MATH 2419 FINAL EXAM REVIEW

31 March 2008

Complex Numbers

DeMoivre's Theorem: If $z = r(\cos\theta + i\sin\theta)$ and n is a positive integer then

$$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

The n roots of $z = r(\cos\theta + i\sin\theta)$ are given by

$$u_k = \sqrt[n]{r}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right) \text{ for } k = 0,1,\dots,n-1$$

1. Compute the indicated power by using DeMoivre's Theorem

   (a) $\left(\frac{i}{1+i}\right)^3$ (b) $\left(\sqrt{3} - i\right)^4$

2. Find the cube roots of $27i$
3. Find the fourth roots of $3 + 3i$

Chapter 8, Section 8

1. Evaluate the improper integral or determine that it diverges.

   (a) $\int_0^3 \frac{x}{(x^2-9)^{4/3}}\,dx$ (b) $\int_0^6 \frac{2x}{x^2-4}\,dx$ (c) $\int_0^\infty \frac{2}{x^2+9}\,dx$

   (d) $\int_0^\pi \tan x\,dx$ (e) $\int_2^{\sqrt{20}} \frac{1}{x^2(x^2-4)^{1/2}}\,dx$

Chapter 9, Sequences and Series

1. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

   (a) $\sum_{k=1}^{\infty}\left(\tan\frac{\pi}{3}\right)^k$ (b) $\sum_{k=3}^{\infty}\frac{5}{k(\ln k^2)}$ (c) $\sum_{n=1}^{\infty}\frac{(-1)^n n}{2^n}$

   (d) $\sum_{n=2}^{\infty}\frac{(-1)^n\sqrt{n^2-1}}{n^2+1}$ (e) $\sum_{n=1}^{\infty}\frac{n}{e^{n^2}}$ (f) $\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}$

   (g) $\sum_{n=2}^{\infty}\frac{(-1)^n n^2\ln n}{2^n}$ (h) $\sum_{n=1}^{\infty}\frac{(2n)!}{3^n(n!)^2}$ (i) $\sum_{k=1}^{\infty}\frac{5\cdot 8\cdot 11\cdots(3k+2)}{3\cdot 7\cdot 11\cdots(4k-1)}$

2. Find the 5th degree Maclaurin polynomial of $f(x) = x\cos x$.

3 Find the interval of convergence of the power series.

(a) $\sum_{k=0}^{\infty} \frac{(-1)^k (x-5)^k}{2^k (k+4)}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n^2 3^n}$

4. Let f be the function $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x)^n}{4^n \sqrt{n}}$. Find the interval of convergence for $\int f(x)\,dx$.

5. Find a power series centered at $c = -2$ for the function $f(x) = \frac{3}{3-x}$ and identify the interval of convergence.

6. Find a power series centered at $c = 1$ for the function $f(x) = \frac{3}{4x-2}$ and identify the radius of convergence.

7. Let $f(x) = \frac{x-5}{x^2 - x - 2}$
   (a) Find the partial fraction decomposition of f.
   (b) Find the power series centered at zero for each term of the decomposition.
   (c) Write the power series for f and identify the interval of convergence.

8. Given the Maclaurin series $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n-1}}{(2n-1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots.$

   for x in $(-\infty, \infty)$
   (a) Find the Maclaurin series for $\sin x^2$

   (b) Use the first 4 terms of the series found in part (a) to approximate $\int_0^{0.3} \sin x^2 \, dx$

   (c) Find the Maclaurin series for $\cos x$

   (d) Given the identity $\cos^2 x = \frac{1 + \cos 2x}{2}$ and the series in part (c), determine the validity of the following statement; $\cos^2 x = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1} x^{2n}}{(2n)!}$.

Chapter 10

1. Let C be the curve represented by the parametric equations $x = \sqrt{t+1}$ and $y = 3 - t$

   (a) Find the corresponding rectangular equation for C by eliminating the parameter.
   (b) Sketch the curve C, indicate the orientation of the curve.
   (c) Find the domain of the rectangular function.

