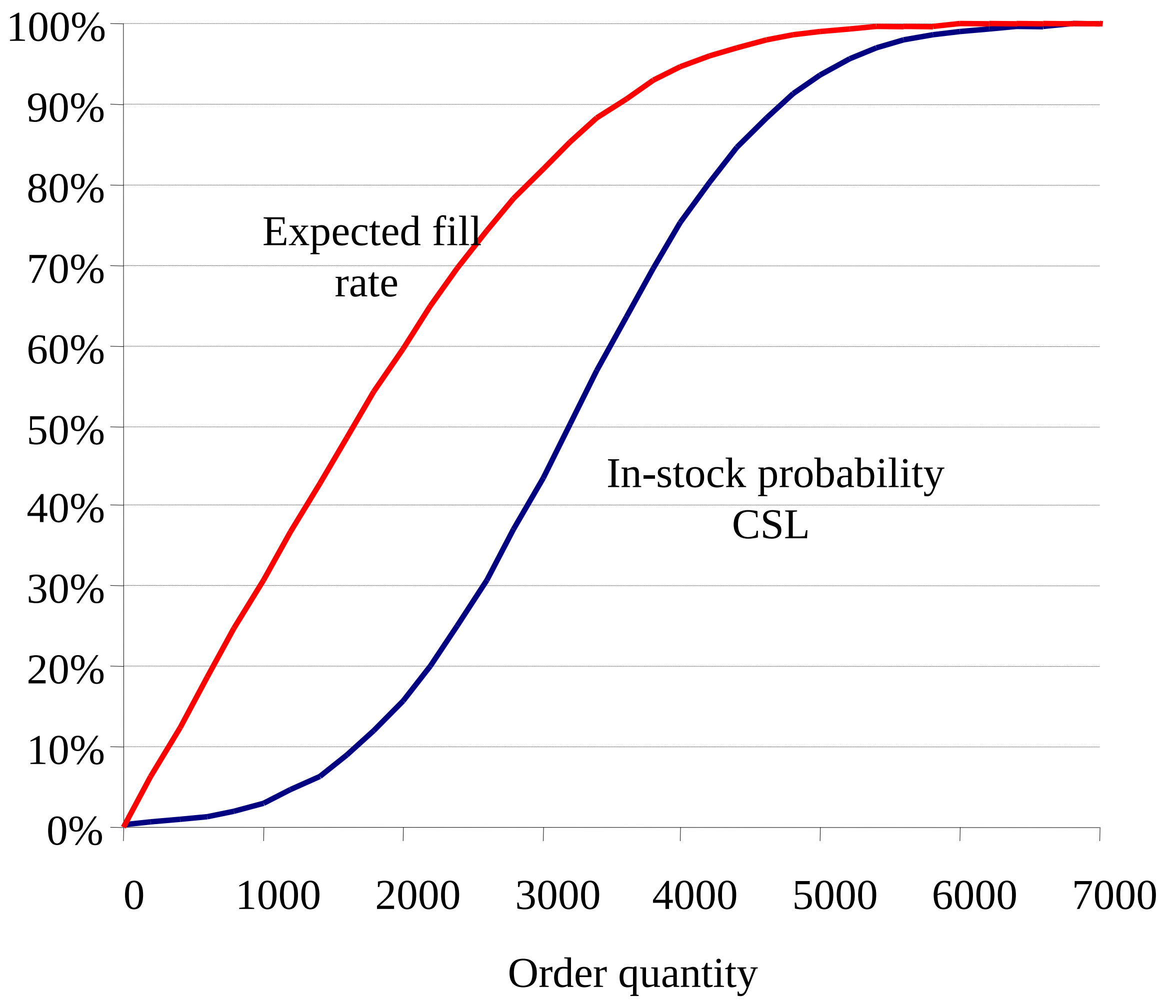
Recall:

