

# Experiment 5: Frequency Modulation

In this experiment, the generation of frequency modulated signals by the direct method and the demodulation of the message signal using a phase-lock loop circuit are considered. The spectra of tone-modulated FM signals are analyzed as well.

## 1 Introduction

In frequency modulation (FM), the phase angle of a carrier signal  $A_c \cos(2\pi f_c t)$  is varied proportionally to the integral of the message signal  $x(t)$ , the frequency modulated signal  $x_c(t)$  is:

$$x_c(t) = A_c \cos \left[ 2\pi f_c t + k_f \int_{-\infty}^t x(\alpha) d\alpha \right] \quad (1)$$

where  $k_f$  is a constant. The instantaneous frequency of the FM signal is given by:

$$f_i = f_c + \frac{k_f}{2\pi} x(t) \quad (2)$$

In the special case of tone-modulated FM, the message signal is a sinusoid

$$x(t) = A_x \cos(2\pi f_x t)$$

the instantaneous phase deviation of the modulated signal is

$$\phi(t) = \frac{k_f A_x}{2\pi f_x} \sin(2\pi f_x t) \quad (3)$$

and the modulated signal is given by:

$$x_c(t) = A_c \cos [2\pi f_c t + \beta \sin(2\pi f_x t)] \quad (4)$$

where the parameter  $\beta$  is called the *modulation index* defined as

$$\beta = \frac{k_f A_x}{2\pi f_x} \quad (5)$$

$$= \frac{\Delta f}{f_x} \quad (6)$$

and represents the maximum phase deviation produced by the message signal. The parameter  $\Delta f$  represents the *peak frequency deviation* of the modulated signal. The modulation index is defined only for tone-modulation.

The calculation of the spectrum of an FM signal is difficult since frequency modulation is a nonlinear process. However, for the special case of tone-modulation an equivalent expression for Eq.(4) is

$$x_c(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos [2\pi(f_c + n f_x)t] \quad (7)$$

This equation describes the modulated signal as a series of sinusoids whose coefficients  $J_n(\beta)$  are given by the Bessel function of the first kind and  $n$ th order.

The bandwidth of the tone-modulated FM signal can be estimated by the Carlson's rule, and it depends on the modulation index and the frequency of the modulating tone:

$$B_{FM} = 2(\beta + 1)f_x \quad (8)$$

From this equation it can be noticed that a small value of  $\beta$  ( $\ll 1$ ) will result in a bandwidth of about  $2f_x$ . Modulated signals with this condition are referred to as *narrowband* FM signals. Large values of the modulation index will produce signals with relatively large bandwidth, or *wideband* FM signals.

The direct generation of FM signals involves a voltage-controlled oscillator (VCO), in which the oscillation frequency varies linearly with the control voltage. When a message signal is used as the control voltage, the output signal of the VCO is frequency modulated.

In this experiment the phase-locked loop (PLL) circuit utilized for the generation of FM signals is the MC14046B. The demodulation of FM signals using PLL circuits is currently the most widely used method. A PLL circuit is constituted by a phase comparator (the MC14046B actually contains two), a VCO, and a loop filter (extern to the MC14046B).

## 2 Prelab instructions

Enclosed in brackets is the number of points assigned to each question/plot.

1. Obtain the spectrum of the FM signal described by Equation (7) by computing its Fourier transform. [2].
2. Create one-sided spectral plots of the spectrum found above. The Bessel function is available in MATLAB using the command `besselj`. For example,  $J_0(1.4)$  and  $J_{-2}(5.2)$  are obtained in Matlab using `besselj(0,1.4)` and `besselj(-2,5.2)`, respectively. The parameters for the FM signal and for the plot are specified next. [4 points each plot].
  - (a)  $A_c=10$  V,  $f_c=2.0$  kHz,  $f_x=500$  Hz,  $\Delta_f=50$  Hz, Noise floor NF=-25 dBV, Freq. span FS=4 kHz.
  - (b)  $A_c=10$  V,  $f_c=2.5$  kHz,  $f_x=500$  Hz,  $\Delta_f=500$  Hz. NF=-25 dBV, FS=5 kHz.
  - (c)  $A_c=10$  V,  $f_c=4.0$  kHz,  $f_x=500$  Hz,  $\Delta_f=1.2$  kHz. NF=-25 dBV, FS=8 kHz.
  - (d)  $A_c=10$  V,  $f_c=5.0$  kHz,  $f_x=500$  Hz,  $\Delta_f=2.5$  kHz. NF=-25 dBV, FS=10 kHz.