2. A curve is defined by the parametric equations
   $x = t^2 + t - 2$, $y = t^3 - 2t - 1$. Find

   (a) all value(s) of t where the curve has horizontal tangents.

(b) all value(s) of t where the curve has vertical tangents.
(c) an equation of the line tangent to the curve at the point (4,3).
(d) the concavity of the curve at the point (4,3).

3. Find and classify any relative extrema for the curve given by the parametric equations $x = t^3 + 2t - 1$ and $y = t^2 - t + 5$.
4. Calculate the arc length of the curve:
$$x = \frac{8}{3}t^{3/2}\ ,\ y = 2t - t^2\text{; on the interval } [1,3]$$
5. Let $r = 1 - \sin\theta$
   (a) Find the area of the region bounded by the polar function.
   (b) Find the length of the curve.
   (c) Find the slope of the tangent to the curve when $\theta = \pi/4$.
6. Calculate the area outside the graph of $r = \sin\theta$ and inside the graph of $r = 2\sin\theta$.
7. Calculate the area inside the graph of $r = 5\cos\theta$ and outside the graph of $r = 2 + \cos\theta$.

Chapter 11

1. Let $\mathbf{u} = 5\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{v} = 3\mathbf{i} - 2\mathbf{k}$, and $\mathbf{w} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$
   (a) Determine if any pair of the three vectors are orthogonal.
   (b) Find all λ such that $\mathbf{r} = 4\lambda\mathbf{i} + \lambda\mathbf{j} - 12\mathbf{k}$ is orthogonal to **v.**
   (c) Find the projection of **w** onto **v.**
   (d) Find the vector component of **w** orthogonal to **v.**
   (e) Determine the angle between **u** and the negative x-axis.
   (f) Find $\mathbf{s} \bullet \mathbf{u}$ if $\|\mathbf{s}\| = 4$ and the angle between **s** and **u** is $5\pi/6$

2. Find two unit vectors orthogonal to $\mathbf{u} = \mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 3\mathbf{i} - \mathbf{k}$.
3. Let $\mathbf{u} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, and $\mathbf{w} = 3\mathbf{i} - \mathbf{k}$.
   (a) compute $(\mathbf{u} \times \mathbf{v}) \bullet \mathbf{w}$
   (b) Is the result in part (a) equivalent to $(\mathbf{u} \times \mathbf{w}) \bullet \mathbf{v}$?
   (c) describe the geometric significance of the triple scalar product in part (a).
4. Determine the area of the triangle with vertices (3, -1, 2), (2, 1, 5), and (1, -2, -2).
5. Write parametric equations for the line through the point (-2, 1, 3) that is perpendicular to the plane $2x - 3y + z - 7 = 0$.
6. Find numbers x and y such that the point (x, y, 1) lies on the line passing through the points (2, 5, 7) and (0, 3, 2).
7. Find an equation of the plane that contains the line $x = \frac{y-1}{3} = \frac{z+1}{2}$ and is perpendicular to the line $x = 1 - 17t,\ y = -5 + t,\ z = 3 + 7t$. Express your result in the form $ax + by + cz + d = 0$.

8. Find the point of intersection of the line $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z-1}{1}$ and the plane $2x + 3y + z - 3 = 0$
9. Calculate the distance from the point (-1, 2, 3) to

(a) the line $x = 2 - t,\ \ y = 1,\ \ z = 1 + t$

(b) the plane $x - 3 + 2(y + 1) - 4z = 0$

10. Find an equation of the plane with axes intercepts (-3,0,0), (0,5,0) and (0,0,6). Express your result in the form $ax + by + cz + d = 0$

Chapter 12 Vector Valued Functions

1. Let $\mathbf{r}(t) = e^{-t}(\mathbf{i} + \mathbf{j} + \mathbf{k})$. Find the unit tangent vector.
2. Let $\mathbf{T}(t) = \dfrac{3t\,\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{\sqrt{9t^2 + 5}}$ be the unit tangent vector of a space curve $\mathbf{r}(t)$.

