

PROBLEM SET 9

MATH 2019 – SPRING 2008

1. Evaluate the iterated integral.

$$(a) \int_0^1 \int_x^{2x^2} (10 + 2x^2 + 3xy^2) dy dx \quad (b) \int_0^1 \int_0^x \cos(x^2) dy dx$$

2. Sketch the region R of integration and switch the order of integration.

$$(a) \int_{-2}^0 \int_0^{4-x^2} f(x, y) dy dx \quad (d) \int_1^{10} \int_0^{\ln y} f(x, y) dx dy$$

$$(b) \int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} f(x, y) dy dx \quad (e) \int_{-2}^1 \int_{x+2}^{4-x^2} f(x, y) dy dx$$

$$(c) \int_0^2 \int_y^{4-y} f(x, y) dx dy \quad (f) \int_0^1 \int_{y^2}^{2-y} f(x, y) dx dy$$

3. Evaluate the iterated integral. (Note that you may need to switch the order of integration.)

$$\int_0^2 \int_{y^2}^4 4y \cos(x^2) dx dy$$

4. Evaluate the iterated integral. Make your life easier by converting to polar coordinates.

$$(a) \int_{-2}^0 \int_0^{\sqrt{4-y^2}} (x^2 + y^2)^{3/2} dx dy$$

$$(b) \int_0^{\sqrt{3}} \int_{\frac{1}{\sqrt{3}}x}^{\sqrt{4-x^2}} e^{4-x^2-y^2} dy dx$$

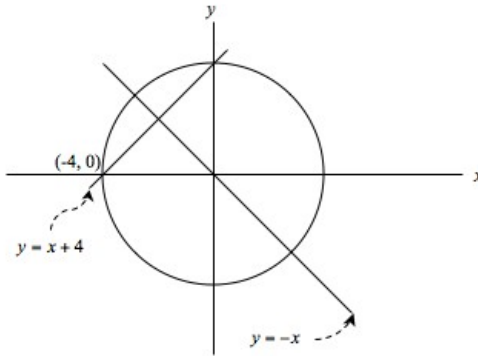
5. Use polar coordinates to combine the sum

$$\int_{\frac{1}{\sqrt{2}}}^1 \int_{\sqrt{1-x^2}}^x xy dy dx + \int_1^{\sqrt{2}} \int_0^x xy dy dx + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} xy dy dx$$

into one double integral. Then evaluate the double integral.

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6. Consider the figure below. Set up an iterated integral(s) to calculate the area of the region inside the circle, below the line $y = x + 4$, and below the line $y = -x$.¹



7. (a) Evaluate $\iint_R xy \, dA$ where R is the region in the plane bounded by the graphs of

$$x - y^2 = 0 \quad \text{and} \quad x + y = 2$$

- (b) Evaluate $\iint_R \frac{4}{\sqrt{100 - x^2 - y^2}} \, dA$ where R is the region in the second and third quadrants inside the circle $x^2 + y^2 = 64$ outside the circle $x^2 + y^2 = 36$ and above the line $y = x$.

¹This problem is courtesy of Dr. Lewis. There are many correct answers. Try to find several of them. Remember that you may use both rectangular and polar coordinates.