These spectra will be taken experimentally in the lab.

3. Using the formulas in page 4 of the MC14046B data sheet, calculate  $f_{min}$  and  $f_{max}$  with the component values of  $R_1$ ,  $R_2$  and  $C_1$  of Fig. 2 [2 points] and Fig. 3 [2 points]. You might need to use these formulas during the lab. Pay attention to the note beside the formulas.

## 3 Lab procedure

GENERAL INSTRUCTIONS:

- After taking each plot make sure to ask your TA to verify that your results are correct. This also serves to monitor your progress and performance.

### 3.1 Spectra of tone-modulated FM signals

In this section the spectra of the signals of Prelab part 2 will be observed using FM signals produced by the function generator, and captured using TIMEFREQ.vi. Notice that since the frequency of an FM signal is time variant, not all the frequencies will be present in the particular interval of the signal recorded by TIMEFREQ.vi. You may expect to observe that some of the sidebands will not appear in the spectrum shown by the VI.

1. Connect the top signal generator (FG1) to the oscilloscope probe of Channel 1. In the FG1, set Amplitude  $A=10$  V, Frequency  $f_c=2$ kHz (this is the carrier frequency). Output an FM signal by pressing **Shift** **FM**. Set the frequency of the message signal (which is a sine wave) to  $f_x=500$  Hz by pressing **Shift** **Freq**, and the peak frequency deviation to  $\Delta f=50$  Hz by pressing **Shift** **Level**. Capture this signal and its spectrum using TIMEFREQ.vi and the parameters Channel=1, Time Span TS=5m, Freq. Span FS=10, a proper file name. [P1, 5 points].
2. Repeat the previous step changing the values of  $f_c$ ,  $\Delta f$ , TS, and FS as follows:
  - (a)  $f_c = 2.5$  kHz,  $\Delta f = 500$  Hz, TS=5m, FS=20. [P2, 5].
  - (b)  $f_c = 4.0$  kHz,  $\Delta f = 1.2$  kHz, TS=2m, FS=20. [P3, 5].
  - (c)  $f_c = 5.0$  kHz,  $\Delta f = 2.5$  kHz, TS=2m, FS=20. [P4, 5].

### 3.2 VCO FM signal generation

In this section you will create an FM signal by the direct method using the VCO included in the MC14046B.

1. Assemble the circuit of Fig. 1. The  $0.1 \mu\text{F}$  capacitor is for decoupling of the power supply and should be connected from pin 16 to pin 8 right on top of the integrated circuit.
2. Use FG1 to apply a sine wave of 1 kHz as the modulating (message) signal with Amplitude=3 V, Offset=3 V. Observe the filtered FM signal using channel 1. The purpose of lowpass filtering the VCO output signal is to remove some of the harmonics of the frequency modulated *square* wave produced by the MC14046B. Ideally, we would like to observe a modulated sine wave, such that its spectrum contained fewer components.
3. Capture this signal using Channel=1, TS=1m, FS=200, Save data=ON and a proper file name [P5, 5]. You should see an *approximation* to the spectrum of the FM signal. Remember that not all the frequencies are present in the particular time record used by TIMEFREQ.vi to compute the Fourier transform. However, it can be observed that the spectrum is composed by the components with magnitude above -15 dBV.
4. **Turn off** the function generator.

### 3.3 FM modulation and demodulation circuits

In this section you will implement an FM modulator (by the direct method) and the corresponding demodulator (using PLL), both based on the MC14046B.

