Experiment 1: Fourier Theory

This experiment verifies in experimental form some of the properties of the Fourier transform using electrical signals produced in the laboratory. Measurements and data collection are done by using computer controlled equipment (virtual instruments).

1 Introduction

In previous courses of engineering school you have been dealing with electrical signals only in the time domain. You are familiar with the concepts of amplitude, period, DC offset level, power, etc., related to a signal. In this course however, you will study the signals from the *frequency* domain perspective. Fourier analysis makes possible the representation of signals and systems in the frequency domain. The analysis and design of communication systems are commonly achieved in the frequency domain.

1.1 Fourier series

In simple words, Fourier theory establishes that a signal[1] can be represented as an infinite sum of sinusoids (a series) over *any* interval T_o. These sinusoids have different amplitudes (c_n) and their frequencies (nf_o) are multiples of the *fundamental* frequency $f_o = 1/T_o$. When $g(t)$ is a *periodic* signal and T_o is equal to its period, the *entire* signal is described by the sum of sinusoids. Mathematically, the *complex exponential Fourier series* of $g(t)$ is expressed as

$$g(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_o t) \tag{1}$$

where the *complex Fourier coefficients* are given by

$$c_n = \frac{1}{T} \int_{-T_o/2}^{T_o/2} g(t) \exp(-j2\pi n f_o t) dt;\ n = 0, \pm 1, \pm 2, \dots \tag{2}$$

Notice in these equations the presence of terms of *negative* frequency and *complex* exponential functions of frequency. In the physical world these two concepts are meaningless, however these equations are an excellent mathematical model to describe a physical signal. You will see during the lab that components of negative frequencies may indeed exist.

1.2 Fourier transform

Aperiodic signals can be represented by a Fourier series, however a more convenient frequency domain representation for aperiodic signals may be obtained by using the *Fourier transform.* From the Fourier integral theorem,

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi f t) df;\ -\infty < t < \infty \tag{3}$$

[1] The signal $g(t)$ must satisfy the *Dirichlet* conditions. The signals generally used in communication systems satisfy these conditions.

where $G(f)$ is the Fourier transform of $g(t)$,

$$G(f) = \int_{-\infty}^{\infty} g(t) \exp(-j2\pi ft)dt; \ -\infty < f < \infty \tag{4}$$

The pair $g(t)$ and $G(f)$ is called a Fourier transform pair and is mathematically expressed as $g(t) \leftrightarrow G(f)$.

1.3 Fourier spectrum

The representation of a by its Fourier transform (or series) determines the *frequency content* of the signal, in other words, it describes the amplitudes, frequencies and relative phases of the sinusoidal waveforms that *compose* the signal. A graphical representation of the frequency content of a signal is the *Fourier spectrum.* The *magnitude spectrum* consists of plotting the *magnitude* of the Fourier transform $G(f)$ (or the coefficients c_n) versus frequency, and the *phase spectrum* plots the angle of $G(f)$ (or c_n) versus frequency. Generally, the magnitude spectrum is referred to simply as the spectrum of the signal.

In this course you will deal only with the magnitude spectrum of signals. Notice that the Fourier transform of aperiodic signals yields a continuous magnitude spectrum, whereas the Fourier series of periodic signals produces spectra with discrete frequency components. Also notice that Eqs. (1)-(4) present the description of the Fourier theory in terms of the frequency variable f as oppose to $\omega (= 2\pi f)$. **All the work in EE3150 must be carried in the domain of the linear frequency f in Hertz**. Refer to Appendix A for a table of Fourier transforms in terms of f.

1.4 Prelab instructions

Enclosed in brackets is the number of points assigned to each question/plot. There are 30 total points. **Follow the procedures of Appendix B to create the required plots**.

1. Compute the Fourier transform $G(f)$ [1] and create time [3] and frequency [3] domain plots of the signal $g(t) = A\sin(2\pi f_o t)$ with the following parameters: $A = 10$ V, $f_o = 1$ kHz, $TS = 2$ ms, $FS = 5$ kHz and $NF = -30$ dBV. [7 points total].

2. Investigate (from a book, web search) the Fourier series of a triangle wave [2] and plot its spectrum [4]. Use the following parameters: $A = 10$ V, $f_o = 1$ kHz, $TS = 2$ ms, $FS = 10$ kHz and $NF = -40$ dBV. [6 points total].

