

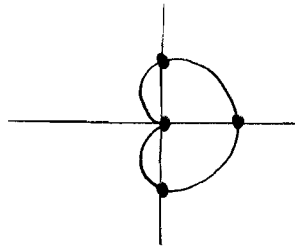
## Polar Graphs

You should be able to recognize and sketch the graphs of the following.

### Limaçons

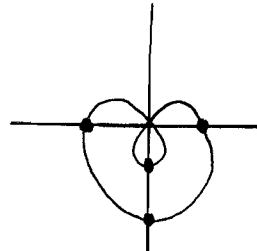
$$r = a \pm b \cos \theta$$

$$r = a \pm b \sin \theta$$



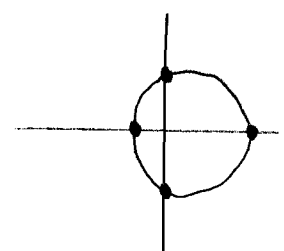
$$|b| = |a|$$

cardioid



$$|b| > |a|$$

limaçon  
with loop



$$|b| < |a|$$

### Note

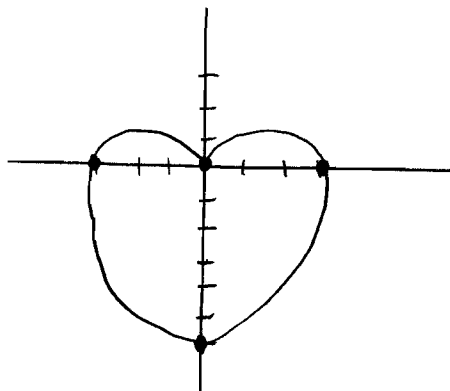
These curves are traced once as  $\theta$  increases from 0 to  $2\pi$ .

### Example

$$r = 3 - 3 \sin \theta$$

We recognize that this is an equation of a limaçon (a cardioid to be exact). We know its general shape. To pin it down more precisely, we plot some important points.

$\theta$	$r$
0	3
$\pi/2$	0
$\pi$	3
$3\pi/2$	6

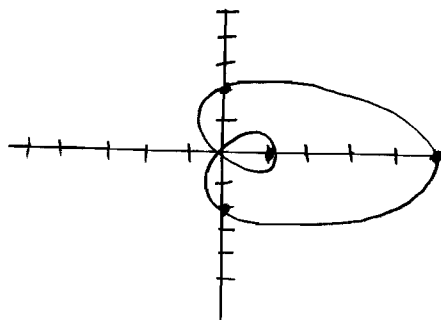


## Example

$$r = 2 + 3 \cos \theta$$

This is an equation of a limacon with loop.

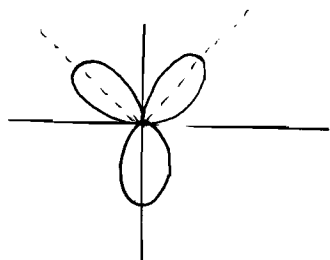
$\theta$	$r$
0	5
$\pi/2$	2
$\pi$	-1
$3\pi/2$	2



## Roses

$$r = a \cos(n\theta)$$

$$r = a \sin(n\theta)$$



1° If  $n$  is odd, there are  $n$  petals and the curve is traced once as  $\theta$  increases from  $0$  to  $\pi$ .

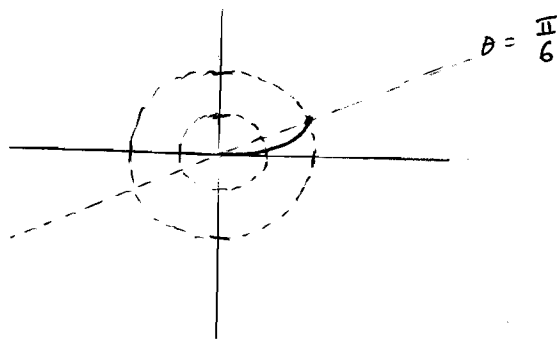
2° If  $n$  is even, there are  $2n$  petals and the curve is traced once as  $\theta$  increases from  $0$  to  $2\pi$ .

## Example

$$r = 2 \sin 3\theta$$

This is a rose with 3 petals.

when  $\theta = 0$ ,  $\sin 3\theta = 0$ . As  $\theta$  increases  $\sin 3\theta$  will increase until it is 1 at  $\theta = \frac{\pi}{6}$ . (study the unit circle). So, as  $\theta$  increases from 0 to  $\frac{\pi}{6}$ ,  $r = 2\sin 3\theta$  increases from 0 to 2. This portion of the curve is shown

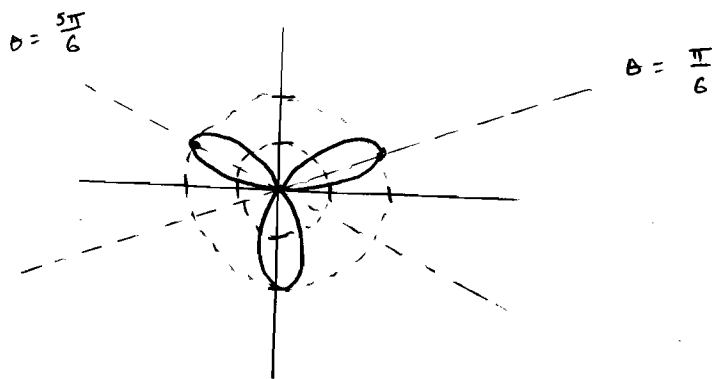


We know there are three equally spaced petals. The tips will be  $\frac{2\pi}{3}$  radians apart.

