13.2

Ex 1 Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - 4y^2}{x^2 + y^2}$$

Soln: To show that the limit does not exist, you have to show that you get two different limits as you approach $(0,0)$ along two different paths.

What two paths do you choose? This is kind of an art form. But common paths are

$$x = 0, \quad y = 0, \quad y = x, \quad y = x^2, \quad x = y^2$$

$$y = x^3, \quad x = y^3, \quad \text{etc.}$$

For this particular problem:

Approach along $x = 0$

$$\frac{x^2 - 4y^2}{x^2 + y^2} = \frac{-4y^2}{y^2} = -4$$

Approach along $y = 0$

$$\frac{x^2 - 4y^2}{x^2 + y^2} = \frac{x^2}{x^2} = 1$$

So, $f(x,y) = \dfrac{x^2 - 4y^2}{x^2 + y^2}$ approaches -4 if we approach $(0,0)$ along the line $x = 0$.

$f(x,y)$ approaches 1 if we approach $(0,0)$ along the line $y = 0$.

Thus, the limit does not exist.

$\blacksquare$

<u>Ex 2</u> Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$$

<u>Soln</u>: Approach along $x = 0$

$$\frac{xy}{x^2 + y^2} = \frac{0}{y^2} = 0$$

If we approach $(0,0)$ along $y = 0$ we also get 0 which does not help us.

Approach along $y = x$

$$\frac{xy}{x^2+y^2} = \frac{x^2}{x^2+x^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$

So, we get two different limits if we approach $(0,0)$ along the two paths $x = 0$ and $y = x$.

Therefore, the limit does not exist.

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<u>other problems</u> for you to try.

Show that the following limits do not exist.

① $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

② $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$

③ $\displaystyle\lim_{(x,y)\to(0,0)} \frac{8x^2y^2}{x^4+y^4}$

④ $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2+3xy+4y^2}{3x^2+5y^2}$