Experiment 6: Pulse Code Modulation

This experiment deals with the conversion of an analog signal into a digital signal, the coding of the digital signal into pulses (Pulse Code Modulation), and the recovery of the analog signal at the receiver end (demodulation). The integrated circuits that are utilized in this experiment are the Analog-to-Digital Converter (ADC) TL5501 and the Digital-to-Analog Converter (DAC) DAC0806.

1 Introduction

From the *sampling theorem*, it is known that a signal $x(t)$ that is bandlimited to B Hz can be reconstructed from its samples taken uniformly at a rate greater than $2B$ samples per second. Ideally, the sampling is performed by multiplying $x(t)$ by a train of (physically nonexistent) impulses $\delta_{T_s}(t)$ spaced T_s, where $T_s = 1/f_s$ is the sampling period and f_s is the sampling rate. The sampled signal $x_s(t)$ is expressed as:

$$\begin{aligned} x_s(t) &= x(t)\delta_{T_s}(t) \\ &= \frac{1}{T_s}\left[x(t) + 2x(t)\cos 2\pi f_s t + 2x(t)\cos 4\pi f_s t + 2x(t)\cos 6\pi f_s t + \ldots\right] \\ &= \frac{x(t)}{T_s}\left[1 + 2\sum_{n=1}^{\infty}\cos(2\pi n f_s t)\right] \end{aligned} \tag{1}$$

therefore the spectrum is given by

$$X_s(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(f - nf_s) \tag{2}$$

Hence the spectrum $X_s(f)$ of a sampled signal consists of repetitions of $X(f)$ every f_s Hz. If $f_s < 2B$, overlapping of the repetitions of $X(f)$ occurs leading to a condition known as *aliasing*. The minimum sampling rate required to avoid aliasing is called the *Nyquist rate*, $f_s = 2B$. However, at the Nyquist rate, an ideal lowpass filter would be necessary to recover $x(t)$ from its samples $x_s(t)$ hence a higher sampling rate is needed in practice.

Sampling allows for the conversion of an analog signal into a digital signal after a *quantization* is made on the amplitude of the samples. Quantizing is the process of rounding off the sample value to the closest permissible discret value (quantizing level). The quantizing levels are determined by the analog-to-digital converter characteristics, specifically, the analog input voltage range (V_{min}, V_{max}) and the number L of quantizing levels. The levels are separated by the *step size*

$$\Delta v = \frac{V_{max} - V_{min}}{L} \tag{3}$$

The sample must be approximated to the closest quantizing level producing a *maximum quantization error* of $\pm\Delta v/2$. The quantization error can be considered as noise introduced to the samples before quantizing, and the power of the quantization noise is

$$N_q = \frac{(\Delta v)^2}{12} \tag{4}$$

The L-ary digital signal (L levels), discret in time and amplitude can be converted into a binary signal (2 levels) by using *pulse coding*. The code is formed by assigning a binary representation of the decimal number to each of the levels from 0 to $L-1$. In a 32-ary signal for example, the binary code for level 0 is 00000_2

and 10100_2 represents level 20. When this pulse-coded signal is transmitted, 5 binary digits (or *bits*) are sent for each sample of $x_s(t)$ every T_s seconds. Generally, L is a power of two, such that $L = 2^n$, where n is the number of bits necessary to pulse-code the signal. The bits are represented by two voltage levels. In the TL5501 ADC circuit, 0 and V_{cc} volts represent bits 0 and 1 respectively.

2 Prelab instructions

Enclosed in brackets is the number of points assigned to each question.

1. From the TL5501 data sheet determine:
   (a) The number of bits n and quantizing levels L. [2].
   (b) The *recommended* input voltage range (V_{min} , V_{max}) at the analog input of this ADC. [2].
   (c) The highest frequency B of the signal that can be sampled with this ADC without aliasing. [2].
   (d) The step size Δv and maximum quantization error $\Delta v/2$. [2].
   (e) Function table in page 3 of the data sheet presents several digital output codes (binary codes) for different analog input voltages, determine the binary code for the following analog input voltages (all in Volts): [2 points each]
      i. 4.101.
      ii. 4.489.
      iii. 4.916.

