

PROBLEM SET 1

MATH 2019 – SPRING 2008

1. Sketch the curve represented by the parametric equations (indicate the orientation of the curve).

(a) $x = 4 + 2 \sin^2 t$, $y = \cos t$, $0 \leq t \leq \pi$

(b) $x = 2 \sin t - 5$, $y = 3 \cos t + 2$, $0 \leq t \leq \pi$

(c) $x = \ln t$, $y = \sqrt{t}$, $t \geq 1$

(d) $x = \cos t$, $y = \cos 2t$, $-\pi \leq t \leq -\frac{\pi}{2}$

2. Find all points (if any) of horizontal and vertical tangency to the curve.

(a) $x = 2t^3 - 3t^2 - 36t + 1$, $y = t^4 - 8t^2 - 5$

(b) $x = \cos \theta - \cos^2 \theta$, $y = \sin \theta - \sin \theta \cos \theta$, $0 \leq \theta < 2\pi$

3. Consider the curve described by the parametric equations

$$x = e^{-3t} \cos^3 2t \quad y = \ln(5t^3 + 2t + 1)$$

(a) Find $\frac{dy}{dx}$.

- (b) Find an equation of the tangent line to the curve at the point corresponding to $t = 0$.

4. Determine whether the curve is concave upward or concave downward at the given value of the parameter.

$$x = t^3 + t^2 + 1, \quad y = t - t^2, \quad t = 1$$

5. (a) Write $z = -\frac{\sqrt{3}}{4} + \frac{1}{4}i$ in polar form.

- (b) Find z^{11} using DeMoivre's Theorem.

(c) If $w = 4 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$, compute $\frac{w}{z}$.

- (d) Plot $u = -3 - 4i$ on the complex plane. Without doing any additional work, describe in words where uz is on the complex plane.