   (a) Find $\mathbf{N}(t)$

   (b) If $\mathbf{r}(0) = \langle 6, -1, 5 \rangle$ find $\mathbf{r}(t)$
3. Let a space curve be given by $\mathbf{r}(t) = e^t \cos t\,\mathbf{i} + e^t \sin t\ \mathbf{j} + e^t\ \mathbf{k}$.

   (a) Find a set of parametric equations for the line tangent to the curve at the point (1, 0, 1).

   (b) Find the length of the space curve on the interval $[0, \ln 8]$
4. The position of a particle moving along a space curve is given by $\mathbf{r}(t) = t\,\mathbf{i} + \dfrac{\sqrt{6}}{2}t^2\mathbf{j} + t^3\ \mathbf{k}$,

   (a) find the tangential component of acceleration $a_{\mathbf{T}}$.

   (b) find the normal component of acceleration $a_{\mathbf{N}}$

   (c) find the curvature at $t = 1$
5. $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j}$.

   (a) Find $\mathbf{T}(2)$

   (b) Find $\mathbf{N}(2)$

   (c) Find the length of the curve on the interval $[0, \dfrac{2}{\sqrt{3}}]$

Chapter 13 Real Valued Functions of more than one variable.

1. Find the indicated partial derivatives:

   (a) $f(x, y) = e^{x \sin y}$; $\quad f_x,\ \ f_y(1,0)$

   (b) $z = \sin(x + at) + e^{x - at}$ $\quad \dfrac{\partial^2 z}{\partial t^2},\ \dfrac{\partial^2 z}{\partial x^2},\ \dfrac{\partial^2 z}{\partial t \partial x}$

   (c) Let $w = xy^2 + xz^2$, $\ x = t^2 - 3s^2$, $\ y = t/s$, and $z = 2 - st$.

   Find $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$ at $t = 2$ and $s = -1$.

2. Use implicit differentiation to find the first partial derivative of z with respect to x:

$$x^3y + y^3z + z^3x = 11$$

3. The length, width, and height of a rectangular chamber are changing at the rate of 3 feet per minute, 2 feet per minute, and 0.5 feet per minute, respectively. Find the rate at which the volume is changing at the instant the length is 10 feet, the width is 6 feet, and the height is 4 feet.
4. The width of a rectangle is increasing at a rate of 0.2 meters per second while the height is increasing at a rate of 1 meter per second. At what rate is the area increasing when the height is 10 meters and the width is 5 meters?

Directional derivative and gradient

5. Let $f(x, y) = e^x \arctan y$. Let points $P(0, 1)$ and $Q(3, 5)$ be given.
   (a) Find the gradient of f at P.
   (b) Find the directional derivative of f at P in the direction $\overrightarrow{PQ}$.
6. Let $f(x, y, z) = z^2e^{xy}$.
   (a) Find $\nabla f(-1,0,3)$.
   (b) Calculate the directional derivative of $f(x, y, z) = z^2e^{xy}$ at the point (-1, 0, 3) in the direction of the vector **v** = 2**i** +3 **j** - 6**k**..
   (c) Find the maximum value of the directional derivative of f at the point (-1, 0, 3).
7. Find all unit vectors **u** such that the directional derivative of $f(x, y) = x^3y^3 - xy$ at the point (1, -1) is zero.
8. A bird is flying through a forest fire at 6 m/sec. in the direction $\langle 2,2,-1 \rangle$. The temperature as a function of position is $T(x, y, z) = xy + 2z^2$. At what rate is the temperature changing when the bird passes the point (5, 20, 10)? Is the bird a dove?
9. A surface is given by the function $f(x, y) = x \tan y$.
   (a) Find an equation of the tangent plane to the surface at the point $(2, \pi/4, 2)$.
   (b) Find parametric equations of the line normal to the surface at $(2, \pi/4, 2)$.
10. Consider the surface given by $xy^2 + zy^2 + 4y - xz^2 = 18$ and the point $P(-2, 0, 3)$ on the surface.
    (a) Find the gradient at the point P.
    (b) Find an equation of the tangent plane to the surface at the point P.
    (c) Find symmetric equations for the normal line to the surface at the point P.
11. Find symmetric equations for the tangent line to the curve of intersection of the surfaces given by $z = x^2 + y^2$ and $x + 2y - z + 5 = 0$ at the point (2, 3, 13).
12. Find the point(s) on the surface $x^2 + y^2 + 2x - 4y + z^2 + 1 = 0$ where the tangent plane is horizontal.
13. Determine any relative extrema or saddle points of
    (a) $f(x, y) = x^3 + y^3 + 3y^2 - 3x - 9y + 2$
    (b) $f(x, y) = x^4 + y^5 - 5y - 32x - 8$
14. Find the absolute extrema of $f(x, y) = y^2 - 3x^2 - 2y + 6x$ on the square with vertices (0, 0), (2, 0), (2, 2), and (0, 2).

