Safety Inventories
Accurate Response to Variability
Methods of Accurate Response to Variability

- Centralization
  - Physical, Laura Ashley
  - Information
    » Virtual aggregation, Barnes&Nobles stores
  - Specialization, what to aggregate

- Product substitution

- Raw material/component commonality
Centralization: Inventory Pooling

Which of two systems provides a higher level of service for a given safety stock? Consider locations and demands:

\[ D_1 \sim (R_1, \sigma_1) \quad D_2 \sim (R_2, \sigma_2) \]
\[ D_3 \sim (R_3, \sigma_3) \quad D_4 \sim (R_4, \sigma_4) \]

With K locations centralized, mean and variance of

\[ D^C = \sum_{i=1}^{K} D_i \]

\[ R^C = \sum_{i=1}^{K} R_i; \quad (\sigma^C)^2 = \sum_{i=1}^{K} \sigma_i^2 + 2 \sum_{i<j}^{K} \text{cov}(D_i, D_j) \]
Sum of Random Variables Are Less Variable

When they are independent, \( \text{cov}(D_i,D_j)=0 \)

When perfectly positively correlated, \( \text{cov}(D_i,D_j)=\sigma_i \sigma_j \)

When perfectly negatively correlated, \( \text{cov}(D_i,D_j)=-\sigma_i \sigma_j \)

\[
\sigma^C = \sqrt{\sum_{i=1}^{K} \sigma_i^2} \leq \sum_{i=1}^{K} \sigma_i
\]

\[
\sigma^C = \left( \sum_{i=1}^{K} \sigma_i^2 + 2 \sum_{i=1}^{K} \sigma_i \sigma_j \right)^{\frac{1}{2}} = \sum_{i=1}^{K} \sigma_i
\]

\[
\sigma^C = \sqrt{\sum_{i=1}^{K} \sigma_i^2 - 2 \sum_{i=1}^{K} \sigma_i \sigma_j} < \sqrt{\sum_{i=1}^{K} \sigma_i^2} \leq \sum_{i=1}^{K} \sigma_i
\]
Factors Affecting Value of Aggregation

◆ When to aggregate? Statistical checks: Positive correlation and Coefficient of Variation.
  – Aggregation reduces variance almost always except when products are positively correlated
  – Aggregation is not effective when there is little variance to begin with. When coefficient of variation of demand is relatively small (variance w.r.t. the mean is small), do not bother to aggregate.

◆ In real life,
  – Is the electricity demand in Arlington and Plano are positively or negatively correlated? Is there an underlying factor which affects both in the same direction? Note that a big portion of electricity is consumed for heating/cooling.
  – Are the Campbell soup sales over time positively or negatively correlated? How many soups can you drink per day?
Impact of Correlation on Aggregated Safety Inventory
Aggregating 4 Outlets

- Safety stocks are proportional to the StDev of the demand.
- With four locations, we have total ss proportional to $4\sigma$
- If four locations are all aggregated,
  - ss proportional to $4\sigma$ with correlation 1
  - ss proportional to $2\sigma$ with correlation 0
- Benefit = SS before - SS after / SS before
EX 11.8: W.W. Grainger a supplier of Maintenance and Repair products

- About **1600 stores** in the US

- Produces large **electric motors** and industrial cleaners

- Each **motor** costs $500; **Weekly Demand** is iid **Normal(20,40x40)** at each store

- Each **cleaner** costs $30; **Weekly Demand** is iid **Normal(1000,100x100)** at each store

- Which demand has a larger coefficient of variation?
- **How much savings** if motors/cleaners **inventoried centrally?**
Use CSL=0.95
Supply lead time L=4 weeks for motors and cleaners

For normally distributed demand : \( ss = \text{Norminv}(\text{CSL}, 0, 1) \cdot \sqrt{L\sigma} \)

For a single store

Motor safety inventory = \( \text{Norminv}(0.95, 0, 1) \cdot 2 \cdot (40) = 132 \)
Cleaner safety inventory = \( \text{Norminv}(0.95, 0, 1) \cdot 2 \cdot (100) = 329 \)
Value of motor \( ss \) = \( 1600 \cdot (132) \cdot (500) = \$105,600,000 \)
Value of cleaner \( ss \) = \( 1600 \cdot (329) \cdot (30) = \$15,792,000 \)

Standard deviation of demands after aggregating 1600 stores

Standard deviation of Motor demand = \( 40 \cdot (40) = 1,600 \)
Standard deviation of Cleaner demand = \( 40 \cdot (100) = 4,000 \)

