Pricing and Production Game under Revenue Sharing and Information Update

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Outline

- Introduction and motivation
- Literature review
- Model: 1) Whole Sale Game; 2) Revenue Sharing Game
- Information Update
- Multiperiod Games: 1) Two Period Games; 2) Finitely Repeated Games
- Concluding remarks

Introduction and Motivation

Industry Initiatives in Supply Chain Management:

- Vendor Managed Inventory System: Dell Inc, Wal Mart Stores Inc, Kraft Inc.
  Retailer decides on the price; with less control for inventory, loss of market share, and increased risk of disruption; sets the higher sale price to make more profit.
  Supplier decides on the inventory; with worrying about the excess inventory cost, produces less inventory.
  Information Technology allows Vendor Managed Inventory.
  EDI, Quick Response(QR), Collaborative Planning Forecasting and Replenishment(CPFR), and Just-in-Time Distribution(JITD).

- Revenue Sharing: Blockbuster and Studios
  Higher tape price and lower rental price lead to poor video availability.

Literature review

- N.C. Petruzzi and M. Dada(1999):- Survey: The Newsvendor Pricing.
- A. Federgruen and A. Heching(1999) and Chen and Simchi-Levi(2003):- Dynamic Pricing, Base Stock List Price Policy. With Fixed Ordering Cost, (s, S, A, p) Policy.
- S.P.Sethi and F.M.Bass(2002):- Optimal pricing with hazard rate model of demand.
- G.P. Cachon and M.A. Lariviere(2001):- Supply Chain Coordination with Revenue-Sharing Contracts.
- Q. Li and D. Atkins(2002):- Coordinating Relenishment and Pricing, Production Game and Service Game.
- Iyer and Bergen(1997):- Traditional Channel, Manufacturer doesnot always benefit from Quick Response.

Some notation

- p, K: Sale price and production quantity;
- I: Initial inventory;
- p_{min}: The lower bound for the retailer's sale price;
- w, c: Whole sale price and production cost;
- λ: Revenue sharing ratio;
- π, Π, Ω: The retailer, supplier and supply chain profit;
- $D(y(p), \epsilon) = y(p) + \epsilon$: Stochastic demand with mean $y(p) = a - bp$;
- ϵ: Additive demand randomness with IFR distribution.

Model: Whole Sale Game

Centralized Supply Chain System:

$$\Omega(p, K) = \max_{p,K} pS(p, K) - cK$$

Expected profit is not joint concave in p and K for most distributions, but the expected sales is.

Decentralized System: whole sale game where the supplier charges a fixed whole sale price for every unit sold.
Retailer Profit Function:

$$\pi^w(p, K) = \max_{p \geq w} (p - w)S(p, K)$$

Supplier Profit Function:

$$\Pi^w(p, K) = cI + \max_{K \geq I} wS(p, K) - cK$$

The Game has unique Nash Equilibrium[Li and Atkins(2002)].

Model: Wholesale Game

Observations:

i) The Nash Equilibrium price p^w decreases with standard deviation σ.

ii) Both retailer and supplier profits and Nash Equilibrium price p^w, quantity K^w decrease with demand sensitivity b.

iii) The double marginalization.

b	K^I	p^I	Ω^I	K^w	p^w	π^w	Π^w	Ω^w
5	94.78	21.95	1590.97	50.33	29.75	1109.38	139.96	1249.34
10	82.69	11.94	618.21	39.83	15.93	341.50	108.46	449.97
15	71.33	8.61	309.51	29.33	11.32	119.85	76.96	196.81
20	60.29	6.94	166.34	18.83	9.01	34.74	45.46	80.21
25	49.40	5.93	89.20	8.33	7.63	4.26	13.96	18.22

Model: Revenue Sharing Game

Decentralized System: revenue sharing game where the supplier sets a fixed revenue sharing ratio of channel revenue.
Retailer Profit Function:

$$\pi^{\lambda}(p, K) = \max_{p \geq p_{min}} (1 - \lambda) p S(p, K)$$

Response function:

$$S(p(K), K) - bp(K)F(K - y(p(K))) = 0$$

Supplier Profit Function:

$$\Pi^{\lambda}(p, K) = cI + \max_{K \geq I} \lambda p S(p, K) - cK$$

Response function:

