Aggregate Planning

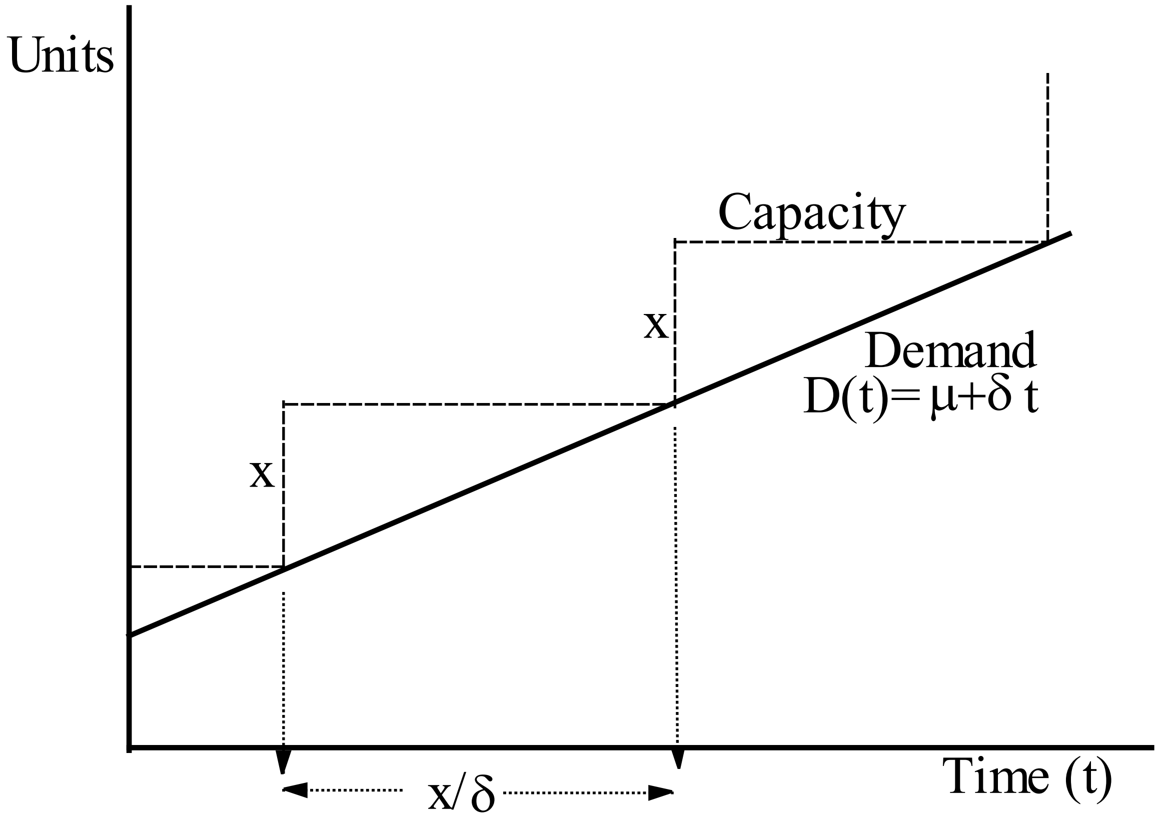
Capacity Planning & Assignment

Updated in Spring 2020

Deterministic Capacity Expansion Issues

- Single vs. Multiple Facilities
  - Dallas and Atlanta plants of Lockheed Martin
- Single vs. Multiple Resources
  - Machines and workforce; or aggregated capacity
- Single vs. Multiple Product Demands
  - Have you aggregated your demand when studying the capacity?
- Expansion only or with Contraction
  - Is there a second-hand machine market?
- Discrete vs. Continuous Expansion Times
  - Can you expand SOM building capacity during the spring term?
- Discrete vs. Continuous Capacity Increments
  - Can you buy capacity in units of 2.313832?
- Resource costs, economies of scale
- Penalty for demand-capacity mismatch
  - Recallable capacity: Electricity block outs vs Electricity buy outs
    - » Happens in Wisconsin and Texas Electricity market for Industrial customers
    - » What if American Airlines recalls my ticket
- Single vs. Multiple decision makers