For the aggregated store

Motor safety inventory = \( \text{Norminv}(0.95, 0, 1) \cdot 2 \cdot (1600) = 5,264 \)
Cleaner safety inventory = \( \text{Norminv}(0.95, 0, 1) \cdot 2 \cdot (4,000) = 13,159 \)
Value of motor \( ss \) = \( 5264 \cdot (500) = \$2,632,000 \)
Value of cleaner \( ss \) = \( 13,159 \cdot (30) = \$394,770 \)
# Impact of $cv$ on Aggregation Benefit

1600-Stores; $h=0.25$

<table>
<thead>
<tr>
<th></th>
<th>Motors</th>
<th>Cleaner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean demand/wk</td>
<td>20</td>
<td>1,000</td>
</tr>
<tr>
<td>SD of demand</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Disaggregate cv</td>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>Value/Unit</td>
<td>$500</td>
<td>$30</td>
</tr>
<tr>
<td>Disaggregate ss value</td>
<td>$105,600,000</td>
<td>$15,792,000</td>
</tr>
<tr>
<td>Aggregate cv</td>
<td>0.05</td>
<td>0.0025</td>
</tr>
<tr>
<td>Aggregate ss value</td>
<td>$2,632,000</td>
<td>$394,770</td>
</tr>
<tr>
<td>Inventory cost savings</td>
<td>$102,968,000</td>
<td>$15,397,230</td>
</tr>
<tr>
<td>Holding Cost Saving</td>
<td>$25,742,000</td>
<td>$3,849,308</td>
</tr>
<tr>
<td>Saving / Unit</td>
<td>$15.47</td>
<td>$0.046</td>
</tr>
</tbody>
</table>

$\text{Saving / Unit} = \frac{\text{Disaggregate ss value}}{(\text{Mean demand/wk}) \times (\text{SD of demand}) \times 52}$
Slow vs Fast Moving Items

- Low demand = Slow moving items, vice versa.
  - Repair parts are typically slow moving items
- Slow moving items have high coefficient of variation, vice versa.
- Stock slow moving items at a central store

Buying a best seller at Amazon.com vs. a Supply Chain book vs. a Banach spaces book, which has a shorter delivery time?

Slow moving items are kept at upstream supply chain:

- Blinder spare parts: “Cylinder connecting the stick to the rotation mechanism” inside the blinder. Not available in major hardware stores. - July 2009.
- “Case Interview books” are not in our s.k.u. list. You must check with our central stores. - Store keeper at Barnes and Nobles at Collin Creek, March 2002.
Product Substitution

- Manufacturer driven or Customer driven
- One-way substitution
  - Downward substitution
    - No regular coffee, get a latte
    - Army boots. What if your boot is large? Aggregate?
- Two-way substitution:
  - Grainger motors; water pumps model DN vs IT.
  - Similar products, can customer detect specifications.

If products are very similar, why not to eliminate one of them?

Dell Example:

- Dell producing **27 products** with **3 components** (processor, memory, hard drive)
- **No product commonality:** A component is used in **only 1 product**. **27 component versions** are required for each component. A total of **3*27 = 81 distinct components** are required.
- Component commonality allows for component inventory aggregation.
Max Component **Commonality**

- Only three distinct versions for each component.
  - Processors: P1, P2, P3. Memories: M1, M2, M3. Hard drives: H1, H2, H3
- Each combination of **components** is a distinct **product**. A **component** is used in 9 **products**.
- Each way you can go from left to right is a product.

![Diagram showing product combinations](diagram.png)

**Product Demands**

- Maximum Commonality
  - 9 products use the same component

**Component Demand**

- Centralization
Standardization

◆ **Standardization:** Extent to which there is an absence of variety in a product, service or process
◆ Standardized products are immediately available to customers
◆ Who wants standardization?
  – The day we sell standard products is the day we lose a significant portion of our profit.
  – A TI manager on November 1, 2005

**Disadvantages**
◆ Decreased variety results in less consumer appeal.
◆ Designs may be frozen with too many imperfections remaining.
◆ High cost of design changes increases resistance to improvements
  – Who likes optimal Keyboards?
◆ Standard systems are more vulnerable to failure: Epidemics, Computer security.