2. In practice, the sampled signal $x_s(t)$ is produced by sampling $x(t)$ with a train of *pulses* because impulses are physically nonexistent. This have some impact on the spectrum of the sampled signal. We will be unable to observe the sampled signal when using the TL5501, as the sampling process is internal to the integrated circuit.

   In the lab, we will see the spectrum of the signal *reconstructed* by the digital-to-analog converter. Although this signal is continuous in time and amplitude it shows the quantization process that occurs in the ADC. An *approximation* to the spectrum of the reconstructed signal is

$$\widehat{X_s}(f) = \text{sinc}(T_s f) \sum_{n=-\infty}^{\infty} X(f - nf_s) \quad (5)$$

   From Eq.(5):

   (a) Determine the spectrum $\widehat{X_s}(f)$ when $x(t) = A\cos(2\pi Bt)$. [2].
   (b) Create one-sided spectral plots of this spectrum for the cases enumerated below. Plot only frequencies between the specified range and neglect components with magnitude below -60 dBV (noise floor is -60 dBV). Hint: You must consider the components that arise from the negative side of the frequency axis, in other words, consider n from -10 to 10 to make sure *all* the components are shown in the positive side of the spectrum. The parameters for the signal and for the plot are specified next. [3 points each plot].
      i. A=1 V, B=1.0 kHz, f_s=10 kHz, FS=100 kHz (Freq. span from 0 to 100 kHz).
      ii. A=1 V, B=1.0 kHz, f_s=95 kHz, FS=100 kHz.
      iii. A=1 V, B=10.2 kHz, f_s=5 kHz, FS=20 kHz.

      These spectra will be taken experimentally in the lab.

3 Lab procedure

GENERAL INSTRUCTIONS:

- **Keep the power supply off during the assembling of any circuit.**
- **After taking each plot make sure to ask your TA to verify that your results are correct. This also serves to monitor your progress and performance.**

3.1 Quantization and pulse coding

In this section the binary codes produced by the ADC TL5501 corresponding to different analog input voltages will be determined.

1. Set the following parameters in the bottom function generator (FG2): Amplitude=2.3 V, Offset=1.2 V, Waveform: square, Frequency=10 kHz. The top function generator (FG1) must be **off** at this time.
2. Assemble the circuit of Fig. 1 **without** connecting node `A` to pin 12 of the ADC yet. Connect FG2 as the sampling signal at pin 7 of the ADC. Use the output labeled `+6 V` of the power supply to power this circuit. Adjust the power supply voltage to 5.000$\pm$0.005 V (use the digital multimeter to do this measurement). This voltage is V_{max} in Eq. (3). It may be cumbersome to adjust the power supply with such accuracy, but it is needed for the purpose of the experiment.
3. Using the multimeter, verify that the voltage at pin 13 of the TL5501 is within the range 4.000$\pm$0.010 V, this voltage is V_{min} in Eq.(3). If this voltage is lower than 3.990 V increase the resistor R1, if it is higher than 4.010 V decrease the resistor R1.
4. Using the multimeter make sure that the analog input voltage V_I (at node `A`) is less than 5 V, if this is not the case you must troubleshoot your circuit to avoid damage to the ADC.
5. Connect node `A` to pin 12 of the ADC and adjust the output of the `+20 V` power supply such that V_I is within the voltage ranges of the following table. Record the corresponding actual input voltage and then with the multimeter read the binary code at the output of the ADC (pins 1 through 6).

   Represent by a "0" any voltage below 2.5 V and by a "1" any voltage above 2.5 V. Complete the following table using 0's and 1's for the entries in columns Pin 6 - Pin 1 (you do not need to record the voltage out of these terminals).

