Regarding Problem set 8

The solutions to most of these problems are in "Relative Extrema" and "Lagrange Multipliers" notes.

The ones that aren't in the notes are done below.

You should probably try to look at all of them since they're all a little different.

The ordering of the problems is not necessarily from easiest to hardest.

Problem 1 a

$$f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$$

$f_x = 6x^2 + y^2 + 10x$

$f_y = 2xy + 2y$

$f_{xx} = 12x + 10$

$f_{yy} = 2x + 2$

$f_{xy} = 2y$

1° Find critical points

we solve

$$6x^2 + y^2 + 10x = 0 \qquad (1)$$

$$2xy + 2y = 0 \qquad (2)$$

From (2) we have

$$2y(x+1) = 0$$

So,

$y = 0$ or $x = -1$

plug into (1)

$6x^2 + 10x = 0$

$x(6x+10) = 0$

$x = 0$ or $x = \frac{-10}{6}$

$(0,0)$ $\left(\frac{-5}{3}, 0\right)$ crit pts

plug into (1)

$6 + y^2 - 10 = 0$

$y^2 = 4$

$y = 2$ or $y = -2$

$(-1, 2)$ $(-1, -2)$ crit pt.

$\underline{\underline{2^\circ}}$ Apply 2nd partials test

$$d = f_{xx} f_{yy} - (f_{xy})^2$$

$$d = (12x+10)(2x+2) - 4y^2$$

$d(0,0) = (10)(2) > 0$

$f_{xx}(0,0) = 10 > 0$ $\cup$ $\Rightarrow$ rel min at $(0,0)$

$d(\frac{-5}{3},0) = (-10)(\text{negative}) > 0$ $\Rightarrow$ rel max at $(\frac{-5}{3},0)$

$f_{xx}(\frac{-5}{3},0) = -10 < 0$ $\cap$

$d(-1,2) = (-2)(0) - 16 < 0 \Rightarrow$ saddle pt at $(-1,2)$

$d(-1,-2) = (-2)(0) - 16 < 0 \Rightarrow$ saddle pt at $(-1,-2)$

$\boxed{E}$

Problem 2f

$f(x,y,z) = 2x + 6y + 10z$

$\underbrace{x^2+y^2+z^2}_{g(x,y,z)} = 35$

To find critical points we solve

$$\begin{cases} \vec{\nabla} f = \lambda \vec{\nabla} g \\ \text{constraint} \end{cases}$$

$$\langle 2, 6, 10 \rangle = \lambda \langle 2x, 2y, 2z \rangle$$

Our system is

$$\begin{cases} 2 = \lambda(2x) & (1) \\ 6 = \lambda(2y) & (2) \\ 10 = \lambda(2z) & (3) \\ x^2+y^2+z^2 = 35 & (4) \end{cases}$$

Note that λ can't be 0 since we would have $2=0$ in (1).

$(1) \Rightarrow \quad x = \frac{1}{\lambda} \quad (*)$

$(2) \Rightarrow \quad y = \frac{3}{\lambda} \quad (**)$

$(3) \Rightarrow \quad z = \frac{5}{\lambda} \quad (***)$

plugging this into (4) gives

$$\frac{1}{\lambda^2} + \frac{9}{\lambda^2} + \frac{25}{\lambda^2} = 35$$

$$\frac{35}{\lambda^2} = 35$$

$$1 = \lambda^2$$

$\lambda = 1$ or $\lambda = -1$

plug into (*), (**), (***)

$x = 1$

$y = 3$

$z = 5$

$(1, 3, 5)$ crit pt

$x = -1$

$y = -3$

$z = -5$

$(-1, -3, -5)$ crit pt

$\boxed{F_c}$

Problem 2c

$f(x,y) = 2x^2 + 32y^2$ $\qquad$ $\underbrace{xy}_{g(x,y)} = 4$

To find critical points we solve

$$\begin{cases} \vec{\nabla} f = \lambda \vec{\nabla} g \\ \text{constraint} \end{cases}$$

$$\langle 4x, 64y \rangle = \lambda \langle y, x \rangle$$

$$\begin{cases} 4x = \lambda y & (1) \\ 64y = \lambda x & (2) \\ xy = 4 & (3) \end{cases}$$

Note that neither x nor y can be 0 since this would contradict (3). We can divide by x and y as much as we want.

$(1) \Rightarrow \frac{4x}{y} = \lambda$

(2) becomes $64y = \frac{4x}{y} x$

$$64y^2 = 4x^2$$

$$16y^2 = x^2 \quad (*)$$

$(3) \Rightarrow y = \frac{4}{x}$

plug this into $(*)$

$$16 \cdot \frac{16}{x^2} = x^2$$

$$256 = x^4$$

$x = 4$ or $x = -4$

$(3) \Rightarrow y = 1$ $\qquad$ $(3) \Rightarrow y = -1$

$(4,1)$ $\quad$ $(-4,-1)$ crit pts

$\boxed{E}$