1. Assemble the circuit of Fig. 2. The  $0.1 \mu\text{F}$  capacitor is for decoupling of the power supply and should be connected from pin 16 to pin 8 right on top of the integrated circuit.
2. Connect VC01in to +10 V. Observe the VC01out signal with channel 1 of the oscilloscope. Press **Autoscale**. If a trace for channel 2 appears in the oscilloscope screen, connect the channel 2 probe to its own ground clip and press **Autoscale** again. You should observe a *square* waveform of about 10 Vpp with an offset voltage of about 5 V.
3. Measure its frequency by pressing in the oscilloscope **Time** and in the screen menu selecting **Frequency**. The frequency should be around 200 kHz. As it is mentioned in the MC14046B data sheet, the frequency variation using the same passive components but a different sample of the MC14046B can be up to  $\pm 20\%$ , so the frequency of your particular circuit could be somewhat different. Record this frequency,  $f_{1_{max}} = \dots$ .
4. Disconnect VC01in from +10 V and now connect it to ground. Now the frequency of the VC01out signal should be around 100 kHz. Again, the frequency of your particular circuit could be somewhat different. Record this frequency,  $f_{1_{min}} = \dots$ . Disconnect VC01in from ground.

5. The interval  $[f_{1_{min}}, f_{1_{max}}]$  is the range of frequencies that the VCO1 can output. Keeping the circuit assembled, assemble the circuit of Fig. 3 using a second MC14046B. This is another VCO circuit which will be part of the demodulator and its frequency range is determined next.
6. Connect VC02in to +10 V. Observe the VC02out signal with channel 1 of the oscilloscope. You should observe a square waveform with a frequency of approximately 250 kHz ( $\pm 20\%$ ). Measure and record its frequency,  $f_{2_{max}} = \text{-----}$ .
7. Disconnect VC02in from +10 V and now connect it to ground. Now the frequency of the VC02out signal should be around 80 kHz ( $\pm 20\%$ ). Measure and record this frequency,  $f_{2_{min}} = \text{-----}$ .
8. Before continuing, make sure that the following relationships are true:  $f_{2_{max}} \geq 1.1 \cdot f_{1_{max}}$  and  $f_{2_{min}} \leq 0.9 \cdot f_{1_{min}}$ . If these conditions are not met, the demodulator might have difficulties locking on to the frequency of the modulator in the next stage of the experiment. If this is the case, try swapping the integrated circuits first (**turn off** the power before doing it), then try swapping the 470 pF capacitors. If that does not help, modify the values of  $R_1$  and/or  $R_2$  in the second VCO according to the formulas studied in Prelab part 3.
9. Once the conditions above are met, cascade both circuits, add the loop filter and make the necessary modifications as shown in Fig. 4.
10. Connect the function generator FG1 to VC01in with the following parameters: Amplitude=3 V, DC Offset level=3 V, Frequency  $f_x=1$  kHz, sine wave. This is the message signal (a tone).
11. Observe the message signal with channel 2 and the *output* signal with channel 1. At this point you should observe that a noisy scaled version of the modulating signal has been recovered by the demodulator. Correct any clipping in the output signal by adjusting the amplitude and/or offset of FG1. Capture the output signal using the following parameters: Channel=1, TS=4m, FS=10, Save data=ON and a proper file name, [P6, 5].
12. Increase the frequency of the message signal in increments of 0.1 kHz and observe when the modulating signal is no longer recovered. Find the maximum frequency at which the signal is recovered. Record this frequency  $f_{x_{max}} = \text{-----}$ .
13. Change the shape of the modulating signal to triangle and set its frequency  $f_x=1$  kHz. Capture the output signal using Channel=1, TS=4m, FS=10, Save data=ON and a proper file name, [P7, 5].
14. Change the resistor  $R_f$  of the loop filter for  $R'_f = 3.0$  k $\Omega$ . Observe and capture the output signal when the message signal is: (a) Sine wave, [P8, 5]. (b) Triangle wave, [P9, 5].
15. Set the message signal to sine wave, increase its frequency in increments of 0.1 kHz and observe when the modulating signal is no longer recovered. Find the maximum frequency at which the signal is recovered. Record this frequency  $f'_{x_{max}} = \text{-----}$ .

Please shut down properly the computer and equipment, and arrange your station.

## 4 Analysis

This section contains questions regarding the observations you made during the lab.