15. Use the method of Lagrange multipliers to maximize the function $f(x, y) = x + 2xy + 2y$ subject to the constraint $x + 2y = 80$.

16. Use the method of Lagrange multipliers to find the extrema of $f(x, y) = 3xy$ subject to the constraint $4x^2 + y^2 = 200$.

17. Maximize $f(x, y, z) = 32xyz$ subject to the constraints $x + y + z = 4$ and $z - x - y = 3$.

Chapter Chapter 14, Double Integrals

1. Evaluate the iterated integral.

   (a) $\int_{-1}^{2} \int_{0}^{1-x} (4 - y)\, dy\, dx$ (b) $\int_{0}^{\sqrt{\pi}} \int_{\pi/6}^{y^2} 2y \cos x\, dx\, dy$

2. Sketch the region of integration and reverse the order.

   $\int_{0}^{2} \int_{0}^{4-x^2} f(x, y)\, dy\, dx$

3. Calculate the iterated integral by first reversing the order of integration.

   $\int_{0}^{1} \int_{\sqrt{y}}^{1} \left(\sin \pi x^3\right) dx\, dy$

4. Write the integral in polar coordinates and evaluate.

   (a) $\int_{0}^{2} \int_{x}^{\sqrt{8-x^2}} \sqrt{x^2 + y^2}\, dy\, dx$ (b) $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$

5. Prove that A(0, 0,1), B(2, 0,4), and C(5,6,2) are the vertices of a right triangle.
   Find the area of the triangle by (a) elementary means, and (b) as a double integral.

6. Let *R* be a closed and bounded region in the *xy* plane over which $z = \ln(x^2 + y^2)$ and $z = \arctan(y/x)$ are defined. Show that these surfaces have the same area over *R*.

7. Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ that is inside the cylinder $x^2 + y^2 = ax$.

Solutions

Complex Numbers

1. (a) $\frac{i}{1+i} = \frac{1}{2} + \frac{1}{2} i = \frac{1}{\sqrt{2}}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$, $\left(\frac{i}{1+i}\right)^3 = \left(\frac{1}{4}\right)(-1+i)$ (b) $(-8)(1+\sqrt{3}\, i)$
2. $z = 27(\cos\pi/2 + i\sin\pi/2)$ $u_0 = 3(\cos\pi/6 + i\sin\pi/6)$, $u_1 = 3(\cos 5\pi/6 + i\sin 5\pi/6)$, and $u_2 = 3(\cos 3\pi/2 + i\sin 3\pi/2)$
3. $u_0 = \sqrt[8]{18}(\cos\pi/16 + i\sin\pi/16)$ $u_1 = \sqrt[8]{18}(\cos 9\pi/16 + i\sin 9\pi/16)$, $u_2 = \sqrt[8]{18}(\cos 17\pi/16 + i\sin 17\pi/16)$ $u_3 = \sqrt[8]{18}(\cos 25\pi/16 + i\sin 25\pi/16)$

Chapter 8, Section 8

1. (a) div (b) div (c) $\frac{\pi}{3}$ (d) div (e) $\frac{\sqrt{3}}{4}$

Chapter 9, Sequences and Series

1. (a) $\left(\sqrt{3}\right)^k$ (Div Geo) (b) Div; (Integral Test) (c) abs conv (ratio test)
   (d) Cond conv (e) abs conv (f) Cond conv
   (g) Abs conv (h) div (ratio test) (i) Abs conv (ratio test)

3 (a) (3, 7] (b) [-1,5]

4. [-4,4] 5. $\sum_{k=0}^{\infty} \frac{3(x+2)^k}{5^{k+1}}$; $-7 < x < 3$ 6. $\sum_{k=0}^{\infty} 3(-1)^k (2)^{k-1} (x-1)^k$; $radius = 1/2$

7. $f(x) = \sum_{n=0}^{\infty} \left[\frac{1}{2^n} + 2(-1)^n\right] x^n$; $|x| < 1$ 8. (b) 0.009 (d) The statement is true.