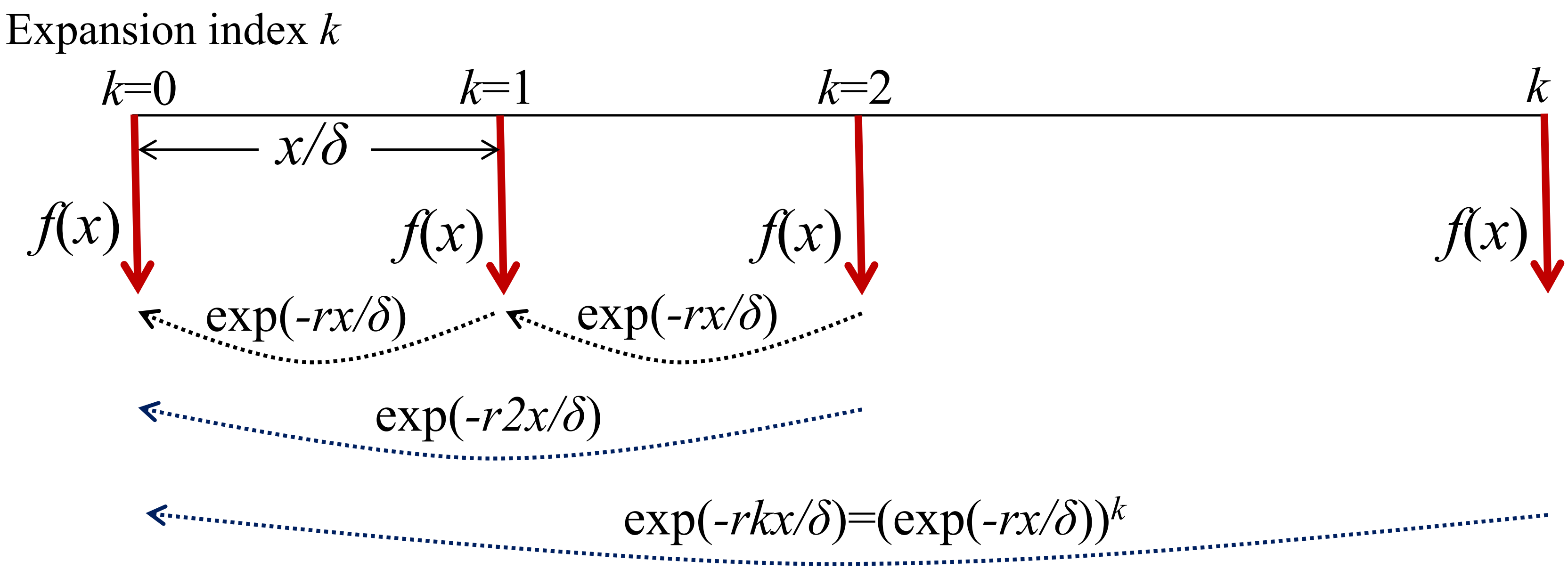
A Simple Model



No stock outs. x is the size of the capacity increments.
δ is the increase rate of the demand.

Infinite Horizon Total Discounted Cost

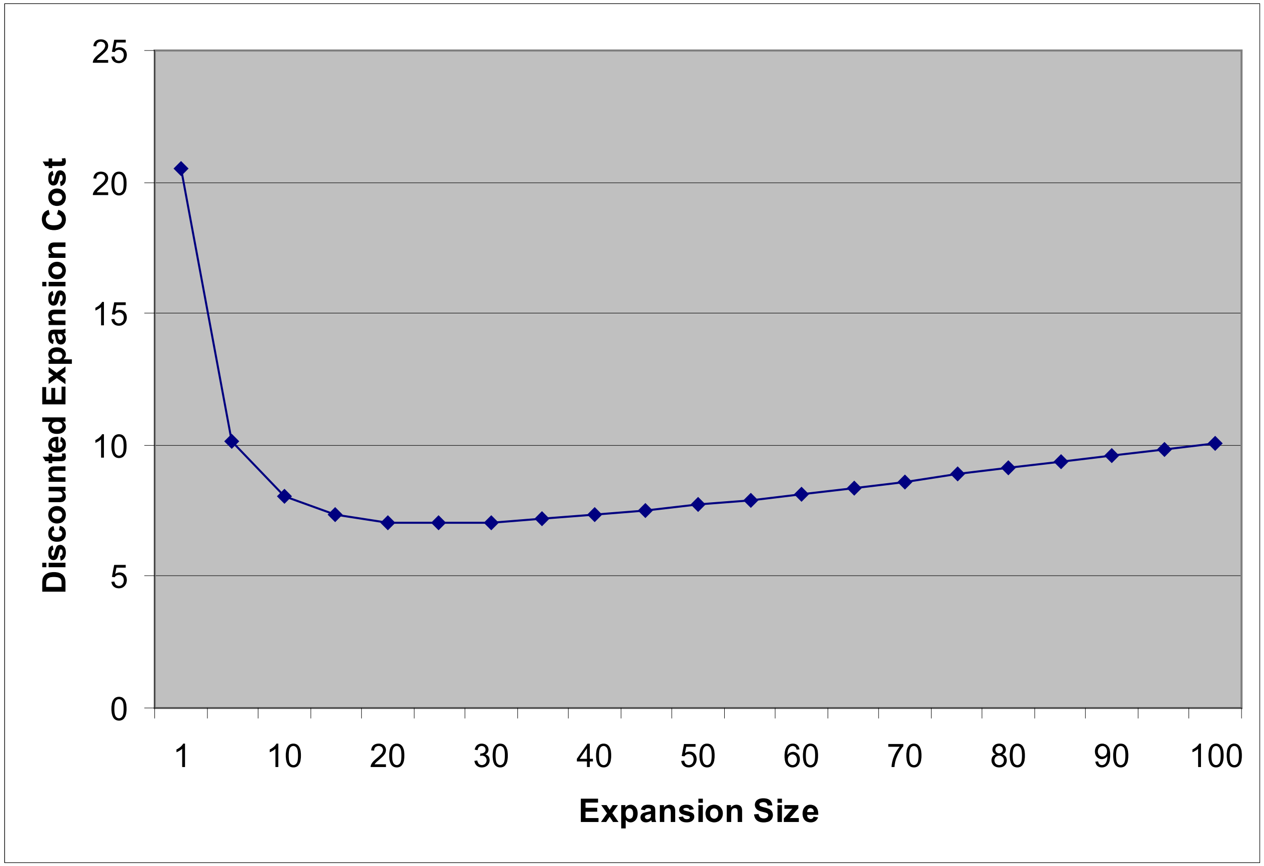
- $f(x)$ is expansion cost of capacity increment x; r is the interest rate