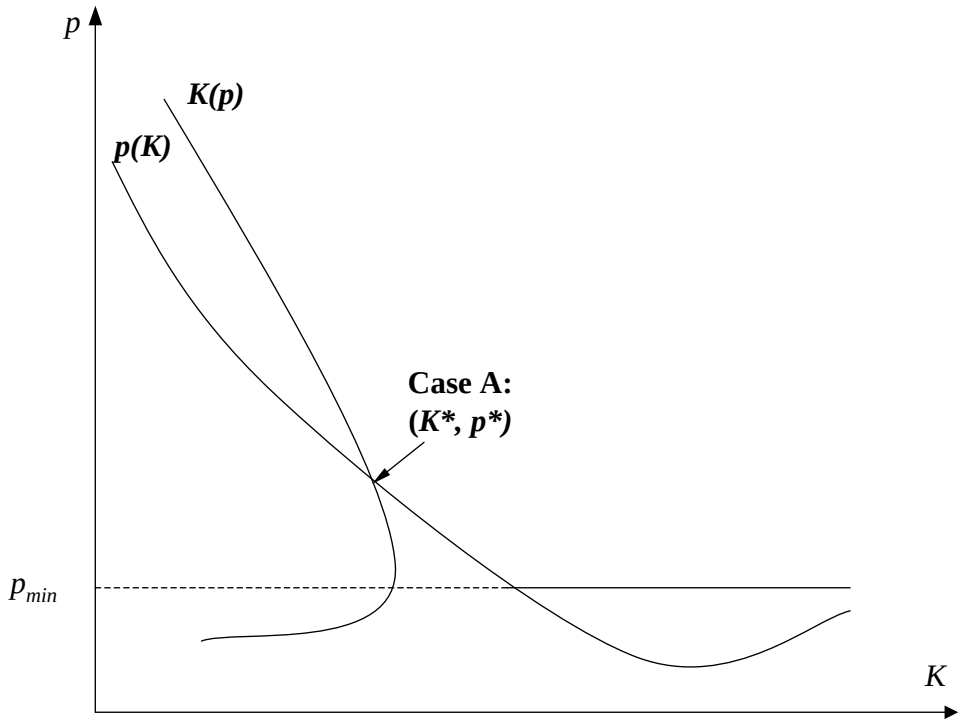
$$F(K(p) - y(p)) = 1 - \frac{c}{\lambda p}$$

The Game also has unique Nash Equilibrium.

Model: Revenue Sharing Game

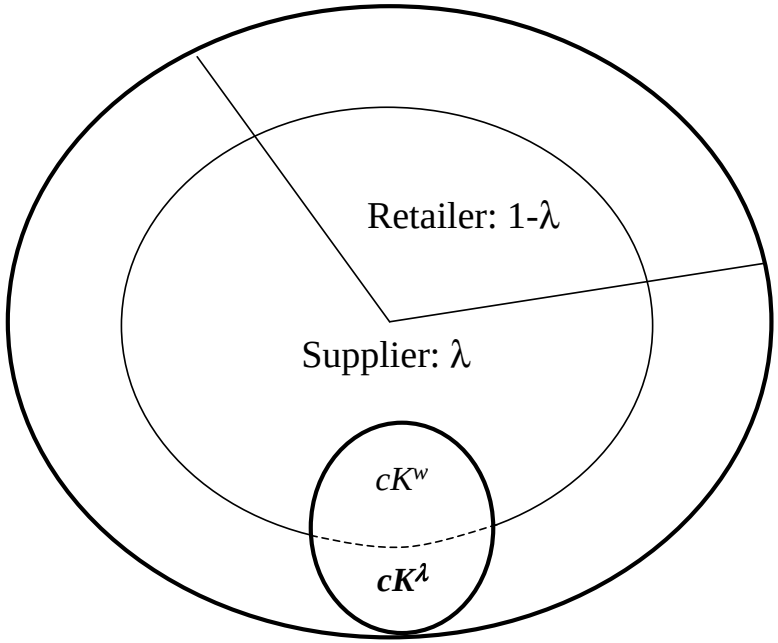
The monotone price property: with uncertain demand, the price decreases with quantity, if

$$p_{min}^2 \, \phi\left(\Phi^{-1}\left(1 - \frac{c}{\lambda} \frac{1}{p_{min}}\right)\right) \geq \frac{c}{\lambda} \frac{\sigma}{b}.$$



Comparison of Whole sale game and Revenue sharing game

If $p_{min} \leq p^w$, the revenue sharing game can achieve pareto improving in terms of expected profit.

