

CHAPTER 11 & 12: VECTORS AND VECTOR-VALUED FUNCTIONS

MATH 2019 – SPRING 2008

- (a) Find the projection of \mathbf{N} onto \mathbf{L} . Then, compute the vector component of \mathbf{N} orthogonal to \mathbf{L} . Show that the answer you found is indeed orthogonal to \mathbf{L} . Draw a picture to illustrate the significance of what you just computed.

$$\mathbf{L} = \langle 6, 3, 2 \rangle \quad \mathbf{N} = 4\mathbf{i} - 4\mathbf{j}$$

- (b) Find the angle between \mathbf{N} and \mathbf{L} .
 - (c) Find the area of the parallelogram determined by the vectors \mathbf{N} and \mathbf{L} .
 - (d) Find two unit vectors orthogonal to both \mathbf{N} and \mathbf{L} .
- (a) A parallelepiped has adjacent edges PQ , PR , and PS . Determine its volume.

$$P(1, 1, 1) \quad Q(2, 0, 3) \quad R(4, 1, 7) \quad S(3, -1, -2)$$

- (b) Pick three of the above points. Explain how you would find the area of the triangle determined by these three points.
- Find parametric equations and symmetric equations for the line.
 - (a) The line that passes through the points $(2, 4, -3)$ and $(3, -1, 1)$
 - (b) The line that passes through the point $(-3, 5, 4)$ and is parallel to the line given by the symmetric equations $\frac{x-1}{3} = \frac{y+1}{-2} = z-3$
 - (c) The line that passes through the point $(-3, 5, 4)$ and is parallel to the line $x = 5 - 2t$, $y = -4 + 2t$, $z = 0$
 - (d) The line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$
 - Find an equation of the plane.

- (a) The plane through the points $(3, -1, 2)$, $(8, 2, 4)$, and $(-1, -2, -3)$
- (b) The plane through the point $(3, 2, 2)$ and perpendicular to the line $\frac{x-1}{4} = y+1 = \frac{z+3}{-3}$
- (c) The plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$

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(d) The plane containing the lines

$$\frac{x-2}{-3} = \frac{y-2}{6} = z-3 \quad \text{and} \quad \frac{x-3}{2} = y+5 = \frac{z+2}{4}$$

(e) The plane that passes through the points $(3, 2, 1)$ and $(3, 1, -5)$ and is perpendicular to the plane $6x + 7y + 2z = 10$

5. (a) Find the distance between the point $(3, 2, 1)$ and the plane $x - y + 2z = 4$

(b) Find the distance between the point $(1, 2, 3)$ and the line $x - 2 = \frac{y - 2}{-3} = \frac{z}{5}$

6. Find the unit vectors, $\mathbf{T}(t)$ and $\mathbf{N}(t)$, and the curvature $K(t)$ for the following space curve.

$$\mathbf{r}(t) = 4t\mathbf{i} + \cos 5t\mathbf{j} + \sin 5t\mathbf{k}$$

7. If $\mathbf{r}(t) = t\mathbf{i} + \frac{\sqrt{3}}{2}t^2\mathbf{j} + t^3\mathbf{k}$ is the position vector of a flying bee,

(a) compute the velocity, speed, and acceleration of the bee.

(b) find the unit tangent vector, the tangential component of acceleration, the normal component of acceleration, and the curvature.

8. For the following space curve $\mathbf{r}(t)$, find the unit tangent vector, the tangential component of acceleration, the normal component of acceleration, and the curvature at the given value of t .

$$\mathbf{r}(t) = (t - \cos t)\mathbf{i} + \ln(t + 1)\mathbf{j} + \frac{e^{-5t}}{5}\mathbf{k} \quad t = 0$$

Then, find a set of parametric equations for the line tangent to the curve at $t = 0$.

9. Find the length of the curve.

(a) $\mathbf{r}(t) = t^2\mathbf{i} + 2t\mathbf{j} + \ln t\mathbf{k} \quad 1 \leq t \leq e$

(b) $\mathbf{r}(t) = \frac{2}{3}t^{3/2}\mathbf{i} + \sqrt{2}t\mathbf{j} + 2\sqrt{t}\mathbf{k} \quad 4 \leq t \leq 9$

(c) $\mathbf{r}(t) = 6\mathbf{i} + 3\sqrt{2}t^2\mathbf{j} + 2t^3\mathbf{k} \quad 1 \leq t \leq \sqrt{2}$

(d) $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k} \quad 0 \leq t \leq 1$