- C(x) is the long run (infinite horizon) total discounted expansion cost

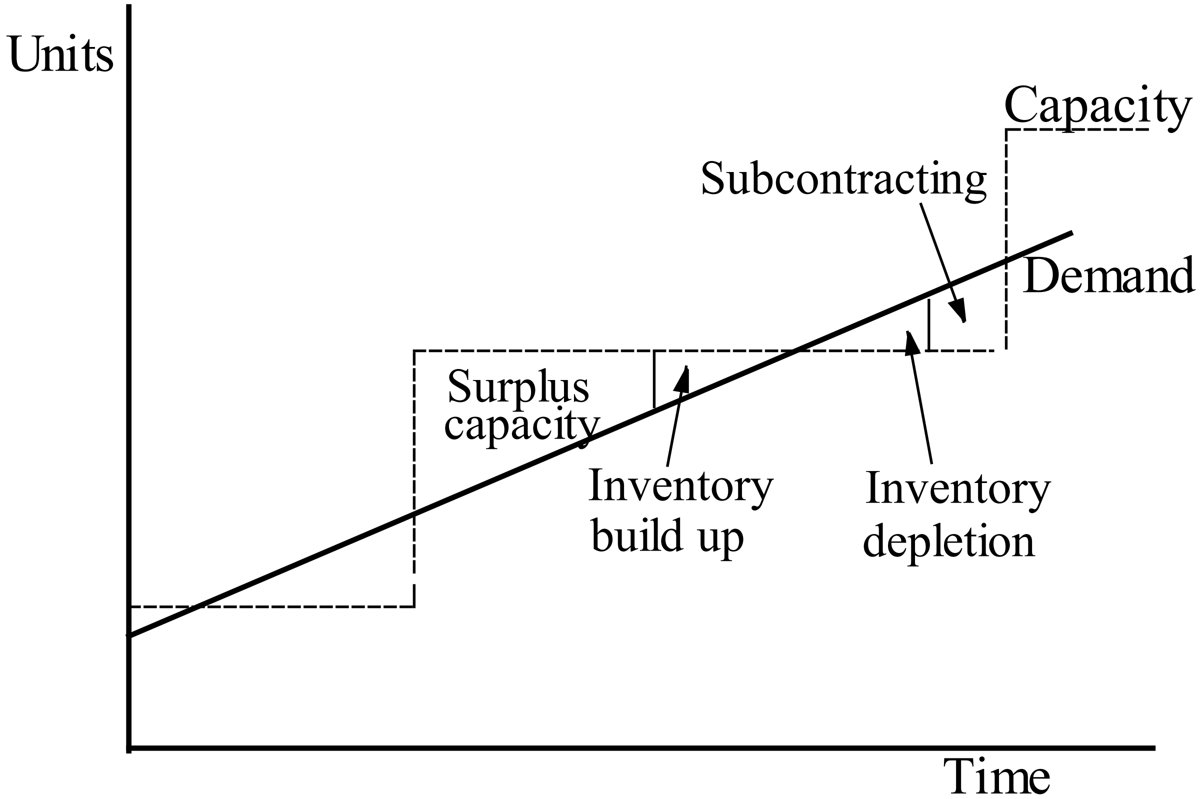
$$C(x)=\sum_{k=0}^{\infty}\exp\left(-r(k\frac{x}{\delta})\right)f(x)=f(x)\sum_{k=0}^{\infty}(\exp(-rx/\delta))^{k}=\frac{f(x)}{1-\exp(-rx/\delta)}$$

Solution of the Simple Model



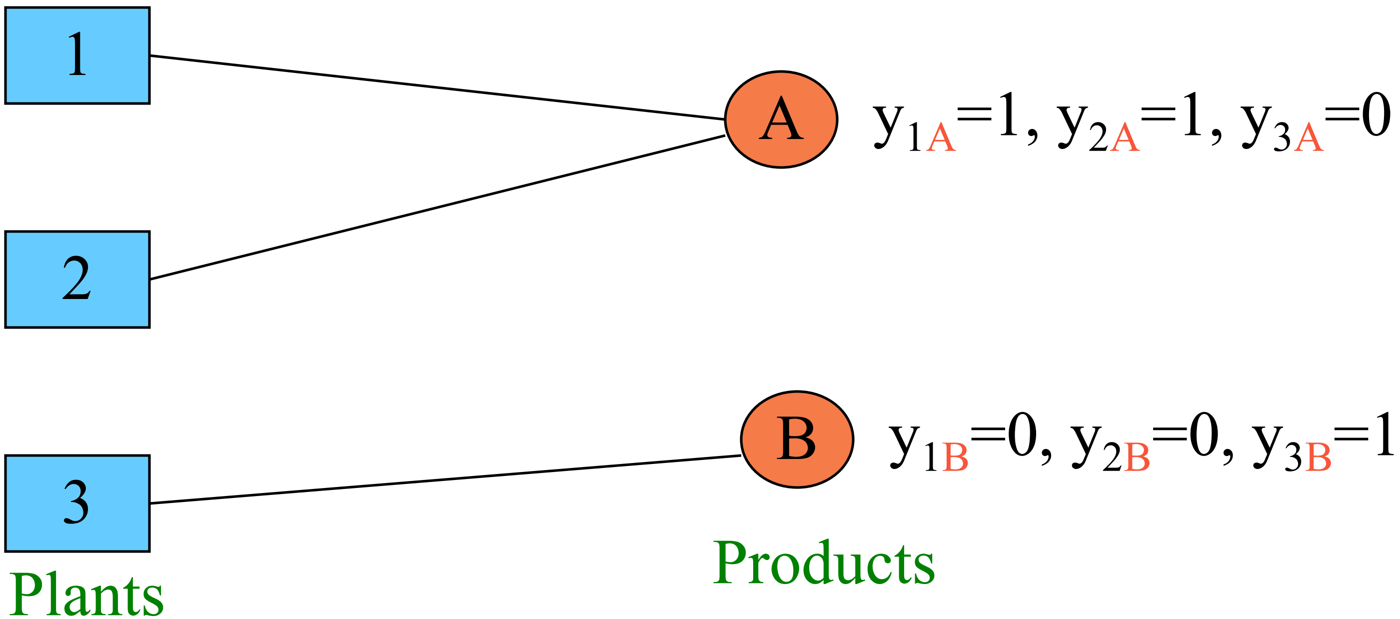
Solution can be: Each time expand capacity by an amount that is equal to 30-week demand.