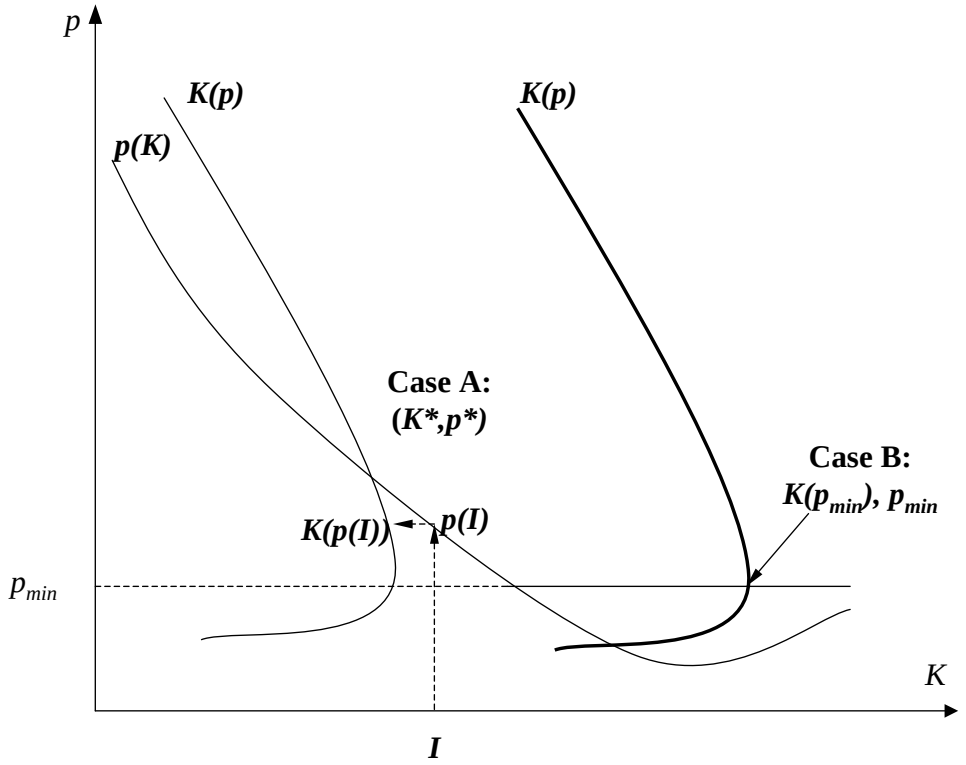


The Games with Initial Inventory

When $I \geq 0$, the whole sale game and the revenue sharing game have unique Nash Equilibrium:

$$\left\{ \begin{array}{ll} (p^*, K^*) & \text{if } I \leq K^* \\ (p(I), I) & \text{otherwise} \end{array} \right\}.$$

Retailer and Supplier reaction curves in revenue sharing game:

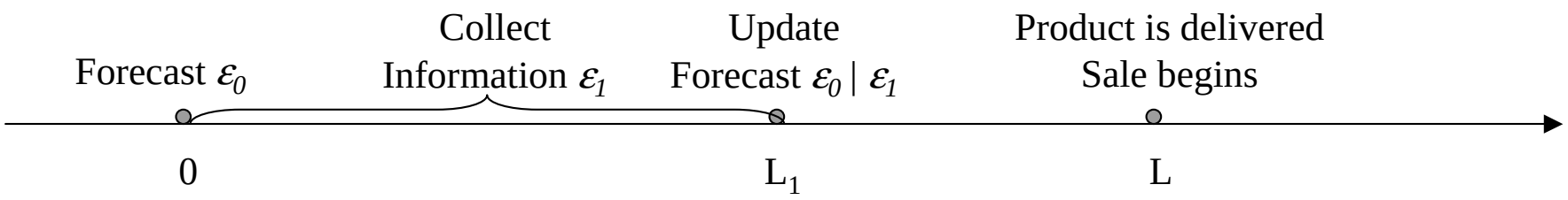


The Effect of Information Update

- The positive effect:
  Better information increases the sales and decrease the excess inventory;

- The negative effect:
  Better information increases the retailer price and decreases the supplier production quantity; the double marginalization is worse off.

- The information update structure:



The Effect of Information Update

- Bivariate normal information update mode:
  ϵ_0, ϵ_1–BIN$(0, 0, \sigma_0, \sigma_1, \rho)$.

