PROBLEM SET 6

MATH 2019 – SPRING 2008

1. Find the length of the curve.

   (a) $\mathbf{r}(t) = t^2\mathbf{i} + 2t\mathbf{j} + \ln t\mathbf{k} \qquad 1 \le t \le e$

   (b) $\mathbf{r}(t) = \frac{1}{t}\mathbf{i} + \frac{t^2}{2}\mathbf{j} - 2\sqrt{2}t^{1/2}\mathbf{k} \qquad 1 \le t \le 2$

   (c) $\mathbf{r}(t) = 6\mathbf{i} + 3\sqrt{2}t^2\mathbf{j} + 2t^3\mathbf{k} \qquad 1 \le t \le \sqrt{2}$

   (d) $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k} \qquad 0 \le t \le 1$

2. Sketch several level curves of the function, and use this information to help you visualize what the graph looks like.

   (a) $f(x,y) = x - y^2$

   (b) $f(x,y) = \dfrac{8}{1 + x^2 + y^2}$

   (c) $f(x,y) = \sqrt{16 - x^2 - 16y^2}$

3. Show that the limit does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2 + 3xy + 4y^2}{3x^2 + 5y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4 + y^4}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^2 + y^8}$

   *(d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4 - y^2}$

4. Let f be a real-valued function defined on $\mathbb{R}^2$. Determine whether the following statements are true or false.

   (a) If $f(x,y) \to 5$ as $(x,y) \to (3,1)$ along every straight line through $(3,1)$, then $\lim_{(x,y)\to(3,1)} f(x,y) = 5$.

   (b) If $\lim_{(x,y)\to(3,1)} f(x,y) = 6$, then $f(3,1) = 6$.

   (c) If f is a continuous at $(3,1)$ and $f(3,1) = 10$, then $\lim_{(x,y)\to(3,1)} f(x,y) = 10$.

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