Shortages, Inventory Holding, Subcontracting



- Use of Inventory and subcontracting to delay capacity expansions

Stochastic Capacity Planning: The case of flexible capacity



- Plant 1 and 2 are tooled to produce product A
- Plant 3 is tooled to produce product B
- A and B are substitute products
  - with random demands $D_A + D_B = \text{Constant}$

Capacity allocation

- Say capacities are $r_1=r_2=r_3=100$
- Suppose that $D_A + D_B = 300$ and $D_A > 100$ and $D_B > 100$

With plant flexibility $y_{1A}=1$, $y_{2A}=1$, $y_{3A}=0$, $y_{1B}=0$, $y_{2B}=0$, $y_{3B}=1$.

Scenario	D_A	D_B	X_{1A}	X_{2A}	X_{3A}	X_{1B}	X_{2B}	X_{3B}	Shortage
1	200	100	100	100				100	0
2	150	150	100	50				100	50 B
3	100	200	100	0				100	100 B

If the scenarios are equally likely, expected shortage is 50.

Capacity allocation with more flexibility

- Say capacities are $r_1=r_2=r_3=100$
- Suppose that $D_A + D_B = 300$ and $D_A > 100$ and $D_B > 100$

With plant flexibility $y_{1A}=1$, $y_{2A}=1$, $y_{3A}=0$, $y_{1B}=0$, **$y_{2B}=1$**, $y_{3B}=1$.

Scenario	D_A	D_B	X_{1A}	X_{2A}	X_{3A}	X_{1B}	X_{2B}	X_{3B}	Shortage
1	200	100	100	100			0	100	0
2	150	150	100	50			50	100	0
3	100	200	100	0			100	100	0

Flexibility can decrease shortages. In this case, from 50 to 0.

A Formulation with Known Demands: $D_j = d_j$

- i denotes plants, $i = 1..n$
- j denotes products, $j = 1..m$
- c_{ij} tooling cost to configure plant i to produce j
- m_j contribution to margin of producing/selling a unit of j
- r_i capacity at plant i
- $D_j = d_j$ product j demand
- $y_{ij} = 1$ if plant i can produce product j, 0 otherwise
- x_{ij} =units of j produced at plant i

Objective

$$\text{Max } -\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}y_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m} m_j x_{ij}$$

st

$$\sum_{j=1}^{m} x_{ij} \le r_i$$

$$\sum_{i=1}^{n} x_{ij} \le d_j$$

$$x_{ij} \le r_i y_{ij}$$

$$x_{ij} \ge 0\ ;\ y_{ij} \in \{0,1\}$$

A Formulation with Known Demands: An instance

- $n = 3$ plants; $m = 2$ products;
- $r_1 = 100$; $r_2 = 100$; $r_3 = 100$.
- $d_A = 200$; $d_B = 100$.
- $m_A = 4$; $m_B = 5$.
- $c_{1A} = 700$; $c_{1B} = 800$;
- $c_{2A} = 600$; $c_{2B} = 500$;
- $c_{3A} = 400$; $c_{3B} = 900$;

Solution depends on scenarios

- If $D_A = d_A = 200$ and $D_B = d_B = 100$, then $y_{1A} = y_{2A} = y_{3B} = 1$.
- If $D_A = d_A = 100$ and $D_B = d_B = 200$, then $y_{1A} = y_{2B} = y_{3B} = 1$.

$$\text{Max } -700y_{1A} - 800y_{1B} - 600y_{2A} - 500y_{2B} - 400y_{3A} - 900y_{3B} + 4(x_{1A} + x_{2A} + x_{3A}) + 5(x_{1B} + x_{2B} + x_{3B})$$

$$x_{1A} + x_{1B} \leq 100$$
$$x_{2A} + x_{2B} \leq 100$$
$$x_{3A} + x_{3B} \leq 100$$

$$x_{1A} + x_{2A} + x_{3A} \leq 200$$
$$x_{1B} + x_{2B} + x_{3B} \leq 100$$

$$x_{1A} \leq 100y_{1A}$$
$$x_{1B} \leq 100y_{1B}$$
$$x_{2A} \leq 100y_{2A}$$
$$x_{2B} \leq 100y_{2B}$$
$$x_{3A} \leq 100y_{3A}$$
$$x_{3B} \leq 100y_{3B}$$

$$x_{ij} \geq 0\ ;\ y_{ij} \in \{0,1\}$$
$$i \in \{1,2,3\}, j \in \{A, B\}$$

Sequence of Decisions and Random Events

- Tranportation problem
  - First demand, then shipments
  - Decision: Shipment; Random event: Demand, already realized
  - Given known (realized) demand, solve for shipment
  - Sequence: Demand → Shipment
  - Random → Decision : Decision anticapatory wrt random
- Flexible vs. Dedicated tire production capacity
  - First type of capacity, then demands and exchange rates
  - Decision: Capacity; Random events: Demands & Exchange rates
  - Given unknown demand (formulated as scenarios), solve for capacity
  - Capacity → Demand
  - Decision → Random : Decision nonanticapatory wrt random
- Configuring plants for production
  - First configuration, second demand, third production
  - Decisions: Configuration & Production; Random event: Demand
  - Configuration → Demand → Production
  - Given unknown demand (formulated as scenarios), solve for configuration
  - Given known (realized) demand, solve for production
  - Configuration Decision → Random Demand→ Production Decision
    - » Configuration decision nonanticapatory wrt random demand
    - » Production decision anticipatory wrt to random demand

Can a decision depend on scenarios?
Yes ⇒ Anticipatory variable ; No ⇒ Nonanticapatory variable

Unknown Demands: $D_j = d_j^k$ with probability p^k

- Random $D_j = d_j^k$ product j demand in scenario k
- x_{ij}^k= units of j produced at plant i in scenario k
- Configuration before demand ⇒ Nonanticipatory
- $y_{ij} = 1$ if i can produce j, 0 otherwise