Chapter 10

1. (a) $y = 4 - x^2$ (b) half a parabola opening downward, vertex (0,4), (c) $x \geq 0$
2. (a) $t = \pm\sqrt{2/3}$ (b) $t = -1/2$ (c) occurs at , $t = 2$; eq: $y - 3 = 2(x - 4)$
3. Relative minimum, (1/8, 19/4)
4. 12
5. (a) $3\pi/2$ (b) 8 (c) $\sqrt{2} - 1$
6. $3\pi/4$
7. $\sqrt{3} + 8\pi/3$

Chapter 11

1. (a) $\mathbf{u} \bullet \mathbf{w} = 0$ (b) $\lambda = -2$ (c) $\frac{-9}{13}(3\mathbf{i} - 2\mathbf{k})$ (d) $\left\langle \frac{-40}{13}, 2, \frac{57}{13} \right\rangle$ (e) $164.21°$ (f) -18
2. $\frac{\pm\langle -2, 1, -6\rangle}{\sqrt{41}}$ 3. (a) $(\mathbf{u} \times \mathbf{v}) \bullet \mathbf{w} = 14$ (b) No, $(\mathbf{u} \times \mathbf{w}) \bullet \mathbf{v} = -14$
   (c) Volume of the parallelepiped determined by the three vectors.
4. $5\sqrt{6}/2$ 5. $x = -2 + 2t$, $y = 1 - 3t$, $z = 3 + t$ 6. $x = -2/5$, $y = 13/5$

7. $17x - y - 7z - 6 = 0$ 8. $\left(\frac{-17}{7}, \frac{33}{14}, \frac{11}{14}\right)$ 9. (a) $\sqrt{3}/2$ (b) $10/\sqrt{21}$

10. $\frac{-x}{3} + \frac{y}{5} + \frac{z}{6} - 1 = 0$ or equivalent.

Chapter 12 (Vector Valued Functions)

1. $\mathbf{T}(t) = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$.

2. (a) $\mathbf{N}(t) = \frac{5\,\mathbf{i} - 3t\,\mathbf{j} - 6t\,\mathbf{k}}{\sqrt{45t^2 + 25}}$ (b) $\mathbf{r}(t) = (3t^2/2 + 6)\mathbf{i} + (t-1)\mathbf{j} + (5 - 2t)\mathbf{k}$

3. (a) $x = 1 + s;\ y = s;\ z = 1 + s$ (b) $7\sqrt{3}$

4. (a) $a_{\mathbf{T}} = 6t$ (b) $a_{\mathbf{N}} = \sqrt{6}$ (c) $\sqrt{6}/64$

5. (a) $\mathbf{T}(2) = \left\langle 1/\sqrt{10}, 3/\sqrt{10} \right\rangle$ (b) $\mathbf{N}(2) = \left\langle -3/\sqrt{10}, 1/\sqrt{10} \right\rangle$ (c) 56/27

Chapter 13 Real Valued Functions of more than one variable.