- The updated demand forecast distribution $H(\epsilon_0|\epsilon_1)$ has mean and standard deviation:

$$\mu = \rho\frac{\sigma_0}{\sigma_1}\epsilon_1$$

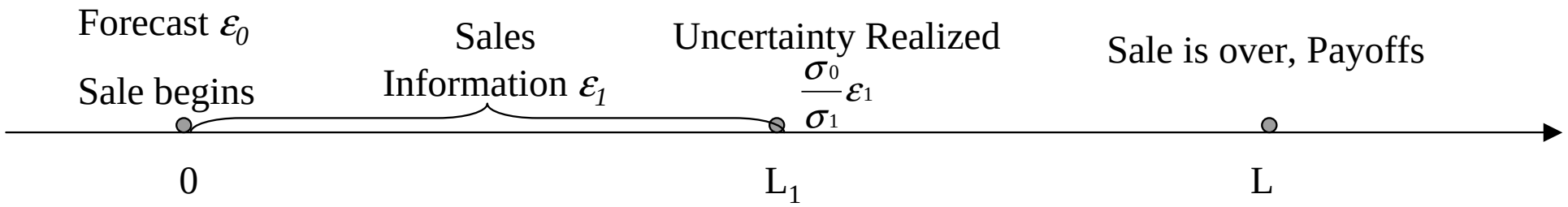
$$\sigma = \sigma_0\sqrt{1-\rho^2}$$

- ϵ_1: the market information related to the product, the higher ϵ_1, the more demand. ρ: the information quality. ρ is bigger, the variance is smaller, the information is better.

In VMI system, the retailer and supplier are better off from information update in both whole sale game and revenue sharing game. Their expected profits increase with information quality ρ.

Two Period Games with perfect information $\rho = 1$

- Sales begins in the first period;
- At the second period, the demand is certain and equal to $y(p) + \frac{\sigma_0}{\sigma_1}\epsilon_1$;
- The sequence of events ;



Two Period Games with perfect information

- At the second period, the Nash Equilibrium is: $(p_2,\ y(p_2) + \frac{\sigma_0}{\sigma_1}\epsilon_1)$;
  where $p_2 = \max\{\frac{a+\frac{\sigma_0}{\sigma_1}\epsilon_1}{2b}, p_{min}\}$

- At the second period, the supplier has three production states: no production, partial production, or total production.

- At the first period;
  i) For given K_1, the retailer's expected profit is concave in p_1. For given p_1, the supplier's expected profit is concave in K_1;
  ii) There exists unique subgame perfect Nash Equilibrium.

Finitely Repeated Games

- Assumptions:
  i) The initial inventory level is zero.
  ii) The demand is nonnegative and stationary;
  iii) The unsatisfied demand is lost;
  iiii) At last period, the leftover inventory can be salvaged at value c.

- Inventory costs:
  h: per unit per time holding cost,
  v: per unit per time shortage cost,

Finitely Repeated Games

Retailer profit function:

$$\pi_n^{\lambda}(I_{n-1}) = \max_{p_n \geq p_{min}} (1-\lambda) p_n S_n(K_n(I_{n-1}), p_n) + \alpha E \pi_{n+1}^{\lambda}(I_n) \tag{1}$$

Supplier profit function:

$$\Pi_n^{\lambda}(I_{n-1}) = cI_{n-1} + \max_{K_n \geq I_{n-1}} \lambda p_n S_n(K_n, p_n(I_{n-1})) - cK_n - hE(K_n - D_n)^+ \\ -vE(K_n - D_n)^- + \alpha E \Pi_{n+1}^{\lambda}(I_n) \tag{2}$$

where $I_n = K_n(I_{n-1}) - D_n(p_n) \vee 0$.

There exists is a myopic Nash Equilibrium (K^*, p^*) in every period.

Numerical Study

- Base cases:
  $a = 200$, $b = 10$, $c = 4$, $w = 7$, $\sigma = 5$.

- Revenue sharing gap (RG):

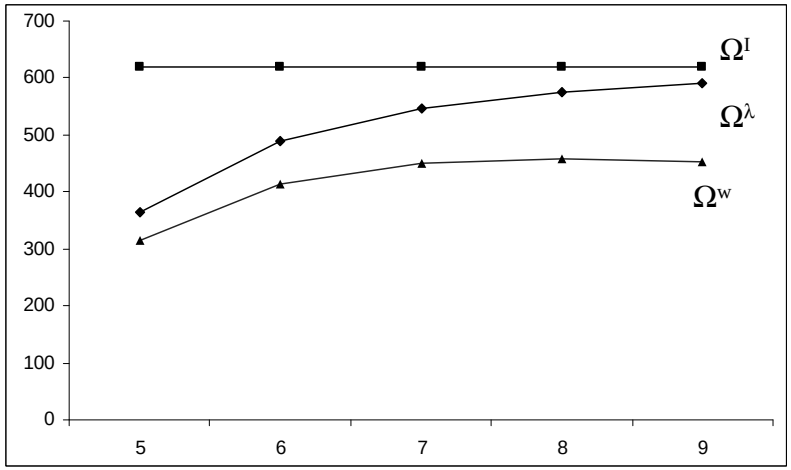
$$RG = \frac{\Omega^{\lambda} - \Omega^{w}}{\Omega^{I} - \Omega^{w}}$$