Expected sales = μ - *Expected lost sales* = 3192 – 334 = 2858

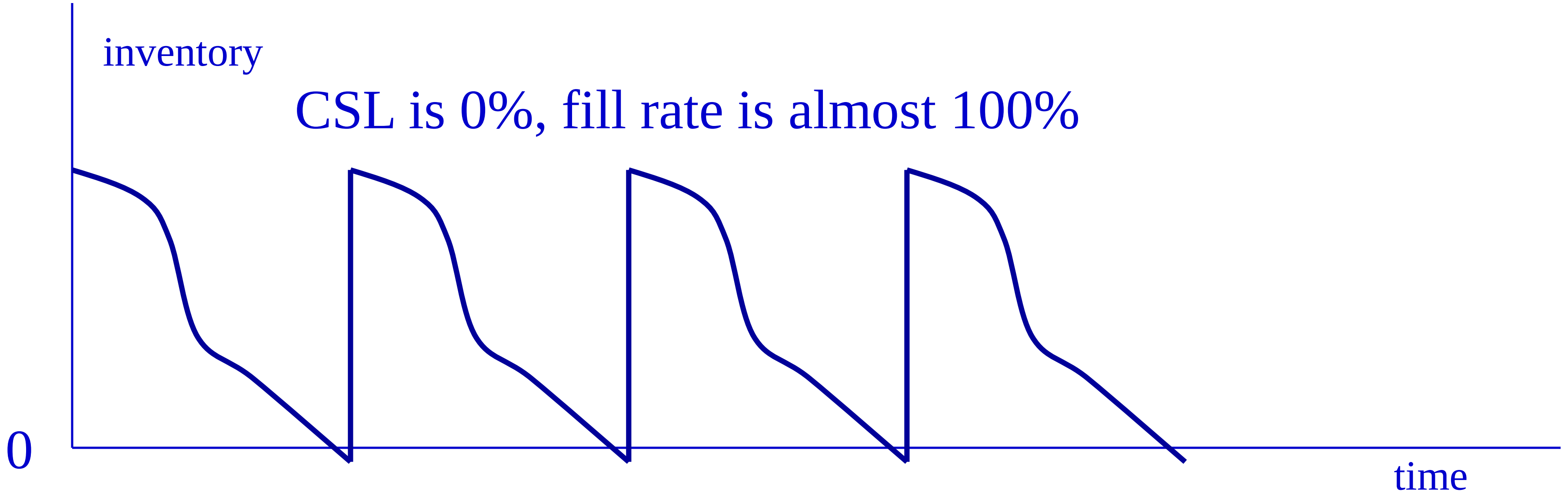
$$\begin{aligned}
\textit{Expected fill rate} &= \frac{\textit{Expected sales}}{\textit{Expected demand}} = \frac{\textit{Expected sales}}{\mu} \\
&= 1 - \frac{\textit{Expected lost sales}}{\mu} = \frac{2858}{3192} \\
&= 89.6\%
\end{aligned}$$

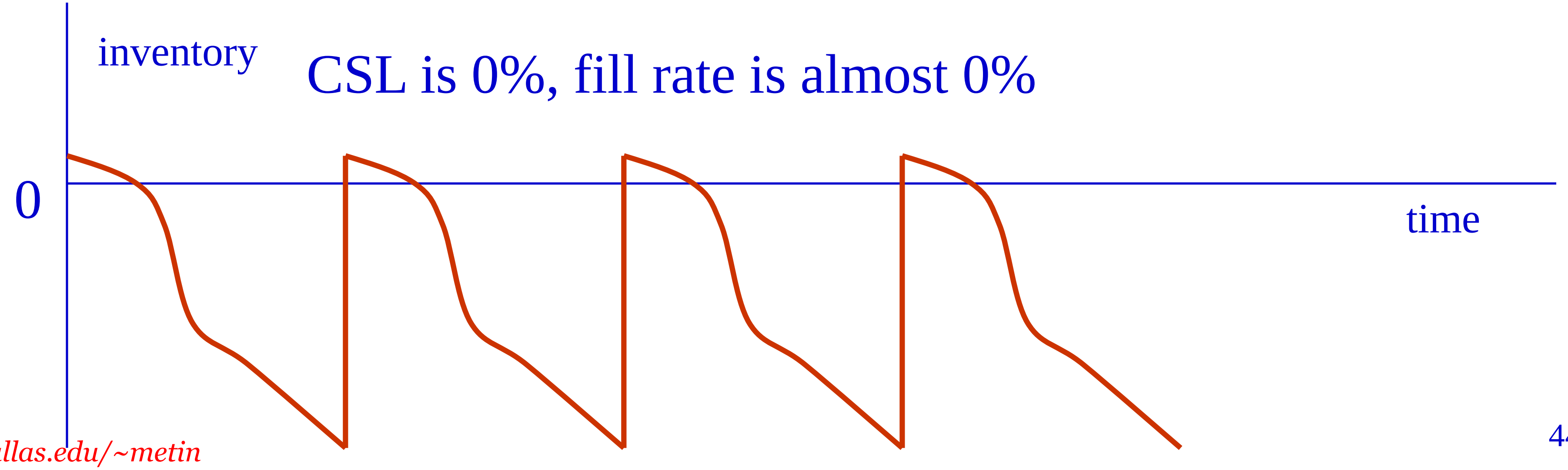
Is this fill rate too low?
Well, lost sales of 334 is with Q=3500, which is less than optimal.

