Appendix A: Fourier transform theorems

This is a list of Fourier transform theorems and pairs expressed in terms of the frequency variable f whose units are Hertz. They differ from the expressions where the frequency variable is $\omega = 2\pi f$ in a scaling factor. In EE3150 ALL the work must be done in terms of the variable f.

Operation	Function	Fourier transform
Linearity	$a_1 g_1(t) + a_2 g_2(t)$	$a_1 G_1(f) + a_2 G_2(f)$
Time delay	$g(t - T_d)$	$G(f)\exp(-j2\pi f T_d)$
Scale change	$g(at)$	$(1/\|a\|)G(f/a)$
Duality	$G(t)$	$g(-f)$
Real signal frequency translation	$g(t)\cos(2\pi f_c t + \theta)$	$\frac{1}{2}[\exp(j\theta)G(f - f_c) + \exp(-j\theta)G(f + f_c)]$
Convolution	$g_1(t) * g_2(t)$	$G_1(f)G_2(f)$
Multiplication	$g_1(t)g_2(t)$	$G_1(f) * G_2(f)$

Function	Time waveform $g(t)$	Spectrum $G(f)$
Rectangular	$\text{rect}(t/\tau)$	$\tau\text{sinc}(\tau f)$
Constant	1	$\delta(f)$
Impulse at $t = t_o$	$\delta(t - t_o)$	$\exp(-j2\pi t_o f)$
Sinc	$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
Phasor	$\exp[j(2\pi f_o t + \phi)]$	$\exp(j\phi)\delta(f - f_o)$
Sinusoid	$\cos(2\pi f_c t + \phi)$	$\frac{1}{2}[\exp(j\phi)\delta(f - f_c) + \exp(-j\phi)\delta(f + f_c)]$
Impulse train	$\sum_{k=-\infty}^{\infty}\delta(t - kT)$	$f_o\sum_{n=-\infty}^{\infty}\delta(f - nf_o),\ f_o = 1/T$

$$\text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & |t| < \frac{\tau}{2} \\ 0 & |t| > \frac{\tau}{2} \end{cases}$$

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$