- Whole sale price (w) effect:

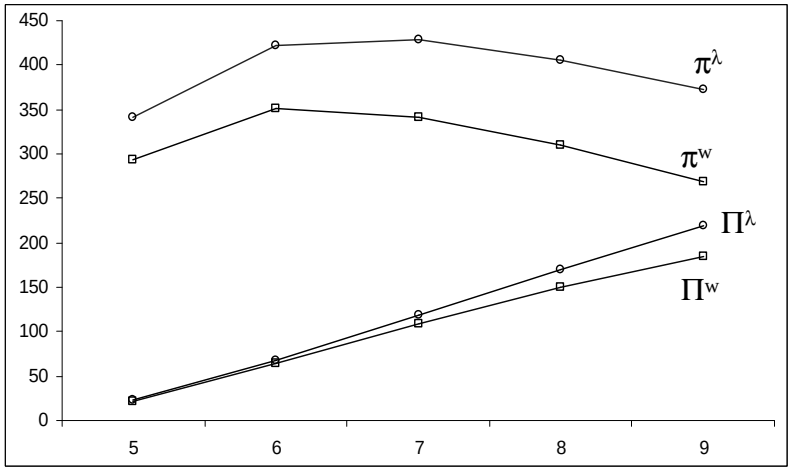
- Demand uncertainty (σ) and sensitivity(b) effect:

- Information Quality (ρ) effect.

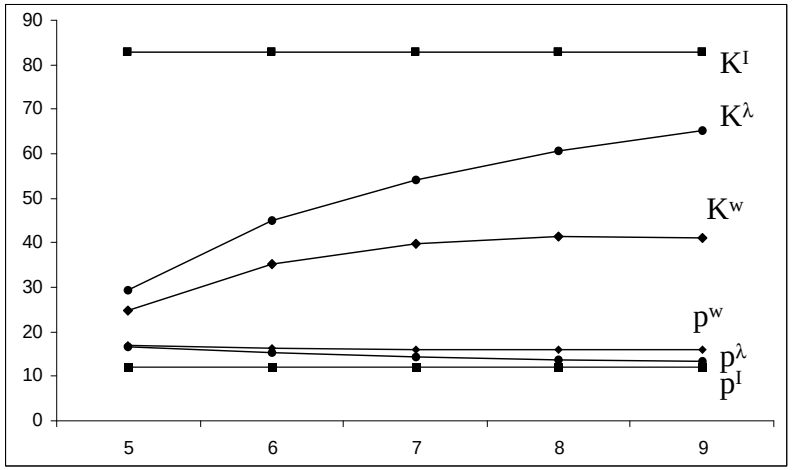
The Effect of Wholesale Price: $w \in (5, 6, 7, 8, 9)$



