

CHAPTER 13 & 14: FUNCTIONS OF SEVERAL VARIABLES

MATH 2019 – SPRING 2008

1. Given

$$w = yz + 3xy^2 + 4z^4$$

$$x = \tan s \sin t$$

$$y = e^{st}$$

$$z = \ln(\sin^2 s + t^4)$$

- (a) Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$. Leave your answer in terms of x, y, z, s, t .
- (b) Find $\frac{\partial w}{\partial t}$ at $s = 0, t = 1$.
- (c) $z = \ln(x - yz) - y^3 z \cos z$ implicitly defines z as a function of x and y . Find the first partial derivatives of z . That is, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

2.

$$f(x, y) = \arctan \frac{x}{y}$$

- (a) Find the directional derivative of f at the point $(2, 2)$ in the direction of the vector $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$.
- (b) Find the maximum value of the directional derivative at the point $(2, 2)$.
- (c) In which direction does this occur? (You should interpret this as the direction in which f increases the fastest.)

3.

$$f(x, y, z) = e^{\sin(xy)} + z^2$$

- (a) Find the directional derivative of f at the point $P(\frac{\pi}{2}, -2, 0)$ in the direction of the point $Q(\frac{\pi}{2}, -1, 6)$.
- (b) Find the maximum value of the directional derivative at the point $P(\frac{\pi}{2}, -2, 0)$.
- (c) In which direction does this occur?

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4. (a) Consider the surface given by the equation $xy^2 + zy^2 + 4y - xz^2 = 18$. Find an equation of the tangent plane and an equation of the normal line to the surface at the point $(-2, 0, 3)$.
- (b) Now, consider the surface described by the function $f(x, y) = x \tan y$. Find an equation of the tangent plane and an equation of the normal line to the surface at the point $(2, \pi/4, 2)$.

5. Examine the following functions for relative extrema and saddle points.

- (a) $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$
 (b) $f(x, y) = x^3 + 12xy + y^3$
 (c) $f(x, y) = 3xy - x^2y - xy^2$
 (d) $f(x, y) = x^3 + y^5 + 3x^2 - 9x - 5y - 8$

6. Use Lagrange multipliers to find the critical points of the function subject to the given constraint.

- (a) $f(x, y) = x^2 - y^2; \quad x^2 + y^2 = 1$
 (b) $f(x, y) = 3xy; \quad 4x^2 + y^2 = 200$
 (c) $f(x, y) = 2x^2 + 32y^2; \quad xy = 4$
 (d) $f(x, y) = x + 2y - 2; \quad x^2y = 432$
 (e) $f(x, y) = x^2y; \quad 4x + 3y = 18$
 (f) $f(x, y, z) = 2x + 6y + 10z; \quad x^2 + y^2 + z^2 = 35$

7. Evaluate the iterated integral. Here, note that you need to first switch the order of integration.

$$\int_0^4 \int_{\sqrt{y}}^2 \cos(x^3) dx dy$$

8. (a) Evaluate $\iint_R 12xy dA$ where R is the region in the plane bounded by the graphs of

$$x - y^2 = 0 \quad \text{and} \quad x + y = 2$$

- (b) Although you would not do this in practice, reverse the order of integration that you used in part (a) and check that you get the same answer.

- (c) Evaluate $\iint_R e^{25-x^2-y^2} dA$ where R is the region in the second and third quadrants inside the circle $x^2 + y^2 = 25$ outside the circle $x^2 + y^2 = 9$ and above the line $y = x$.

9. Evaluate the iterated integral by converting to polar coordinates.

$$\int_{-\sqrt{2}}^0 \int_{-x}^{\sqrt{4-x^2}} 15xy^2 dy dx$$

10. Find the area of the surface given by $z = f(x, y)$ over the region R .

- (a) $f(x, y) = 15 + 2x - 3y$ R : triangle with vertices $(0, 0)$, $(3, 0)$, $(3, 3)$
(b) $f(x, y) = xy$ $R = \{(x, y) : x^2 + y^2 \leq 16\}$