CHAPTER 9: POWER SERIES

MATH 2019 – SPRING 2008

1. Find the radius of convergence and the interval of convergence for the following power series

$$\sum_{k=0}^{\infty} \frac{(-1)^{k-1}\,(x-5)^k}{6^k\,\sqrt{k+4}}$$

2. Find a power series centered at $c = 3$ for the function $f(x) = \dfrac{2}{3x-18}$ and identify the interval of convergence.

3. (a) Find a power series centered at $c = -1$ for $f(x) = \dfrac{2}{3x+9}$ and identify the interval of convergence.
   (b) Use part (a) to find a power series centered at $c = -1$ for $g(x) = \ln(3x+9)$ and identify the interval of convergence.
   (c) Use part (a), again, to find a power series centered at $c = -1$ for $h(x) = \dfrac{1}{(3x+9)^2}$. Here, just state the radius of convergence.

4. Find the 4^{th}-degree Taylor polynomial for $f(x) = \ln(3x+9)$ at $c = -1$. Compare your answer to part (b) of Problem 3.

5. Given the Maclaurin series

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n\,x^{2n+1}}{(2n+1)!}$$

   (a) Find the Maclaurin series for $\cos x$.
   (b) Using part (a), find the Maclaurin series for $\cos(x^2)$.
   (c) Use the first 4 terms of the series you found in part (b) to approximate

$$\int_0^{0.3} \sqrt{x}\cos(x^2)\,dx$$

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Problem 5 illustrates the importance of power series. First of all, it was easy. In the end, we were just integrating a polynomial. In principle, we would get the exact answer if we worked with the full series. This really just involves putting "$\cdots$" in the appropriate places. Secondly, there is no other way. No matter how hard you try integration-by-parts, you will not be able to integrate $\sqrt{x}\cos(x^2)$.