1. Include in your report all the plots obtained during the lab. **Plot** P1-P4 in a frequency span of  $2f_c$ , where  $f_c$  is the corresponding carrier frequency. Make sure to give a **meaningful** title to all the plots. The number of points assigned to each plot is specified in the lab procedure. Refer to Appendix B section 3.4 for instructions regarding the plotting of experimental results.
2. Compute the modulation index  $\beta$  for each of the FM signals of plots P1-P4. [2 each, 8 total].
3. The carrier frequency of the FM signal of plot P3 was  $f_c = 4$  kHz. From this plot observe that the carrier component does not appear in the spectrum, (a) explain the reason for this. [2]. (b) Since a carrier component is not available at the receiver, can the FM signal be demodulated? Justify your answer. [2].
4. From plot P5 estimate: [2 each, 8 total]
  - (a) the bandwidth  $B_{FM}$  of the FM signal, (consider only the components above -15 dBV),
  - (b) the carrier frequency  $f_c$  (which is at the middle of the spectrum),
  - (c) the peak frequency deviation  $\Delta_f$  (since  $f_x = 1$  kHz,  $\Delta_f$  is approximately half the bandwidth),
  - (d) the modulation index  $\beta$ .
5. From the above data, (a) estimate the bandwidth using Carson's rule [2]. (b) Classify this signal as narrowband FM or wideband FM [2].
6. Explain the effect of changing the value of the resistor  $R_f$ , (a) on the output signal (observe plots P6 and P8) [2] and (b) on the value of  $f_{x_{max}}$  [2]. [Hint: The information contained in pages 4 and 5 of the MC14046B data sheet may help you to understand and explain the effect of the loop filter].

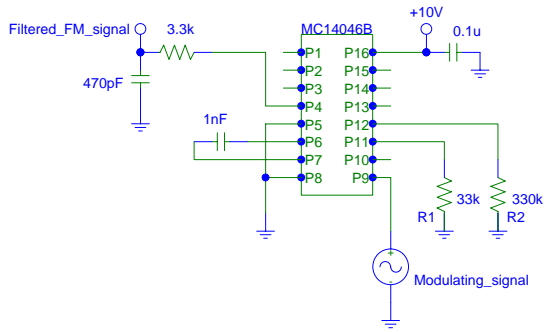


Fig. 1 Test circuit for the generation of an FM signal.

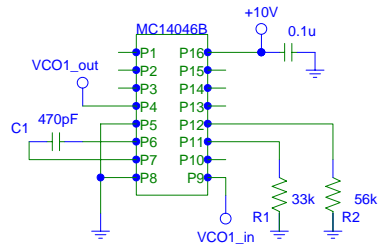


Fig. 2 VCO1 test circuit.

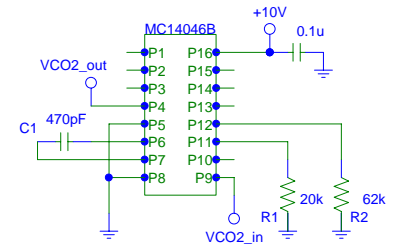


Fig. 3 VCO2 test circuit.

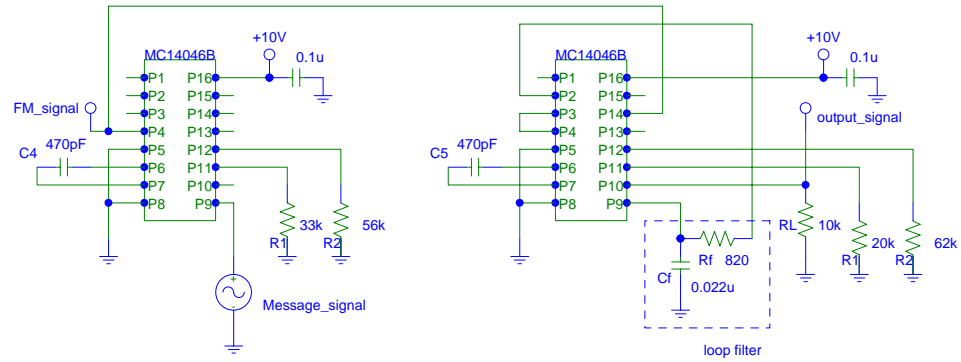


Fig. 4 FM Modulator and Demodulator.