Learning Objectives

- Interarrival and Service Times and their variability
- Obtaining the average time spent in the queue
- Pooling of server capacities
- Priority rules
Where are the queues?

Americans spend > 100 M hours/day waiting in a line.
T. Heymann in his book “On an average day”
A Queue is made of a server and a queue in front

**Input:**
Passengers in an airport  
Customers at a bank  
Patients at a hospital  
Callers at a call center

**Resources:**
Check-in clerks at an airport  
Tellers at a bank  
Nurses at a hospital  
Customer service representatives (CRS) at a call center

**Arrival rate**

**Capacity**

We are interested in the waiting times in the queue and the queue length.
At peak, 80% of calls dialed received a busy signal.

Customers getting through had to wait on average 10 minutes

Extra phone line expense per day for waiting was $25,000.

**Financial consequences**

- Lost throughput
- Holding cost
- Lost goodwill
- Cost of capacity
- Cost per customer
- $$ Revenue $$
A Somewhat Odd Service Process
Constant Arrival Rate (0.2/min) and Service Times (4 min)

<table>
<thead>
<tr>
<th>Patient</th>
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<th>Service Time</th>
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<tbody>
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Where is Variability?

- There certainly is significant (actually infinite) amount waiting when the arrival rate is greater than the service rate
  - Equivalently, the processing capacity is less than the flow rate
- More interestingly, variability can cause long waiting times. Variability in
  - Arrival process
  - Processing times
  - Availability of resources
  - Routing of flow units; Recall the Resume Validation Example.
Variability: Where does it come from? Examples

Input:
- Unpredicted Volume swings
- Random arrivals (randomness is the rule, not the exception)
- Incoming quality
- Product Mix

Especially relevant in service operations (what is different in service industries?):
- emergency room
- air-line check in
- call center
- check-outs at cashier

Buffer

Processing

Tasks:
- Inherent variation
- Lack of Standard Operating Procedures
- Quality (scrap / rework)

Resources:
- Breakdowns / Maintenance
- Operator absence
- Set-up times

Routes:
- Variable routing
- Dedicated machines

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A More Realistic Service Process
Random Arrival Rate and Service Times

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Variability Leads to Waiting Time

Average Arrival Rate (0.2/min) and Service Times (4 min)

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Inventory
(Patients at lab)
Is the incoming call rate stationary?
How to test for stationary?

![Graphs showing cumulative number of customers over time, with expected arrivals and actual cumulative arrivals.

Expected arrivals if stationary

Actual, cumulative arrivals

Cumulative Number of Customers

Cumulative Number of Customers

Time

Time

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Exponential distribution for Interarrival times

\[
\text{Prob}\{IA \leq t\} = 1 - \exp(-t/a)
\]
\[
E(IA) = a, \text{ expected time between arrivals}
\]
\[
\text{StDev}(IA) = a
\]
Comparing empirical and theoretical distributions

Distribution Function

Exponential distribution

Empirical distribution

Interarrival time
Analyzing the Arrival Process

- $CV_a = \text{StDev}(IA)/\text{E}(IA)$

Stationary Arrivals?

YES

- Exponentially distributed inter-arrival times?
  
  YES

  - Compute $a$: average interarrival time
  - $CV_a = 1$
  - All results of chapters 7 and 8 apply

  NO

  - Compute $a$: average interarrival time
  - $CV_a = \text{St.dev. of interarrival times} / a$
  - Results of chapter 7 or 8 do not apply, require simulation or more complicated models

NO

- Break arrival process up into smaller time intervals
Analyzing the Service Times
Seasonality and Variability

CV_p: Coefficient of variation of service times

Std. Dev = 141.46
Mean = 127.2
N = 2061.00
Call durations [seconds]

Call duration [minutes]

Frequency

Time of the day

Week-end averages
Week-day averages
Computing the expected waiting time $T_q$

- Inflow
- Inventory waiting $I_q$
- Entry to system
- Waiting Time $T_q$
- Flow Time $T = T_q + p$
- Service Time $p$
- Inventory in service $I_p$
- Begin Service
- Departure
- Outflow
Utilization

Utilization \( n = u = \frac{\text{FlowRate}}{\text{Capacity}} = \frac{1}{a} = \frac{p}{a} \)

Example: Average Activity time = \( p = 90 \) seconds
Average Interarrival time = \( a = 300 \) seconds

Utilization = \( \frac{90}{300} = 0.3 = 30\% \)
The Waiting Time Formula

**Waiting Time Formula for Exponential Arrivals**

\[
\text{Time in queue} = \text{Activity Time} \times \left( \frac{\text{utilization}}{1 - \text{utilization}} \right) \times \left( \frac{\text{CV}_a^2 + \text{CV}_p^2}{2} \right)
\]

Variability factor

Utilization factor

Service time factor

**Example:**
Average Activity time = p = 90 seconds
Average Interarrival time = a = 300 seconds
\(\text{CV}_a = 1\) and \(\text{CV}_p = 1.333\)

Average time in queue = 90 \(\times\) \(\left( \frac{0.3}{1 - 0.3} \right) \times \left( \frac{1^2 + 1.333^2}{2} \right) = 53.57\) sec
An average of 10 customers per hour come to a bank teller who serves each customer in 4 minutes on average. Assume exponentially distributed interarrival and service times.

