OPRE 6302. OM : Revenue Management

1 Solved Exercises

1.1 Statement of Exercises

1. [Hotel Reservations] A hotel near a university always fills up on the evening before football games. History has shown that when the hotel is fully booked, the number of last-minute cancellations has a mean of 5 and standard derivation of 3. The average room rate is $80. When the hotel is overbooked, policy is to find a room in a nearby hotel any to pay for the room for the customer. This usually costs the hotel approximately $200 since rooms booked on such late notice are expensive. How many rooms should the hotel overbook?

2. [HR Block on April 15] The H&R Block Office at the intersection of Arapaho road and Route 75 wants to make the most tax preparation profit on April 15. For this purpose, it allows clients to reserve 30 min blocks from 7 am until 5 pm. The clients who make reservations on April 10 or before are those who prepare in advance for tax filing. Moreover, they have several alternative companies other than H & R Block to file their tax. On the other hand, the clients who make reservations on April 11 and after do not have many alternatives. Thus, these clients can be charged more. Specifically, H & R Block determines to charge $75 for 30 min sessions if they are reserved before April 10, the same number is $100 for reservations after April 11. From the previous years, we konw the distribution of the number of reservations made over April 11-15:

Number of reservations over April 11-15	15	16	17	18	19	20	21	22	23	24	25
Probability	0.05	0.08	0.12	0.13	0.20	0.15	0.11	0.06	0.05	0.04	0.01

a) How many sessions should be protected for late reservers?
b) What is the expected number of unfilled sessions?
c) What is the expected number of filled (reserved) sessions?

1.2 Solutions

ANSWER for Exercise 1:

The cost of underestimating the number of the cancellations is $80 and the cost of overestimating cancellations is $200.

$$\text{P(Cancellations} \le \text{Overbooking)} = \frac{c_u}{c_o + c_u} = \frac{80}{200 + 80} = 0.2857.$$

Using Normsinv(0.2857) in Excel gives a z-value of -0.56599. The negative value indicates that we should overbook by a value less than the average of 5. Number of overbooked rooms = 5 + 3(-0.566) = 3.302.

Another common method for analyzing this type of problem is with a discrete probability distribution found using actual data and marginal analysis. For our hotel, consider that we have collected data and our distribution of no-shows is as follows.

Cancellations	0	1	2	3	4	5	6	7	8	9	10
Probability	0.05	0.08	0.10	0.15	0.20	0.15	0.11	0.06	0.05	0.04	0.01

Using these data, a table showing the impact of overbooking is created. Total expected cost of each overbooking option is the calculated by multiplying each possible outcome by its probability and summing the weighted costs. The best overbooking strategy is the one with minimum cost.

Number Cancelled	Probability	Number of Overbookings										
		0	1	2	**3**	4	5	6	7	8	9	10
0	0.05	0	200	400	600	800	1000	1200	1400	1600	1800	2000
1	0.08	80	0	200	400	600	800	1000	1200	1400	1600	1800
2	0.10	160	80	0	200	400	600	800	1000	1200	1400	1600
3	0.15	240	160	80	0	200	400	600	800	1000	1200	1400
4	0.20	320	240	160	80	0	200	400	600	800	1000	1200
5	0.15	400	320	240	160	80	0	200	400	600	800	1000
6	0.11	480	400	320	240	160	80	0	200	400	600	800
7	0.06	560	480	400	320	240	160	80	0	200	400	600
8	0.05	640	560	480	400	320	240	160	80	0	200	400
9	0.04	720	640	560	480	400	320	240	160	80	0	200
10	0.01	800	720	640	560	480	400	320	240	160	80	0
	Total Cost	337.6	271.6	228	**212.4**	238.8	321.2	445.6	600.8	772.8	958.8	1156

From the table, the minimum total cost is **212.4** when **3** extra reservations are taken. This approach using discrete probability is useful when valid historic data are available. □

ANSWER for Exercise 2:

a) If too few sessions are protected, H&R Block loses c_u=\$25=100-75 per session. If too many sessions are protected, H & R Block loses c_o=\$75. Then we solve for $F(Q) = 25/100 = 0.25$ to get $Q = 17$ for the protection level from the probability table given in the question.

b) If the demand turns out to be 15 or 16 during April11-15, there will respectively be 2 or 1 unfilled sessions. Multiplying these with the probabilities we get, 0.05*2+0.08*1=0.18.

c) When 0.18 sessions are unfilled, the remaining 20-0.18=19.82 are filled. □

2 Exercises

1. **[Price for Offloading]** While coming home from her spring break mania in Daytona beach, Beatrice was told that her airline seat was overbooked. She was asked to wait for 4 hours for the next flight, and was given a discount coupon of $100 to be used for another flight.
   a) Why does an airline overbook its seat inventory?
   b) What is the minimum amount of discount coupon that you would be willing to accept to wait for four hours?