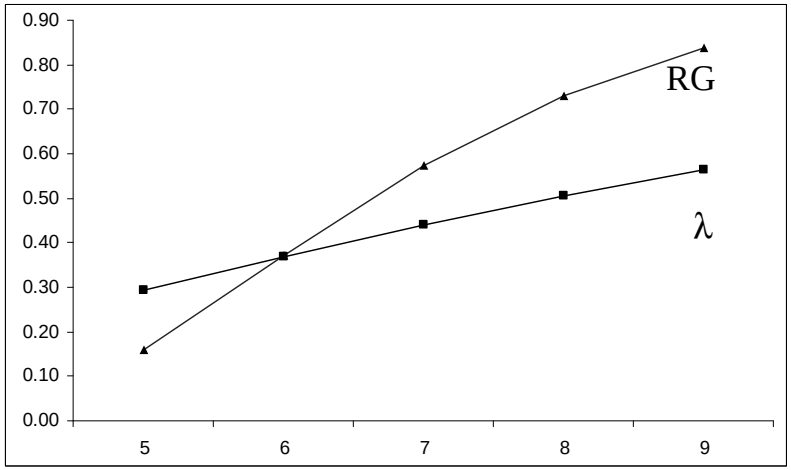
Supply Chain Profit



Retailer and Supplier Profit

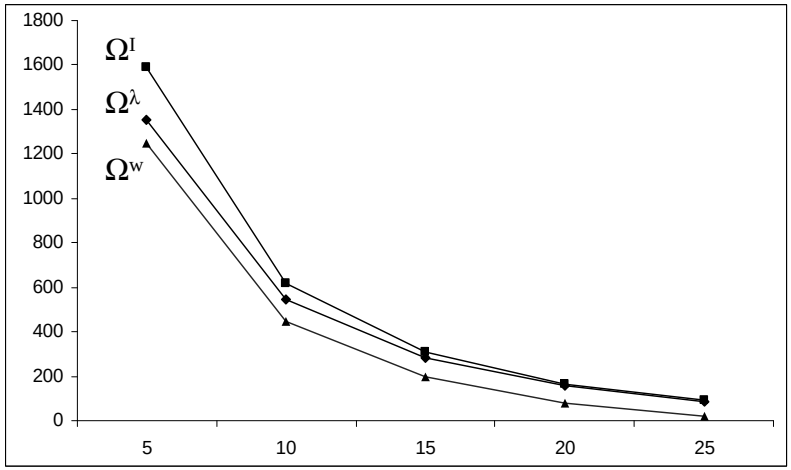


Equilibrium Price and Quantity

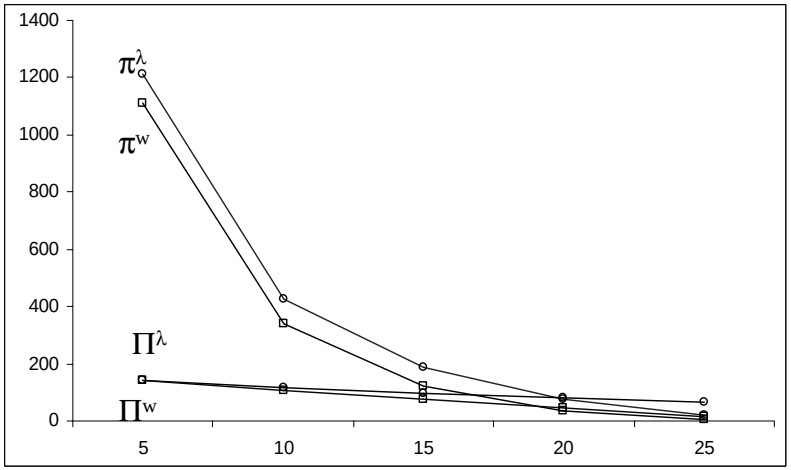


Revenue sharing ratio and Gap

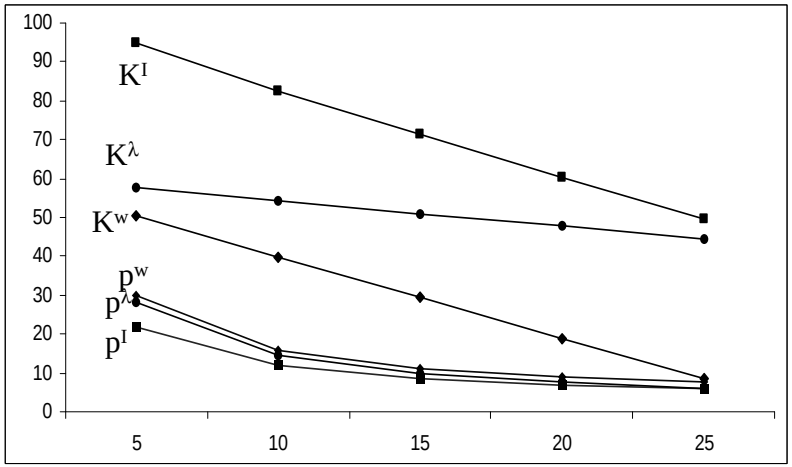
The Effect of Demand Sensitivity: $b \in (5, 10, 15, 20, 25)$