1. (a) $f_x(x,y) = e^{x\sin y}\sin y$; $f_y(1,0) = e^{x\sin y}x\cos y\Big|_{(1,0)} = 1$

   (b) $z_{tt} = -a^2\sin(x+at) + a^2e^{x-at}$, $z_{xx} = -\sin(x+at) + e^{x-at}$, $z_{xt} = -a\sin(x+at) - ae^{x-at}$

   (c) $\left.\frac{\partial w}{\partial s}\right|_{\substack{t=2\\ s=-1}} = 112$ $\left.\frac{\partial w}{\partial t}\right|_{\substack{t=2\\ s=-1}} = 92$

2. $\frac{\partial z}{\partial x} = \frac{-(3x^2y + z^3)}{y^3 + 3z^2x}$

3. $V = LWH$ all variables are time dependent. $\frac{dV}{dt} = 182\ ft^3/\text{sec}$ 4. $\frac{dA}{dt} = 7\ m^2/\text{sec}$

5. (a) $\left\langle \frac{\pi}{4}, \frac{1}{2} \right\rangle$ (b) $\frac{3\pi + 8}{20}$

6. (a) $\nabla f(-1,0,3) = \langle 0,-9,6\rangle$ (b) $D_{\mathbf{u}}f(-1,0,3) = -9$ (c) $\sqrt{117}$

7. $\nabla f(1,-1) = \langle -2,2\rangle$; $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$

8. $D_{\mathbf{u}}T(5,20,10) = 10/3$, Is the bird a dove? Who knows, regardless of species, it should change course!

9. Let $F(x,y,z) = z - x\tan y$ then $\nabla F(x,y,z) = \langle -\tan y, -x\sec^2 y, 1\rangle$.

   (a) $-(x-2) - 4(y - \pi/4) + (z-2) = 0$ (b) $x = 2 - t$, $y = \pi/4 - 4t$, $z = 2 + t$

10. Consider the surface given by $xy^2 + zy^2 + 4y - xz^2 = 18$ and the point P(-2, 0, 3) on the surface.

    (a) $\langle -9,4,12\rangle$ (b) $-9(x+2) + 4y + 12(x-3) = 0$ (c) $\frac{x+2}{-9} = \frac{y}{4} = \frac{z-3}{12}$

11. The point is in the zero level surface of both functions $f(x,y,z) = z - x^2 - y^2$ and $g(x,y,z) = x + 2y - z + 5$. A direction vector of the tangent line is $\nabla f(2,3,13) \times \nabla g(2,3,13)$.

$$\frac{x-2}{16} = \frac{y-3}{-5} = \frac{z-13}{6}$$

12. Let $F(x,y,z) = x^2 + y^2 + 2x - 4y + z^2 + 1 = 0$ and assume horizontal means parallel to the xy plane. The desired points must satisfy the vector equation $\nabla F = \pm \mathbf{k}$, ans: $(-1, 2, \pm 1)$

13. (a) *f* has saddle points at (-1,1,-1) and (1,-3,27), a rel max of 31 at (-1,-3) and a rel min of -5 at (1,1).
    (b) *f* has a saddle point at (1,-1,-35) and a rel min of -43 at (1,1)

14. *f* has a saddle point on the interior of the square at (1,1,2) hence the absolute extrema occur on the boundary of the square. The absolute maximum of *f* is 3 and occurs at the points (1,0) and (1,2). The absolute minimum of *f* is -1 and occurs at (2,1) and (0,1).

15. The constrained critical point is (40,20). You can verify that *f* has a maximum here by simply substituting 80-2y for x in *f* then use the techniques of calculus 1.

16. Critical points (5, 10), (5, -10), (-5, 10), (-5, -10). the extrema of *f* are 150 and -150.

17. $f(x,y,z) = 32xyz$, let $g(x,y,z) = x + y + z - 4$ and $h(x,y,z) = z - x - y - 3$.

    Solve the system $\nabla f = \lambda \nabla g + \mu \nabla h$, $g = 0$, and $h = 0$ (5 equations and 5 unknowns). The constrained critical point is $(1/4, 1/4, 7/2)$ which gives a value of 7 for *f*.

Chapter 14, Double Integrals

1. (a) 9/2 (b) $2 - \frac{\pi}{2}$ 2. $\int_0^4 \int_0^{\sqrt{4-y}} f(x,y)\,dx\,dy$

3. $2/3\pi$ 4. (a) $\frac{4\sqrt{2}\pi}{3}$ (b) $\frac{\pi}{2}(e-1)$

5. Hint: the Pythagorean theorem is an if and only if statement.

7. $2(\pi - 2)a^2$