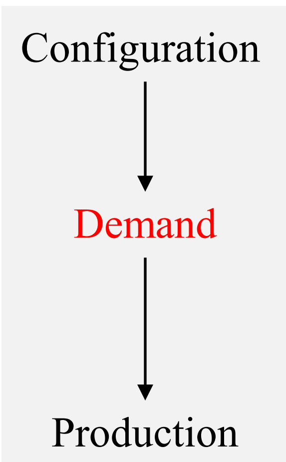
$$\text{Max} \; -\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}y_{ij} + \sum_{k=1}^{K} p^k \sum_{i=1}^{n}\sum_{j=1}^{m} m_j x_{ij}$$

$$\sum_{j=1}^{m} x_{ij}^k \leq r_i$$ For each plant i & each scenario k

$$\sum_{i=1}^{n} x_{ij}^k \leq d_j^k$$ For each product j & each scenario k

$$x_{ij}^k \leq r_i y_{ij}$$ For each plant i & product j & each scenario k

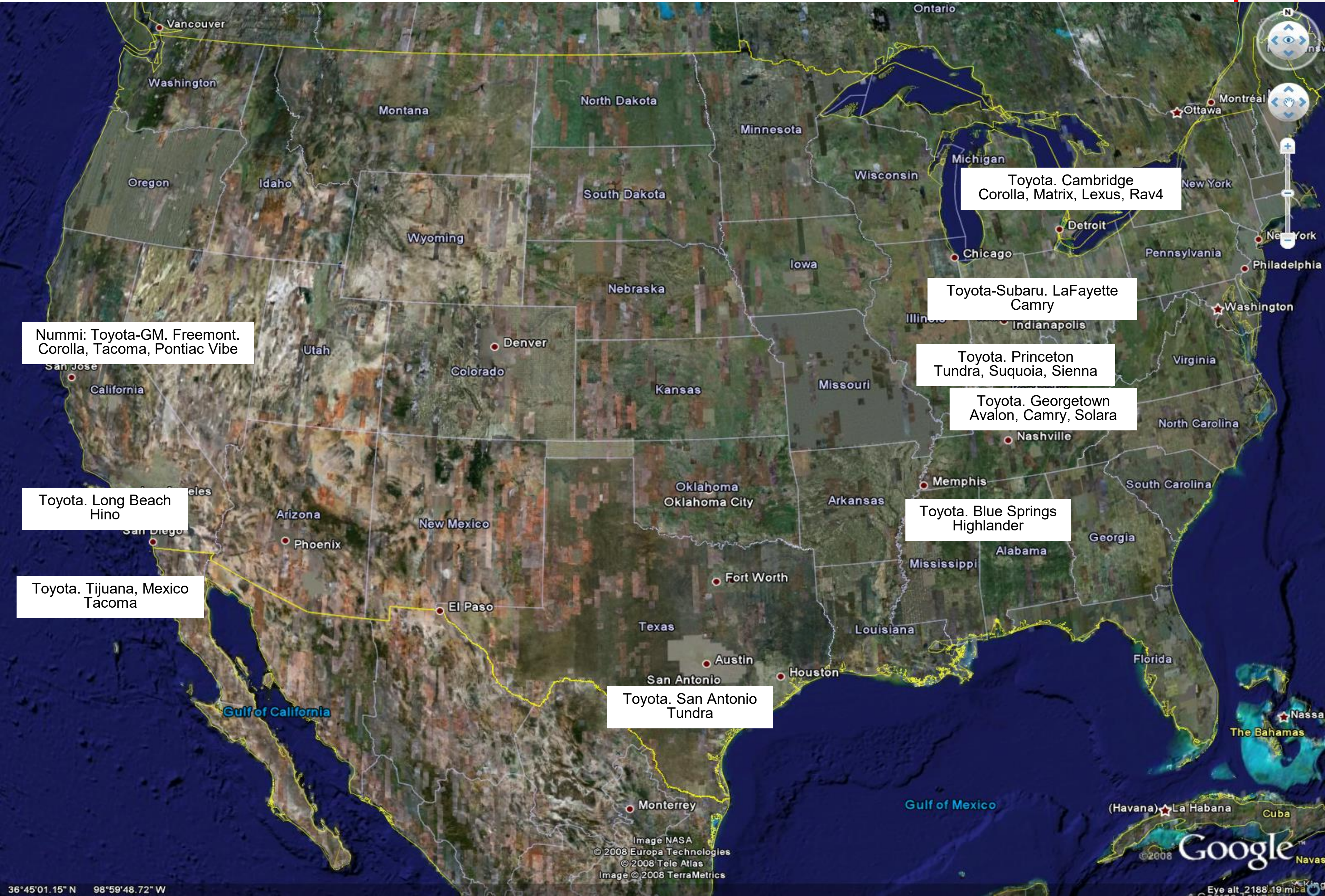
$$x_{ij}^k \geq 0 \, ; \; y_{ij} \in \{0,1\}$$