3. Consider the following signal (you may want to sketch it, however it is not required) where the units are V and μs:

$$g(t) = \begin{cases} 0, & t < 175 \\ 10, & 175 \leq t \leq 225 \\ 0, & t > 225 \end{cases}$$

   Using the Fourier transform pair $A\text{rect}(t/\tau) \leftrightarrow A\tau\text{sinc}(\tau f)$ and the time delay property of the Fourier transform, find $G(f)$ [3] and plot its spectrum [4] in the frequency span $FS = 100$ kHz with $NF = -100$ dBV. [7 points total].

4. From the previous transform pair and by applying the duality property of the Fourier transform (see Appendix A), find $g(t)$ for $G(f) = \text{rect}(f/W)$. [2 points total].

5. By applying the real signal frequency translation property of the Fourier transform, obtain $G(f)$ [2] and create time [3] and frequency [3] domain plots of the signal $g(t) = A\sin(2\pi f_o t)\cos(2\pi f_c t)$. Use the following parameters: $A = 10$ V, $f_o = 1$ kHz, $f_c = 18$ kHz, $TS = 2$ ms, $FS = 20$ kHz and $NF = -30$ dBV. [8 points total].

These signals and their spectra will be observed experimentally in the lab. A **floppy disk will be required** to collect the results during the lab.

2 Lab procedure

The number of points assigned to each measurement or plot is enclosed in brackets and the plots that will be obtained are labeled P1, P2,..., just for reference purposes.

GENERAL INSTRUCTIONS:

- Load the virtual instrument TIMEFREQ.VI by double-clicking the shortcut located in the computer desktop.
- **After taking each plot make sure to ask your TA to verify that your results are correct. This also serves to monitor your progress and performance.**

2.1 Spectrum of a sine wave

In this section the spectrum of the sine wave of Prelab 1.1 will be obtained experimentally. [4 points, P1].

1. Connect the output of the top function generator (FG1) to the oscilloscope probe of Channel 1. In FG1 set Amplitude $A = 10$ V and Frequency $f_o = 1$ kHz.
2. In the panel of the virtual instrument (VI) `TIMEFREQ.VI` enter the following parameters: `Channel=1`, `Time span=2m` (meaning 2 ms), `Frequency span=5` (the default units are kHz), `Save data=ON` and a proper filename (use always drive A to collect your data).
3. Run the VI by pressing `Ctrl-R`. The VI reads the time doamin signal from the oscilloscope, the computer calculates the Fourier transform numerically and creates time and frequency domain plots. Compare this plots with the ones you created in Prelab 1.1. Observe that the spectrum shows a noise floor at about -30 dBV. The data points are saved once you run the VI, you DO NOT have to use the `File` menu of the VI to save the data.

2.2 Spectrum of a triangle wave

In this section the spectrum of the triangle wave of Prelab 1.2 will be obtained experimentally. [4 points, P2].

1. In FG1, change the waveform to triangle.
2. In the panel of the VI enter the following parameters: `Channel=1`, `Time span=2m`, `Frequency span=10`, `Save data=ON` and a proper filename.
3. Run the VI by pressing `Ctrl-R`. Compare the spectrum with your result of Prelab 1.2.

2.3 Spectrum of a rectangular pulse

In this section the spectrum of the rectangular pulse of Prelab 1.3 will be obtained experimentally. [5 points, P3].

1. In FG1, change the waveform to square. Set Amplitude=5 V, Offset=2.5 V, Frequency=2.4 kHz. Set the duty cycle to 20% by pressing `Shift` `Offset` . Use the burst mode of the FG by pressing `Shift` `Burst` (this mode allows for a narrower pulse at the output).
2. In the panel of the VI enter the following parameters: `Channel=1`, `Time span=1m`, `Frequency span=100`, `Save data=ON` and a proper filename.
3. Run the VI and compare the spectrum with your result of Prelab 1.3.

2.4 Duality property of the Fourier transform

In this section the duality property of the Fourier transform is verified experimentally using the functions of Prelab 1.4. [5 points, P4].