Supply Chain Profit



Retailer and Supplier Profit



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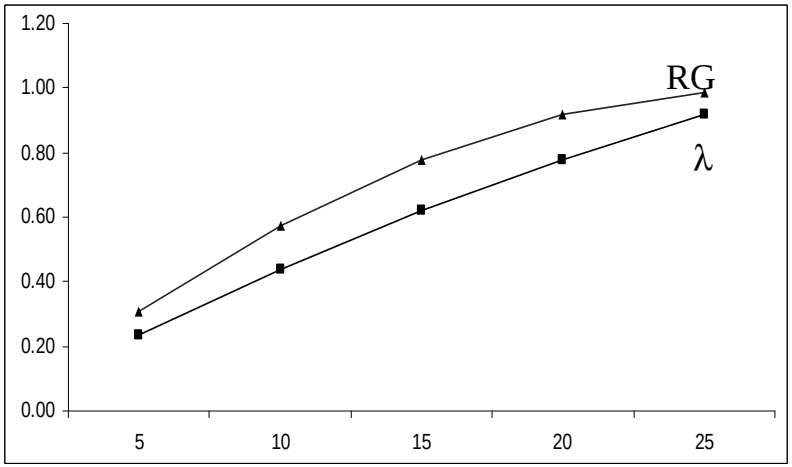
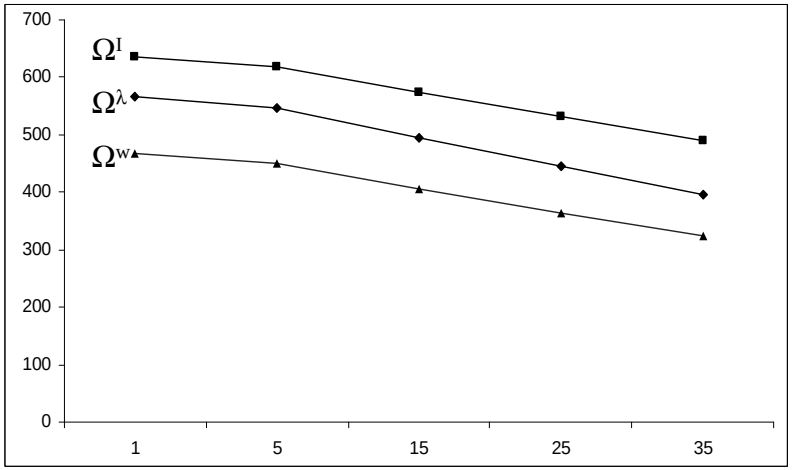
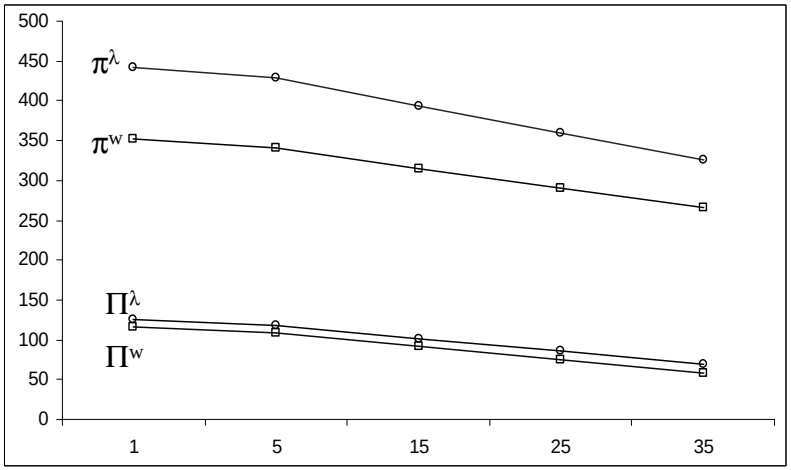


Revenue sharing ratio and Gap

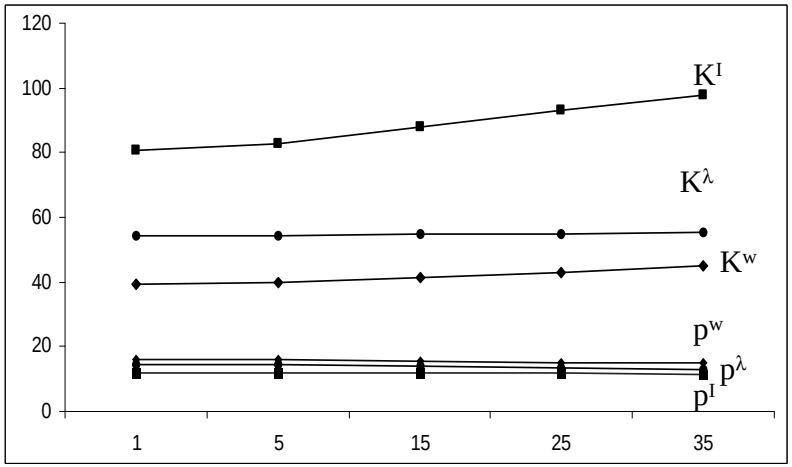
The Effect of Demand Uncertainty: $\sigma \in (1, 5, 15, 25, 35)$



