PROBLEM SET 5

MATH 2019 – SPRING 2008

1. Consider a space curve given by

$$\mathbf{r}(t) = e^t \cos t\, \mathbf{i} + e^t \sin t\, \mathbf{j} + e^t\, \mathbf{k}$$

   Find a set of parametric equations for the line tangent to the curve at the point $P(1, 0, 1)$.

2. If $\mathbf{r}(t) = t\mathbf{i} + \frac{\sqrt{3}}{2}t^2\mathbf{j} + t^3\mathbf{k}$ is the position vector of a flying bee,

   (a) compute the velocity, speed, and acceleration of the bee.

   (b) find the unit tangent vector, the tangential component of acceleration, the normal component of acceleration, and the curvature.

3. Find the unit vectors, $\mathbf{T}(t)$ and $\mathbf{N}(t)$, and the curvature $K(t)$ for the following space curve.

$$\mathbf{r}(t) = 4t\mathbf{i} + \cos 5t\mathbf{j} + \sin 5t\mathbf{k}$$

4. For the following space curve $\mathbf{r}(t)$, tell me everything you know about the curve at the given value of t.

$$\mathbf{r}(t) = (\sin t - t\cos t)\mathbf{i} + \ln(2t+1)\mathbf{j} + \frac{e^{-5t}}{5}\mathbf{k} \qquad t = 0$$

   In particular,

   (a) Find the unit tangent vector, the tangential component of acceleration, the normal component of acceleration, and the curvature at the given value of t.[1]

   (b) Using your answers in part (a), find the principal unit normal vector at the given value of t.[2]

   (c) Find a set of parametric equations for the line tangent to the curve at $t = 0$.

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[1] You are only interested in these quantities at a single point $t = 0$. This should not be messy.

[2] Use the equation $\mathbf{a}(t) = a_{\mathbf{T}}(t)\mathbf{T}(t) + a_{\mathbf{N}}(t)\mathbf{N}(t)$