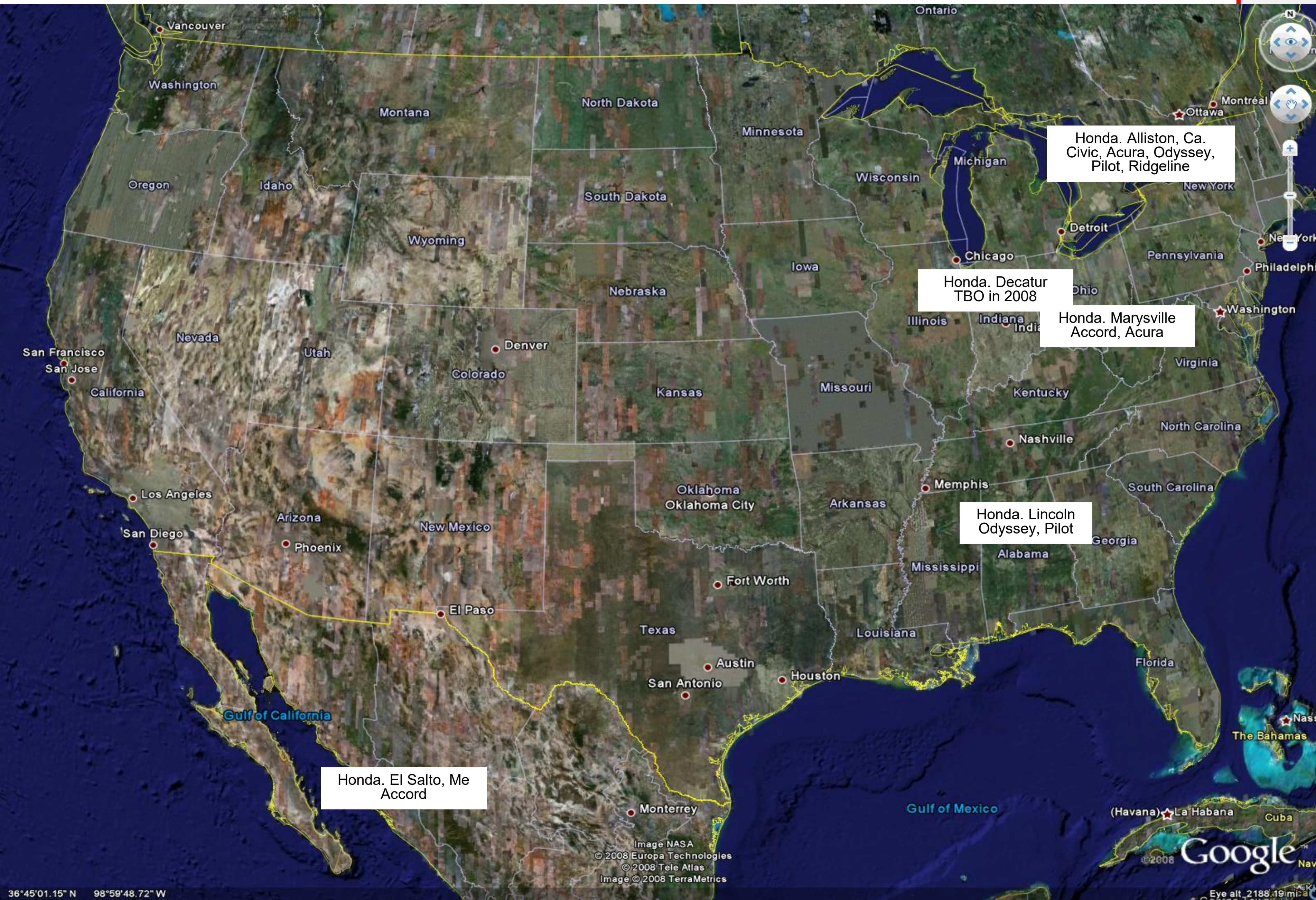
Reality Check:
How do car manufacturers assign products to plants?

- The last formulation treats the problem of assigning products to plants.
- This type of assignment is called for tooling/preparation of each plant so that it can produce the car type it is assigned to.
- These tooling (nonanticipatory) decisions are made at most once a year and manufacturers work with the current assignments to meet the demand.
- When market conditions change, the product-to-plant assignment is revisited.
  - Almost all car manufacturers in North America are retooling their previously truck manufacturing plants to manufacture compact cars as consumer demand basically disappeared for trucks with high gas prices.
  - Also note that the profit margin made from a truck sale is 2-5 times more than the margin made from a car sale. No wonder why manufacturers prefer to sell trucks!
- In the following pages, you will find the product to plant assignment of all major car manufacturers in the North America. These assignments were updated in the summer of 2008 just about the time when manufacturers started talking about retooling plants to produce compact cars.