Voltage range (V)	Actual voltage V_I (V)	Pin 6	Pin 5	Pin 4	Pin 3	Pin 2	Pin 1
4.000-4.330							
4.330-4.660							
4.660-5.000							

Table 1: Output binary codes for different analog input voltages.

3.2 Pulse code modulation and demodulation

In this section you will implement a PCM modulator (with the ADC TL5501) and the corresponding demodulator using the digital-to-analog converter DAC0806. Since this circuit is sensitive to noise and involves a large number of connections, use short wires and do a neat assembling of the circuit to facilitate troubleshooting.

1. Assemble the circuit of Fig. 2 **without** connecting node `B` to pin 12 of the ADC yet. Use FG2 as the sampling signal at pin 7 of the ADC with the parameters of the previous section. Using the multimeter, verify the following voltages with a tolerance of $\pm$50 mV: at pin 9: 5 V, pin 13: 4 V, node `B`: 4.5 V. If one of these voltages is not correct troubleshoot your circuit.

2. Connect node B to pin 12 of the ADC and use FG1 as the message signal with the following parameters: Amplitude=0.45 V, Frequency=1 kHz, Waveform=sine.

3. Using channel 1 of the oscilloscope verify the presence of rectangular pulses at the digital outputs of the ADC (pins 1 through 6). You may need to adjust the Time/Div setting of the oscilloscope to observe the train of pulses in each of the digital outputs.

4. Turn the power supply **off** and complete the assembling of the *modem* by adding the digital-to-analog converter as shown in Fig. 3. To avoid connection errors, begin connecting the power and ground connections to the DAC0806 and then the connections to the ADC.

5. Turn on the power supply and use channel 1 to observe the demodulated output signal [1]. Use channel 2 to observe the message signal from FG1. At the output you should observe a signal that looks like a staircase, clearly this is the effect of the quantization process that occurs in the ADC.

   Notice that the LPF at the output of the op-amp has a very high cut-off frequency since it is used to reduce high frequency *noise* while keeping the spectral components of interest unchanged.

6. Increase the sampling frequency to 50 kHz, you should see that the message signal has been recovered at the output. If clipping is observed at the output reduce the amplitude of the message signal.

7. Using TIMEFREQ.vi capture the signals and their magnitude spectra for the cases listed in Table 2. Use a different file name for each plot. **Important:** The spectral components should be well defined, it may be necessary to run TIMEFREQ.vi several times in order to get a clear picture of the spectrum for each case in Table 2.

Plot	Message waveform	Message freq. (kHz)	Sampling freq. (kHz)	Channel	TS	FS
P1	sine	1.0	10.0	1	4E-3	100
P2	sine	1.0	35.0	1	4E-3	100
P3	sine	1.0	40.0	1	4E-3	100
P4	sine	1.0	95.0	1	4E-3	100
P5	triangle	1.0	N/A	2	4E-3	100
P6	triangle	1.0	20.0	1	4E-3	100
P7	sine	10.2	5.0	1	20E-3	20
P8	sine	15.2	5.0	1	20E-3	20

Table 2: Signal and plot parameters.

3.3 Analysis

This section contains questions regarding the results you obtained in the lab.

1. Present all the plots obtained during the lab. Make sure to label properly all the plots. The number of points assigned to each plot is specified in the lab procedure. Refer to Appendix B section 3.4 for instructions regarding the plotting of experimental results.

2. Obtain the theoretical binary codes of the actual analog input voltages of Table 1 and compare these values with the experimental results. [3].

3. Observe the spectrum of plot P1, explain why the repetitions of the spectrum of the sine wave get smaller as the frequency increases. [2]

[1] Note: No *voltage* signal is produced at the output of the DAC0806 since it requires an external current at pin 4, therefore a current-to-voltage converter implemented with an op-amp is necessary.

4. Observe the spectrum of plot P2, you will agree that the repetitions of $X(f)$ that occur at 35 and 70 kHz are expected as these frequencies are multiples of $f_s = 35$ kHz for this particular case. Explain however, the presence of the repetitions of $X(f)$ at 95 kHz and another (it may not be clear in your plot) at 60 kHz, and why the spectrum of plot P3 does not show any repetition at any frequency other than multiples of $f_s = 40$ kHz. [4]. [Hint: the oscilloscope is also sampling the signal at a rate of 200 kHz.]