**Advantages**
◆ Fewer parts to deal with in inventory & manufacturing
◆ Reduced training costs and time
◆ More routine purchasing, handling, and inspection procedures
◆ Opportunities for long production runs, automation
◆ Fewer parts justify increased expenditures on perfecting designs and improving quality control procedures.
Summary

- Inventory Centralization
- Component Commonality
- Standardization
Inventory–Transportation Costs: Eastern Electric Corporation: p.427

- Major appliance manufacturer, buys motors from Westview motors in Dallas
- Annual demand = 120,000 motors
- Cost per motor = $120; Weight per motor 10 lbs.
- Current order size = 3,000 motors
  - 30,000 pounds = 300 cwt
    - 1 cwt = centum weight = 100 pounds; Centum = 100 in Latin.
- Lead time = 1 + the number of days in transit
- Safety stock carried = 50% of demand during delivery lead time
- Holding cost = 25%
- Evaluate transportation mode for all unit discounting based on shipment weight
AM Rail proposal:
Over 20,000 lbs at 0.065 per lb in 5 days

◆ For the appliance manufacturer
  – No fixed cost of ordering besides the transportation cost
  – No reason to transport at larger lots than 2000 motors, which make 20,000 lbs.
    » Cycle inventory=$\frac{Q}{2}=1,000$
    » Safety inventory=$(6/2)(\frac{120,000}{365})=986$
    » In-transit inventory
      ◆ All motors shipped 5 days ago are still in-transit
      ◆ 5-days demand=$(\frac{120,000}{365})5=1,644$
  – Total inventory held over an average day=3,630 motors
  – Annual holding cost=3,630*120*0.25=$108,900
  – Annual transportation cost=120,000(10)(0.065)=$78,000
Inventory—Transportation trade off: Eastern Electric Corporation, see p.426-8 for details

<table>
<thead>
<tr>
<th>Alternative (Lot size)</th>
<th>Transport Cost</th>
<th>Cycle Inventory</th>
<th>Safety Inventory</th>
<th>Transit Inventory</th>
<th>Inventory Cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM Rail (2,000)</td>
<td>$78,000</td>
<td>1,000</td>
<td>986</td>
<td>1,644</td>
<td>$108,900</td>
<td>$186,900</td>
</tr>
<tr>
<td></td>
<td>120000(0.65)</td>
<td></td>
<td></td>
<td>120000(5/365)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northeast Trucking (1,000)</td>
<td>$90,000</td>
<td>500</td>
<td>658</td>
<td>986</td>
<td>$64,320</td>
<td>$154,320</td>
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<tr>
<td>Golden (500)</td>
<td>$96,000</td>
<td>250</td>
<td>658</td>
<td>986</td>
<td>$56,820</td>
<td>$152,820</td>
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<tr>
<td></td>
<td>120000(0.80)</td>
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<td></td>
<td>120000(3/365)</td>
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<tr>
<td>Golden (2,500)</td>
<td>$86,400</td>
<td>1,250</td>
<td>658</td>
<td>986</td>
<td>$86,820</td>
<td>$173,220</td>
</tr>
<tr>
<td>Golden (3,000)</td>
<td>$78,000</td>
<td>1,500</td>
<td>658</td>
<td>986</td>
<td>$94,320</td>
<td>$172,320</td>
</tr>
<tr>
<td>Golden (4,000)</td>
<td>$67,500</td>
<td>2,000</td>
<td>658</td>
<td>986</td>
<td>$109,320</td>
<td>$176,820</td>
</tr>
</tbody>
</table>

If fast transportation not justified cost-wise, need to consider rapid response
Physical Inventory Aggregation: Inventory vs. Transportation cost: p.428

- HighMed Inc. producer of medical equipment sold directly to doctors
- Located in Wisconsin serves 24 regions in USA
- As a result of physical aggregation
  - Inventory costs decrease
  - Inbound transportation cost decreases
    » Inbound lots are larger
  - Outbound transportation cost increases
Inventory Aggregation at HighMed

Highval ($200, .1 lbs/unit) demand in each of 24 territories
- $\mu_H = 2$, $\sigma_H = 5$

Lowval ($30/unit, 0.04 lbs/unit) demand in each territory
- $\mu_L = 20$, $\sigma_L = 5$

UPS rate: $0.66 + 0.26x$  \{for replenishments\}
FedEx rate: $5.53 + 0.53x$  \{for customer shipping\}

Customers order 1 H + 10 L
# Inventory Aggregation at HighMed

<table>
<thead>
<tr>
<th></th>
<th>Current Scenario</th>
<th>Option A</th>
<th>Option B</th>
</tr>
</thead>
<tbody>
<tr>
<td># Locations</td>
<td>24</td>
<td>24</td>
<td>1</td>
</tr>
<tr>
<td>Reorder Interval</td>
<td>4 weeks</td>
<td>1 week</td>
<td>1 week</td>
</tr>
<tr>
<td>Inventory Cost</td>
<td>$54,366</td>
<td>$29,795</td>
<td>$8,474</td>
</tr>
<tr>
<td>Shipment Size</td>
<td>8 H + 80 L</td>
<td>2 H + 20 L</td>
<td>1 H + 10 L</td>
</tr>
<tr>
<td>Transport Cost</td>
<td>$530</td>
<td>$1,148</td>
<td>$14,464</td>
</tr>
<tr>
<td>Total Cost</td>
<td>$54,896</td>
<td>$30,943</td>
<td>$22,938</td>
</tr>
</tbody>
</table>