Service measures of performance



Service measures: CSL and fill rate are different





The Newsvendor Model:

Measures that follow from lost sales

Measures that follow from expected lost sales

- Demand=Sales+Lost Sales

  D=min{D,Q}+max{D-Q,0} or min{D,Q}=D- max{D-Q,0}

  Expected sales = μ - Expected lost sales

  = 3192 – 334 = 2858

- Inventory=Sales+Leftover Inventory

  Q=min{D,Q}+max{Q-D,0} or max{Q-D,0}=Q-min{D,Q}

  Expected Leftover Inventory = Q - Expected Sales

  = 3500 – 2858 = 642

Measures that follow from expected lost sales

Economics:

- Each suit sells for p = \$180
- TEC charges c = \$110/suit
- Discounted suits sell for v = \$90

Expected total underage and overage cost with (Q=3500)

=70*334 + 20*642

$$\begin{aligned} \textit{Expected profit} &= \left[(\textit{Price-Cost}) \times \textit{Expected sales}\right] \\ &\quad -\left[(\textit{Cost-Salvage value}) \times \textit{Expected left over inventory}\right] \\ &= (\$70 \times 2858) - (\$20 \times 642) = \$187,221 \end{aligned}$$

What is the relevant objective? Minimize the cost or maximize the profit?

Hint: What is profit + cost? It is 70*(3192=334+2858)=C_u*μ, which is a constant.

Profit or [Underage+Overage] Cost; Does it matter?

$$(p: \text{price};\ v: \text{salvage value};\ c: \text{cost}) \text{ per unit.}$$

$$D: \text{demand};\ Q: \text{order quantity.}$$

$$\text{Profit}(D,Q) = \begin{Bmatrix} (p-c)D - (c-v)(Q-D) & \text{if } [D \le Q] \equiv \text{Overage} \\ (p-c)Q & \text{if } [D > Q] \equiv \text{Underage} \end{Bmatrix}$$

$$\text{Cost}(D,Q) = \begin{Bmatrix} (c-v)(Q-D) & \text{if } [D \le Q] \equiv \text{Overage} \\ (p-c)(D-Q) & \text{if } [D > Q] \equiv \text{Underage} \end{Bmatrix}$$

$$\text{Profit}(D,Q) + \text{Cost}(D,Q) = \begin{Bmatrix} (p-c)D & \text{if } D \le Q \\ (p-c)D & \text{if } D > Q \end{Bmatrix} = (p-c)D$$

$$E[\text{Profit}(D,Q)] + E[\text{Cost}(D,Q)] = (p-c)E(\text{Demand}) \equiv \text{Constant in } Q$$

$$\underset{Q}{\text{Max}}\, E[\text{Profit}(D,Q)] \text{ and } \underset{Q}{\text{Min}}\, E[\text{Cost}(D,Q)] \text{ are equivalent;}$$

Because they yield the same optimal order quantity.

Computing the Expected Profit with Normal Demands

$$\text{Expected Profit} = \int_{-\infty}^{\infty} \text{Profit}(D,Q)\, f(D)\, dD$$

Suppose that the demand is Normal with mean μ and standard deviation σ

$$\begin{aligned}\text{Expected Profit} &= (p-v)\ \mu\ \text{normdist}(Q,\mu,\sigma,1)\ -\ (p-v)\ \sigma\ \text{normdist}(Q,\mu,\sigma,0) \\ &\quad -(c-v)\ Q\ \text{normdist}(Q,\mu,\sigma,1)\ +\ (p-c)\ Q\ (1-\text{normdist}(Q,\mu,\sigma,1))\end{aligned}$$

Example: Follett Higher Education Group (FHEG) won the contract to operate the UTD bookstore. On average, the bookstore buys textbooks at \$100, sells them at \$150 and unsold books are salvaged at \$50. Suppose that the annual demand for textbooks has mean 8000 and standard deviation 2000. What is the annual expected profit of FHEG from ordering 10000 books? What happens to the profit when standard deviation drops to 20 and order drops to 8000?

Expected Profit is \$331,706 with order of 10,000 and standard deviation of 2000:

= (150-50) *8000*normdist(10000,8000,2000,1)-(150-50)*2000*normdist(10000,8000,2000,0)

-(100-50)*10000*normdist(10000,8000,2000,1)+(150-100)*10000*(1-normdist(10000,8000,2000,1))

Expected Profit is \$399,960 with order of 8000 and standard deviation of 20:

= (150-50)*8000*normdist(8000,8000,20,1)-(150-50)*20* normdist(8000,8000,20,0)

-(100-50)*8000*normdist(8000,8000,20,1)+(150-100)*8000*(1-normdist(8000,8000,20,1))

Summary

- Determine the optimal level of product availability
  - Demand forecasting
  - Profit maximization / Cost minimization
- Other measures
  - Expected shortages = lost sales
  - Expected left over inventory
  - Expected sales
  - Type I service measure: Instock probability = CSL
  - Type II service measure: Fill rate
  - Expected cost is equivalent to expected profit