5. Giving arguments from the spectrum in plot P4 (compare it with P1), explain why the time domain signal for this case resembles more closely the analog input signal (a sine wave). [2]

6. Observe the spectrum in plot P5, (a) estimate the bandwidth B of this signal.[2] (b) From plot P6, Was the Nyquist rate satisfied in this case? What sampling frequency f_s would you have used? [3].

7. Observe the plots P7 and P8. (a) Estimate the frequency f_m of the time domain signal.[2] (b) Explain why these two plots are similar despite the fact they were taken from input signals with different frequencies.[2].

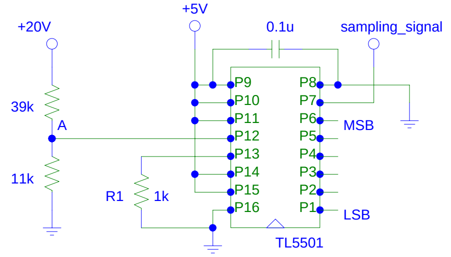

Fig. 1 Analog-to-Digital Converter

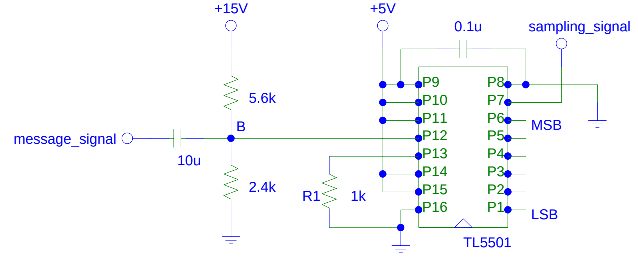

Fig. 2 PCM modulator

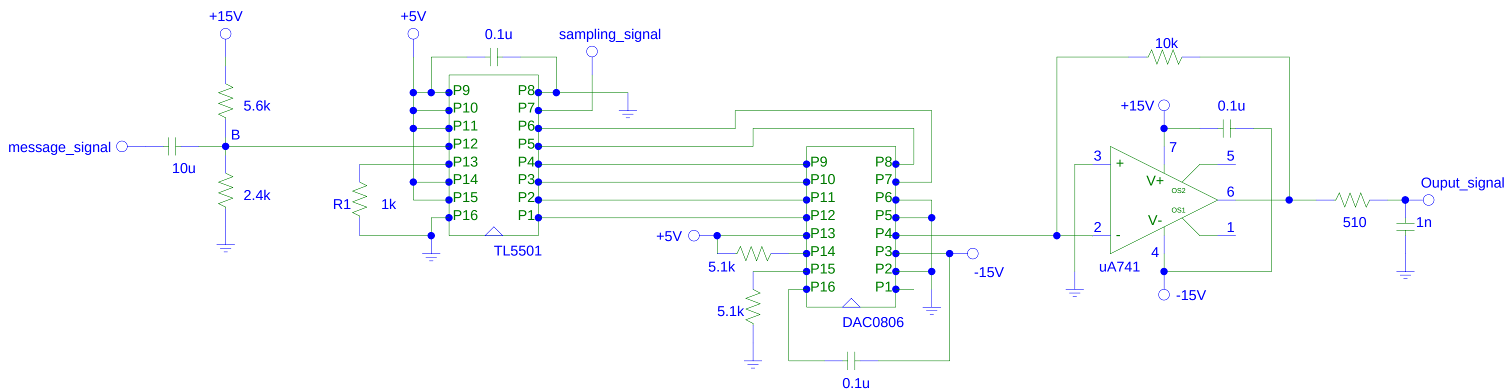

Fig. 3 PCM modem

University of Texas at Dallas
EE3150 Communication Systems Lab
Experiment 6. Pulse code modulation
E. Cura

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