*If shipment size to customer is 0.5H + 5L, total cost of option B increases to $36,729.*
Summary of Cycle and Safety Inventory

Match Supply & Demand

Reduce Buffer Inventory

Economies of Scale
- Reduce fixed cost
- Aggregate across products
- Volume discounts
- Promotion on Sell thru

Cycle Inventory

Supply / Demand Variability
- Quick Response measures
  - Reduce Info Uncertainty
  - Reduce lead time
  - Reduce supply uncertainty

Safety Inventory

Seasonal Variability
- Accurate Response measures
  - Aggregation
  - Component commonality and postponement

Seasonal Inventory
Mass Customization

Mass customization:
- A strategy of producing standardized goods or services, but incorporating some degree of customization
- Modular design
- Delayed differentiation
Mass Customization I: **Customize Services** Around Standardized Products

Warranty for contact lenses:

Source: B. Joseph Pine

- **DEVELOPMENT**
  - Continue developing standardized products or services

- **PRODUCTION**
  - Continue producing standardized products or services

- **MARKETING**
  - Market customized services with standardized products or services

- **DELIVERY**
  - Deliver customized services as well as standardized products and services
Mass Customization II: Create Customizable Products and Services

Customizing the look of screen with windows operating system
Gillette sensor adjusting to the contours of the face

<table>
<thead>
<tr>
<th>DEVELOPMENT</th>
<th>PRODUCTION</th>
<th>MARKETING</th>
<th>DELIVERY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deliver standard (but customizable) products or services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market customizable products or services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Produce standard (but customizable) products or services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Develop customizable products or services</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Mass Customization III: Provide Quick Response Throughout Value Chain

Skiing parkas manufactured abroad vs. in the U.S.A.:

- Reduce development cycle time
- Reduce Production cycle time
- Reduce selection and order processing cycle times
- Reduce Delivery Cycle Times
Mass Customization IV: Provide **Point of Delivery Customization**

- Develop products where point of delivery customization is feasible
- Produce standardized portion centrally
- Market customized products or services
- Deliver standardized portion
- Point of delivery customization

Paint mixing
Lenscrafters for glasses.
Mass Customization V: Modularize Components to Customize End Products

Computer industry, Dell computers:

- Develop modularized products
- Produce modularized components
- Market customized products or services
- Deliver customized product
Modular Design

*Modular design* is a form of standardization in which component parts are subdivided into *modules that are easily replaced or interchanged*.

- **Good** example: Dell uses same components to assemble various computers.
- **Bad** example: Earlier Ford SUVs shared the lower body with Ford cars.
- **Ugly** example:

It allows:

- easier diagnosis and remedy of failures
- easier repair and replacement
- simplification of manufacturing and assembly
Types of Modularity for Mass Customization

Component Sharing Modularity, Dell

Cut-to-Fit Modularity, Gutters that do not require seams

Bus Modularity, E-books

Mix Modularity, Paints

Sectional Modularity, LEGO
Periodic Review

Order at fixed time intervals (T apart) to raise total inventory (on hand + on order) to Order up to Level (OUL)

Inventory

OUL

OUL must cover the Demand during \(T + LT\)

\(T\)
Periodic Review Policy: Safety Inventory

T: Reorder interval
\( \sigma_R: \) Standard deviation of demand per unit time
\( \sigma_{L+T}: \) Standard deviation of demand during \( L+T \) periods

\( OUL: \) Order up to level

\[
R_{T+L} = (T + L)R
\]
\[
\sigma_{T+L} = \sigma \sqrt{L + T}
\]
\[
ss = F^{-1}(CSL;0,1) \cdot \sigma_{T+L}
\]
\[
OUL = R_{T+L} + ss
\]
Example: Periodic Review Policy

\[ R = 2,500/\text{week}; \sigma_R = 500 \]
\[ L = 2 \text{ weeks}; \ T = 4 \text{ weeks}; \ CSL = 0.90 \]

What is the required safety inventory?

\[ ss = F^{-1}(CSL;0,1) \cdot \sigma_{T+L} = 1570 \]

Factors driving safety inventory

- Demand uncertainty
- Replenishment lead time
- Reorder interval
Periodic vs Continuous Review

- Periodic review ss covers the uncertainty over $[0,T+L]$, $T$ periods more than ss in continuous case.
- Periodic review ss is larger.
- Continuous review is harder to implement, use it for high-sales-value per time products