1. Reset the FG1 by turning it off and on. Change the output waveform to `SINC` by pressing `Shift` `Arb` `Enter`. Set Amplitude=6 V, Frequency=2 kHz.
2. In the panel of the VI enter the following parameters: `Channel=1`, `Time span=1m`, `Frequency span=200`, `Save data=ON` and a proper filename.
3. Run the VI to collect the data.

2.5 Frequency translation property of the Fourier transform

In this section the real signal frequency translation property of the Fourier transform is verified experimentally using the functions of Prelab 1.5. [6 points, P5].

1. In FG1 set the output waveform to sine, Amplitude=10 V, Frequency=1 kHz. Turn on the bottom function generator (FG2). Set its output amplitude to 10 V and Frequency=18 kHz.
2. Turn on the power supply and set the `+20 V` and `-20 V` outputs to ± 15 V. Notice that the ground terminal of the power supply is the one labeled `COM` and not the one labeled $\perp$. Turn the power supply off.
3. Assemble the circuit of Fig. 1 of the PSPICE schematic[2]. This is a multiplier circuit whose output is the product of the applied input signals divided by 10 V. Connect the oscilloscope probe of channel 1 to the output of the circuit at pin 7 of the AD633.
4. Turn on the power supply and press `Autoscale` in the oscilloscope. You should see a similar signal as your plot of Prelab 1.5.
5. Run the VI with the following parameters: `Channel=1`, `Time span=2m`, `Frequency span=20`, `Save data=ON` and a proper filename.

2.6 Spectra of modulated signals

In this section the spectra of amplitude modulated (AM) [3 points, P6] and frequency modulated (FM) [3 points, P7] signals are obtained and compared.

1. Connect the output of FG1 to the oscilloscope probe of channel 1. Set Amplitude=10 V and Frequency=2 kHz. Output an AM signal by pressing `Shift` `AM`. Press `Shift` `Freq` and enter 500 Hz. Press `Shift` `Level` and enter 40%. These are parameters of an AM signal that you will consider in detail in Experiment 4. Press `Autoscale` in the oscilloscope.
2. Run the VI with the following parameters: `Channel=1`, `Time span=10m`, `Frequency span=5`, `Save data=ON` and a proper filename. Observe the AM signal and its spectrum.
3. Now, output an FM signal by pressing `Shift` `FM`. Press `Shift` `Freq` and enter 500 Hz. Press `Shift` `Level` and enter 100 Hz. These are parameters of an FM signal that you will consider in detail in Experiment 5. Press `Autoscale` in the oscilloscope.
4. Run the VI changing just the filename. Observe the FM signal and its spectrum.

[2]The schematic has been drawn in PSPICE, however there is no PSPICE model of the integrated circuit AD633.

3 Analysis

The discussion of the experimental results is the purpose of this section. Answers to the questions must be included in the lab report.

3.1 Questions

Enclosed in brackets is the number of points assigned to each question.

1. Include in your report all the plots obtained during the lab. Make sure to label properly all the plots. The number of points assigned to each plot is specified in the lab procedure. Refer to Appendix B section 3.4 for instructions regarding the plotting of experimental results.

2. (a) What is the value in dBV of the component shown in the spectrum of plot P1? [2]. (b) What value would have the component if the amplitude of the signal were reduced by half? [2]. [4 points total].

3. From the spectrum of plot P2, what is the magnitude of the *fifth* harmonic? [2]

4. Observe that in the spectrum of plot P3 the deep peaks occur about 20 kHz apart. If they would occur every 10 kHz, (a) would the rectangular pulse be wider or narrower? [2]. (b) What would be the pulse width (in μs)? [2]. [4 points total].

5. What is the value of W (refer to Prelab 1.4) for the spectrum of plot P4? [2].

6. In plot P5 the spectrum of plot P1 (one component at 1 kHz) was translated to 18 kHz resulting in a component at 19 kHz. Explain the presence of the component at 17 kHz. [2].

7. Compare the spectra of plots P6 and P7. Does this mean the Fourier transforms of the AM signal and FM signal are equal? Explain. [2].

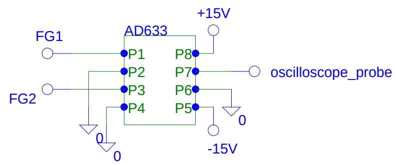

Fig. 1 Test circuit to demonstrate the frequency translation property

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