All of Toyota Plants in the North America



All of Honda Plants in the North America

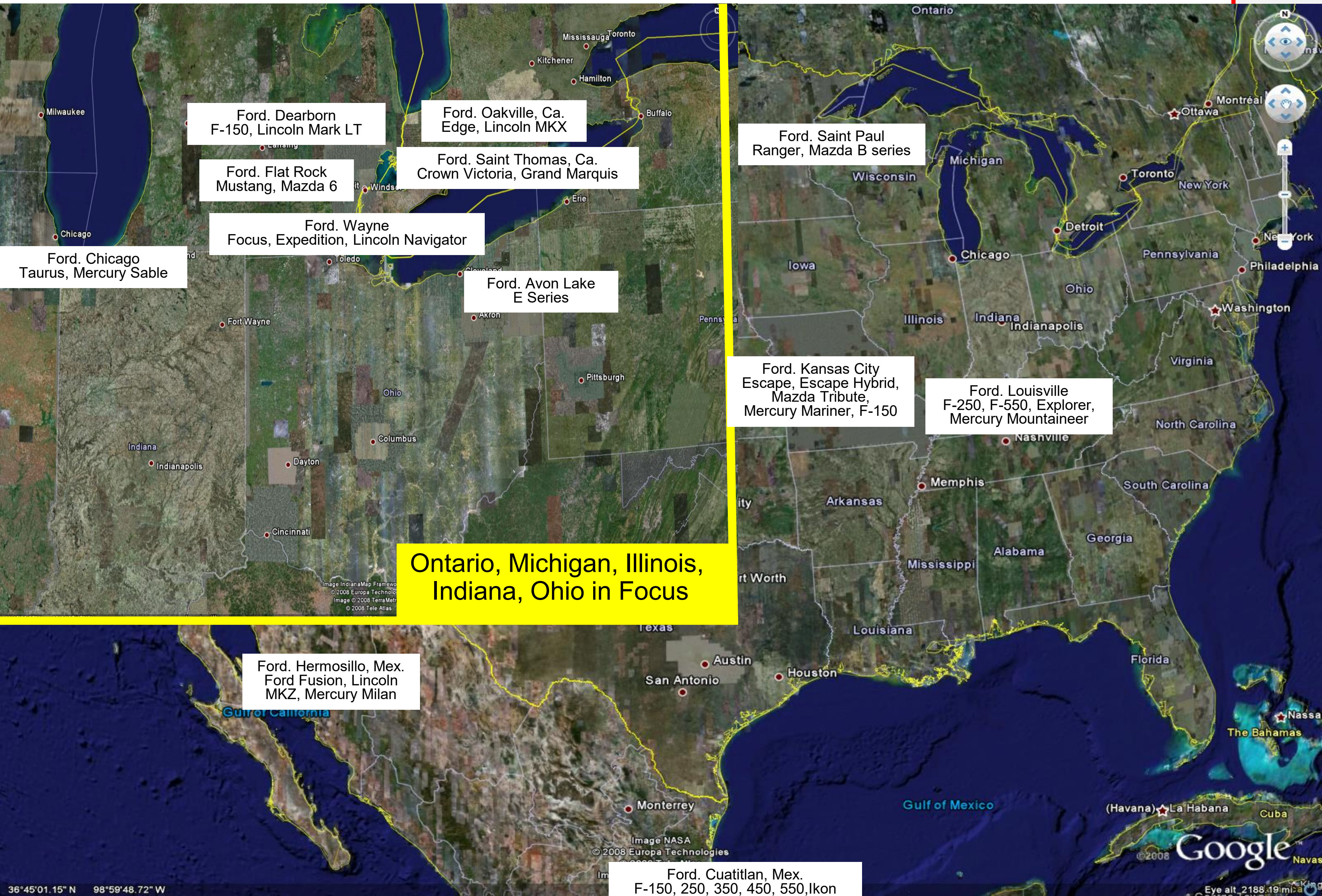


All of Nissan Plants in the North America

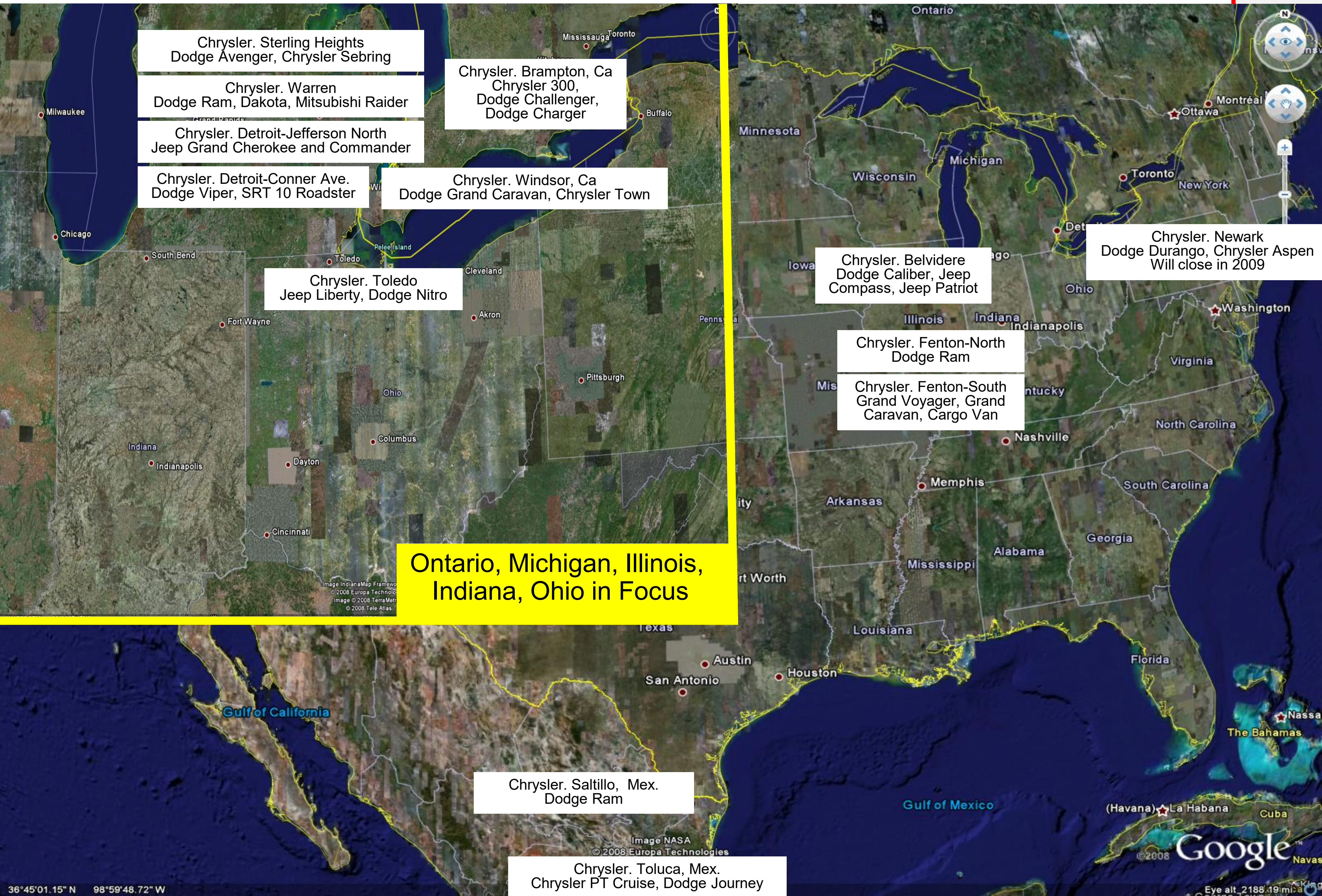
All of Hyundai-Kia Plants in the North America