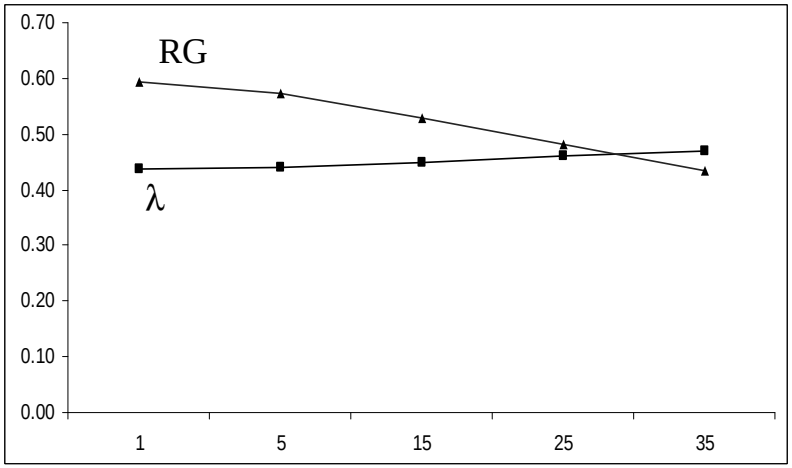
Supply Chain Profit



Retailer and Supplier Profit

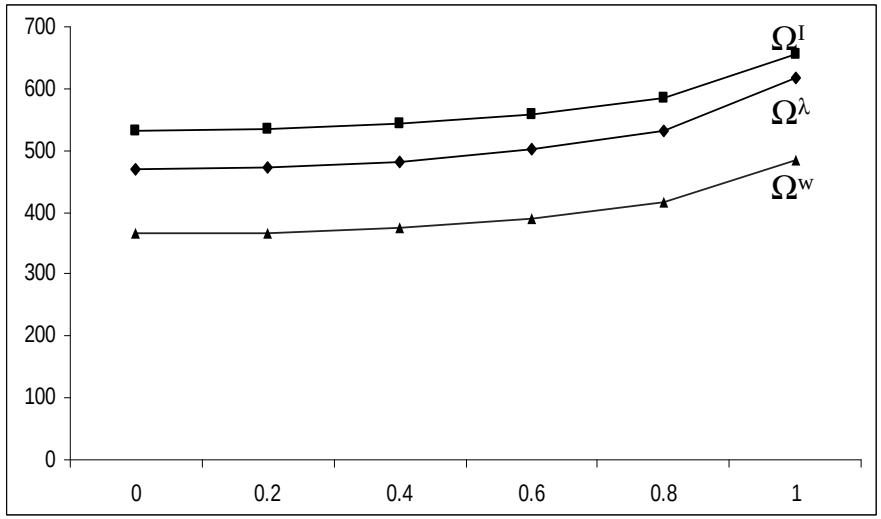


Equilibrium Price and Quantity

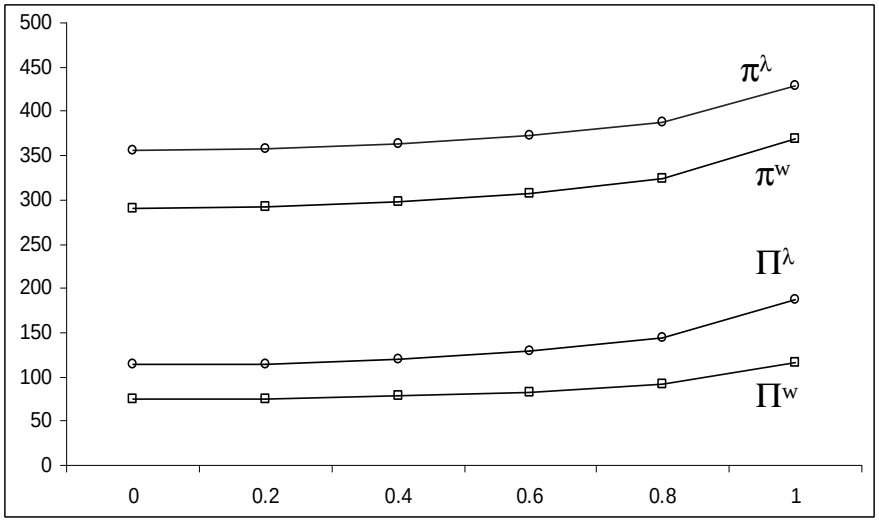


Revenue sharing ratio and Gap

The Effect of Information Update: $\rho \in (0, 0.2, 0.4, 0.6, 0.8, 1.0)$



Supply Chain Profit



Retailer and Supplier Profit

p^λ

Concluding remarks and future research

Conclusion:

- Revenue Sharing: Pareto Improving.
- VMI benefit from information update.
- Subgame perfect N.E exists in two period game with perfect information
  Myopic N.E exists in finitely repeated games.

Future Research:

- Two period model with nonperfect information update.
- Nonlinear demand model and multiplicative uncertainty.
- Coordination Mechanism.