(a) What is the teller’s utilization?
(b) What is the average time spent in the queue waiting?
(c) How many customers would be waiting for this teller or would be serviced by this teller on average?
(d) On average, how many customers are served per hour?

Answer: \( p = 4 \text{ mins}; \ a = 6 \text{ mins} = 60/10 \)

a) \( u = \frac{p}{a} = \frac{4}{6} = 0.66 \)

b) \( T_q = 4 * \left( \frac{0.66}{1 - 0.66} \right) * \left( \frac{1^2 + 1^2}{2} \right) = 8 \)

c) \( I = \left( \frac{1}{a} \right) * (T_q + p) = \left( \frac{1}{6} \right) * (8 + 4) = 2 \)

d) If teller is busy \( wp2/3, \ outputs15 \text{ per hour.} \)
   If teller is idle \( wp1/3, \ outputs0 \text{ per hour.} \)

\[ \text{Averageoutput} = (2/3) * 15 + (1/3) * 0 = 10 \text{ per hour} = \text{Averageinput}! \]
The Flow Time Increase Exponentially in Utilization

Average flow time $T$

Utilization 100%

Increasing Variability

Theoretical Flow Time
Computing $T_q$ with $m$ Parallel Servers

Inventory in the system $I = I_q + I_p$

Inventory waiting $I_q$

Entry to system

Begin Service

Departure

Waiting Time $T_q$

Service Time $p$

Flow Time $T = T_q + p$
Utilization with m servers

\[
Utilization = u = \frac{FlowRate}{Capacity} = \frac{1/a}{m/p} = \frac{p}{a m}
\]

Example: Average Activity time = \(p\) = 90 seconds
Average Interarrival time = \(a\) = 11.39 secs over 8-8:15
\(m\) = 10 servers

Utilization = \(90/(10 \times 11.39)\) = 0.79 = 79%
Waiting Time Formula for Parallel Resources

Approximate Waiting Time Formula for Multiple (m) Servers

\[
\text{Time in queue} \approx \left( \frac{\text{Activity time}}{m} \right) \left( \frac{\text{utilization} \sqrt{2(\text{m}+1)-1}}{1-\text{utilization}} \right) \left( \frac{CV_a^2 + CV_p^2}{2} \right)
\]

Example: Average Activity time=\(p=90\) seconds
Average Interarrival time=\(a=11.39\) seconds
\(m=10\) servers
\(CV_a=1\) and \(CV_p=1.333\)

\[
\text{Time in queue} \approx \left( \frac{90}{10} \right) \left( \frac{0.79 \sqrt{2(10+1)-1}}{1 - 0.79} \right) \left( \frac{1^2 + 1.333^2}{2} \right) = 24.94 \text{ sec}
\]

\[
T = T_q + p = 24.94 + 90 = 114.94 \text{ sec} = 1.916 \text{ min}
\]
Customers send emails to a help desk of an online retailer every 2 minutes, on average, and the standard deviation of the inter-arrival time is also 2 minutes. The online retailer has three employees answering emails. It takes on average 4 minutes to write a response email. The standard deviation of the service times is 2 minutes.

(a) Estimate the average customer wait before being served.

(b) How many emails would there be -- on average -- that have been submitted to the online retailer, but not yet answered?

Answer: a=2 mins; CV_a=1; m=3; p=4 mins; CV_p=0.5
a) Find T_q.  b) Find I_q=(1/a)T_q
Service Levels in Waiting Systems

- Target Wait Time (TWT)
- Service Level = Probability\{Waiting Time ≤ TWT\}; needs distribution of waiting time
- Example: Deutsche Bundesbahn Call Center
  - now (2003): 30% of calls answered within 20 seconds
  - target: 80% of calls answered within 20 seconds

90% of calls had to wait 25 seconds or less
Bank of America’s Service Measures

Customer Service and Support

Our Guiding Principles: Commitment, Passion, Learning, Integrity, Respect, Balance, Family, Fun and Service Excellence

About Us

Customer Service and Support is an integral part of Bank of America, employing more than 9,500 highly skilled associates in contact centers located in twenty cities across the United States. These associates provide service and financial solutions to more than 130 million phone customers and 1.74 million e-mail customers each year, making our contact centers among the busiest in the country.

Customer Service and Support is working to build a world-class customer service organization. The nine guiding principles listed above and the Bank of America Spirit provide the foundation for our daily work routine. Our associates are brand ambassadors whose hard work and determination will be the driving force behind our goal to make Bank of America the most admired company in the world.

Customer Service and Support is focused on building better, stronger and deeper relationships with our customers. Our associates have a passion for reaching a Higher Standard, achieving results and winning for our customers. It is important to all of us that we strive to provide the highest level of service to ensure that all of our customers are “delighted” with their Bank of America experience.