The next tip will occur at  $\theta = \frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6}$

The last tip will occur at  $\theta = \frac{5\pi}{6} + \frac{2\pi}{3} = \frac{9\pi}{6} = \frac{3\pi}{2}$

Using this information and symmetry, we can draw the rest of the curve.

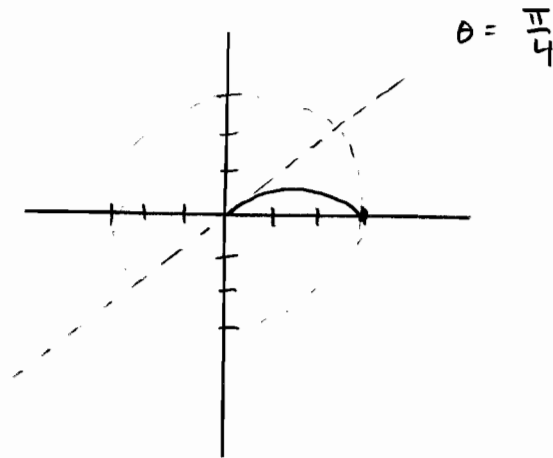


## Example

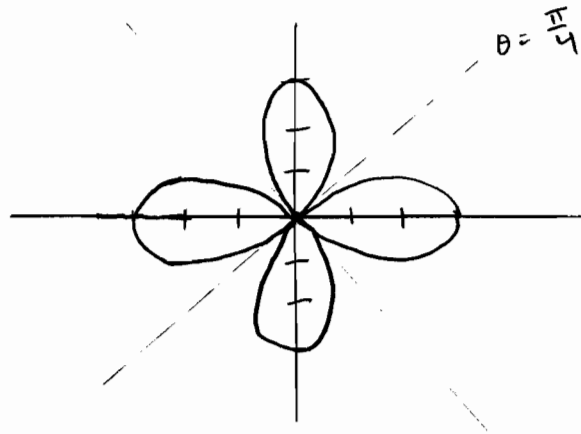
$$r = 3 \cos 2\theta$$

This is a rose with 4 petals.

When  $\theta = 0$ ,  $\cos 2\theta = 1$ . As  $\theta$  increases  $\cos 2\theta$  will decrease until it is 0 at  $\theta = \frac{\pi}{4}$ . So, as  $\theta$  increases from 0 to  $\frac{\pi}{4}$ ,  $r = 3 \cos 2\theta$  will decrease from 3 to 0. This portion of the curve is shown



There are 4 equally spaced petals. The tips will be  $\frac{2\pi}{4} = \frac{\pi}{2}$  radians apart. The full curve is:



## Example

Try to graph  $r = -5 \sin 3\theta$ .

## Circles

$$r = a \cos \theta$$

$$r = a \sin \theta$$

You can think of these as 1-petal roses ( $n=1$ ).

They are traced once as  $\theta$  increases from 0 to  $\pi$ .

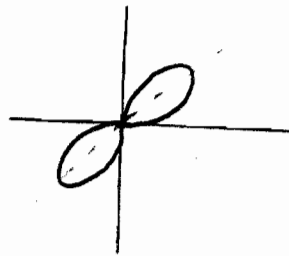
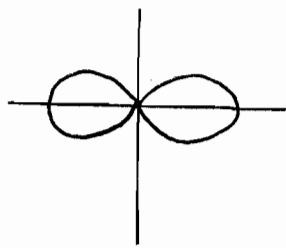
Graph these the same way you would a rose.  
Just remember that these are circles.

What is the radius?

## Lemniscates

$$r^2 = a^2 \cos 2\theta$$

$$r^2 = a^2 \sin 2\theta$$

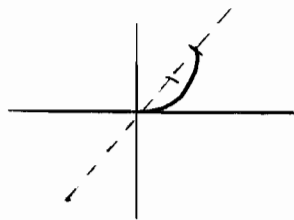


## Example

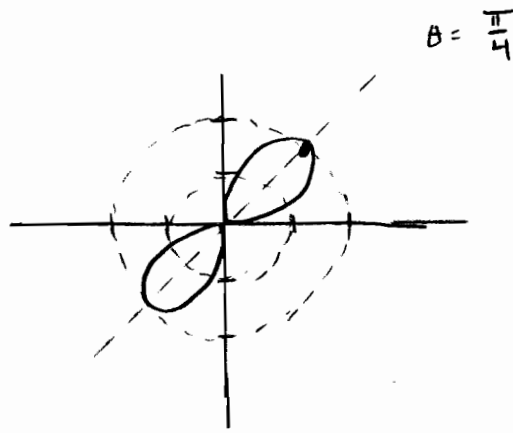
$$r^2 = 4 \sin 2\theta$$

When  $\theta = 0$ ,  $\sin 2\theta = 0$ . As  $\theta$  increases,  $\sin 2\theta$  will increase until it is 1 at  $\theta = \frac{\pi}{4}$ .

Thus,  $r$  will increase from 0 to 2 (why 2 and not 4?)



Draw the full  
curve using  
symmetry.



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Try the other lemniscate on your own.

Ex  $r^2 = 9 \cos 2\theta$