All of Mercedes and BMW Plants in the North America

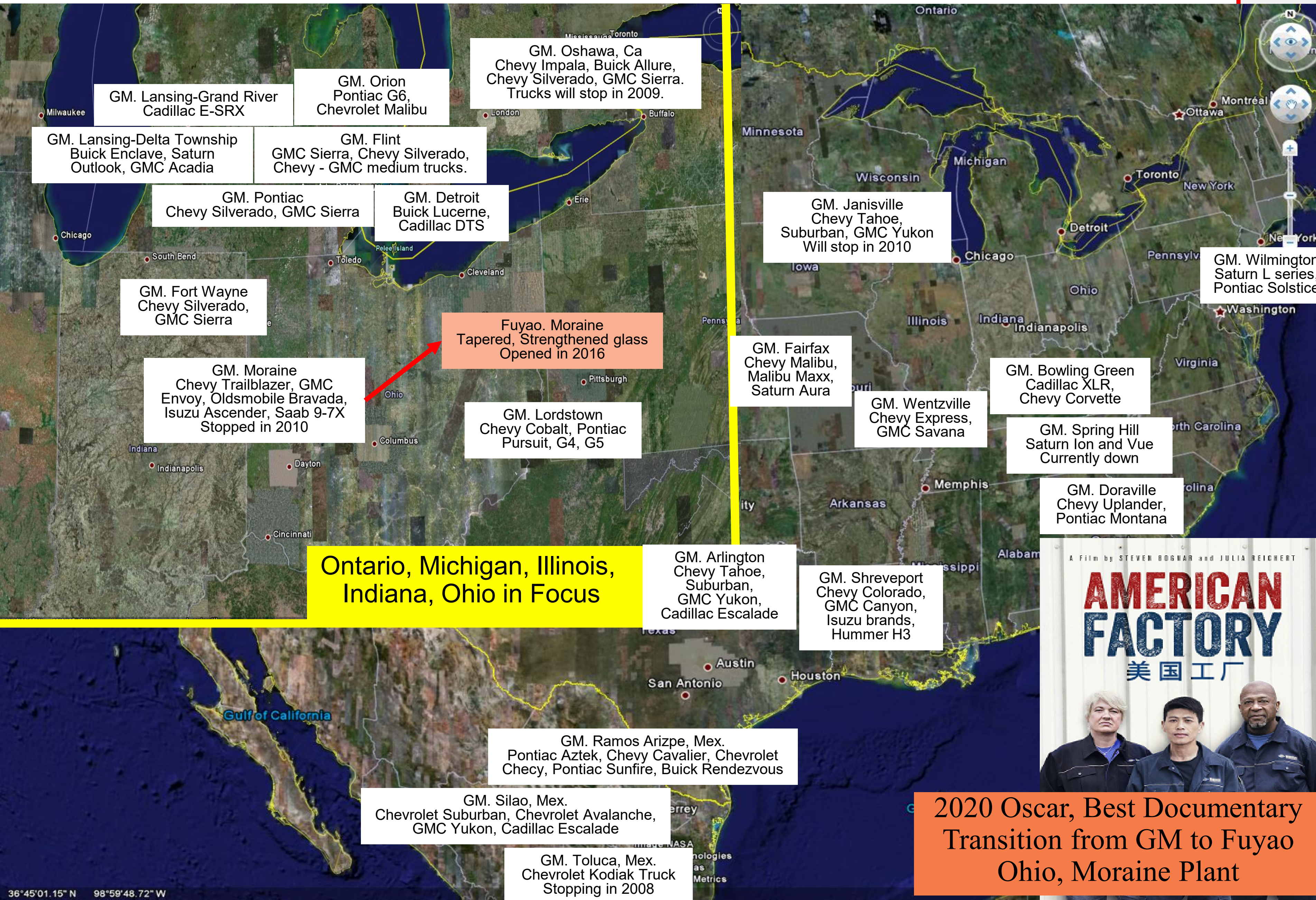
All of Ford Plants in the North America



All of Chrysler Plants in the North America



All of GM Plants in the North America



2020 Oscar, Best Documentary
Transition from GM to Fuyao
Ohio, Moraine Plant

GM’s Restructuring Plan from 2009

- **GM’s Plan details a return to sustainable profitability in 24 months**
  - Demonstrates GM‘s viability under conservative economic assumptions
  - Expands and accelerates the Plan submitted on December 2
  - Lowers the Company‘s breakeven to a U.S. market of 11.5-12.0M units annually
- **GM is comprehensively transforming its business, globally**
  - Brands, nameplates and dealer networks streamlined and focused
  - Productivity and flexibility gains enabling more facility consolidations
  - Shared global vehicle architectures creating substantial cost savings
  - Unprofitable foreign operations addressed
- **GM’s Plan emphasizes the Company’s continued focus on great products**
  - Fewer, better vehicles in U.S. : supporting Chevrolet, Cadillac, Buick and GMC
  - Renewed commitment to lead in fuel efficiency, hybrids, advanced propulsion
  - All major U.S. introductions in 2009-2014 are high-mileage cars and crossovers
- **GM’s Plan calls for considerable sacrifice from all stakeholders**
  - Bondholders and other debtors
  - Hourly and salaried employees, executives and retirees
  - Dealers and suppliers
  - Shareholders

Summary

- Capacity Planning
- Product-to-plant Assignment

Aside: Continuous Compounding

- If my \$1investment earns an interest of r per year, what is my interest+investment at the end of the year?

  Answer: (1+r)
- If I earn an interest of r/2 per six months, what is my interest+ investment at the end of the year?

  Answer: $(1+r/2)^2$
- If I earn an interest of (r/m) per (12/m) months, what is my interest+investment?

  Answer: $(1+r/m)^m$
- Think of continuous compounding as the special case of discrete-time compounding when m approaches infinity.
- What if I earn an interest of (r/infinity) per (12/infinity) months?

$$\text{Answer: } \lim_{m\to\infty}\left(1+\frac{r}{m}\right)^m = e^r \quad \text{where}$$

$$e=\sum_{n=0}^{\infty}\frac{1}{n!}=\frac{1}{1}+\frac{1}{1}+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}$$

See the appendix of scaggregate.pdf for more on continuous compounding.