Functional Scope Areas

Customer Service and Support

National Consumer Service Centers
- Consumer and Consumer Card
- Dealer Financial Services
- IBCC
- NDS
- Plus
- Prime

Associate Experience and Communications

Client Service and Support
- Associate Banking
- Commercial
- Merchant and Commercial Card Services
- Premier
- Small Business

Multicultural Services

Customer Service Process and Operations
- Resolution Services and Support

Risk Management

Customer Delight

Strategy and Marketing

Customer Contact Management

Factoids:

Annualized

Customer Calls Received by VRU in 2002

508,500,000

Customer Calls Handed by VRU in 2002

503,500,000

Customer Calls Offered to Associates in 2002

147,000,000

Customer Calls Handed by Associates in 2002

130,000,000

Avg. Speed to Answer: 96.54 secs

E-mails Received in 2002: 1,750,000

E-mails Processed in 2002: 1,740,000

2002 Customer Delight: 54.3%

Certified Green Belts through 3/03: 203

Certified Black Belts through 3/03: 2

Associate Satisfaction in 2002: 72%

Associate Retention in 2002: 78%

2003 Performance Plan

Bank of America Vision:
Be recognized as the world’s most admired company

Customer Service and Support Vision:
A Passion to Delight

To reach our goal of being the world’s most admired company, we must do the following:

• Execute on our Hoshin Plan
• Live the Bank of America Spirit
• Communicate accurately and consistently
• Execute reliable, repeatable, consistent processes
• Focus on delivering world-class service for our customers

The focus for 2003 is: 65 / 75 / 64

• 65% Customer Delight
• 75% Associate Delight
• $64 million in productivity benefits (Shareholder Delight)
Waiting Lines: Points to Remember

• Variability is the norm, not the exception
  - understand where it comes from and eliminate what you can
  - accommodate the rest

• Variability leads to waiting times although utilization<100%

• Use the Waiting Time Formula to
  - get a qualitative feeling of the system
  - analyze specific recommendations / scenarios

• Adding capacity is expensive, although some safety capacity is necessary

• Next case:
  - application to call center
  - use CV=1
  - careful in interpreting March / April call volume
Summary of the formulas

1. Collect the following data:
   - number of servers, $m$
   - activity time, $p$
   - interarrival time, $a$
   - coefficient of variation for interarrival ($CV_a$) and processing time ($CV_p$)

2. Compute utilization: $u = \frac{p}{a \times m}$

3. Compute expected waiting time
   $$T_q = \left( \frac{Activity\ time}{m} \right) \times \left( \frac{utilizatio\ n \sqrt{2(m+1)} - 1}{1 - \text{utilizatio}\ n} \right) \times \left( \frac{CV_a^2 + CV_p^2}{2} \right)$$

4. Based on $T_q$, we can compute the remaining performance measures as
   Flow time $T = T_q + p$
   Inventory in service $I_p = m \times u$
   Inventory in the queue $I_q = T_q / a$
   Inventory in the system $I = I_p + I_q$
Staffing levels
Cost of direct labor per serviced unit

Ex: $10/hour wage for each CSR; m=10
Activity time= p=90 secs; Interarrival time=11.39 secs
1-800 number line charge $0.05 per minute

Utilization = \( u = \frac{p}{m \times a} = \frac{90}{10 \times 11.39} = 0.79 \)

Cost of Direct Labor = \( \frac{1.5 \text{ min/call} \times (16.66 \text{ cents/min})}{0.79} = 31.64 \text{ cents/call} \)

Recall \( T = 1.916 \); Cost of line charge per call = \( 1.916 \times 0.05 = $0.0958 / \text{call} \)
# Cost of Staffing Levels

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The optimal staffing level m=10
Staffing Levels under various Interarrival Times
The Power of Pooling

**Independent Resources**
2x(m=1)

**Pooled Resources**
(m=2)

**Implications:**
+ balanced utilization
+ Shorter waiting time (pooled safety capacity)
- Change-overs / set-ups
**Service-Time-Dependent Priority Rules**

- Flow units are sequenced in the waiting area (triage step)
- Shortest Processing Time (SPT) Rule
  - Minimizes average waiting time
  - Do you want to wait for a short process or a long one?

---

**Service times:**
- A: 9 minutes
- B: 10 minutes
- C: 4 minutes
- D: 8 minutes

---

- Problem of having “true” processing times
Service-Time-Independent Priority Rules

- Sequence based on importance
  - emergency cases; identifying profitable flow units
- First-Come-First-Serve
  - easy to implement; perceived fairness
- The order in which customers are served does **not** affect the average waiting time.
  - $W(t)$: Work in the system
  - An arrival at $t$ waits until the work $W(t)$ is completed

![Diagram showing work in the system over time with arrivals and departures marked as $p_1$, $p_2$, First arrival, Second arrival, and Last departure.](utdallas.edu/~metin)
Service-Time-Independent Order does not affect the waiting time.

Work is conserved even when the processing order changes. No matter what the order is, the third arrival finds the same amount of work $W(t)$. 
Summary

- Interarrival and Service Times and their variability
- Obtaining the average time spent in the queue
- Pooling of server capacities
- Priority rules