EXAM 1 REVIEW

MATH 2419 – SPRING 2008

1. Consider the curve described by the parametric equations

$$x = 2t^3 - 3t^2 - 36t + 1, \quad y = t^4 - 8t^2 - 5$$

   (a) Find all points of horizontal and vertical tangency to the curve.
   (b) Find the slope and concavity of the curve at $t = 1$.

2. Find all points of horizontal and vertical tangency to the curve

$$x = 3\cos\theta - 3\cos^2\theta, \quad y = 3\sin\theta - 3\sin\theta\cos\theta, \quad 0 \le \theta < 2\pi$$

3. You should be able to recognize the equations of limaçons (cardioid, with loop, etc.), roses, circles, and lemniscates, and you should be able to sketch their graphs.

4. (a) Find the arc length of the curve described by the parametric equations
      i. $x = 2t^2 \qquad y = (4t+9)^{3/2} \qquad 0 \le t \le 1$
      ii. $x = t^2 + 1, \quad y = 4t^3 + 3, \quad 0 \le t \le 1$

   (b) Find the arc length of the polar function over the indicated interval.

$$r = \cos^3(\theta/3) \qquad 0 \le \theta \le 3\pi$$

5. (a) Find the slope of the polar curve at $\theta = \pi/4$

$$r = 3 - 3\cos\theta$$

   (b) Find all points of horizontal and vertical tangency to the polar curve (cf. Problem 2).
   (c) Find the length of the curve.

6. Find the area inside one petal of $r = 4\cos 23\theta$.

7. (a) Find the area of the region inside $r = 2\sin 12\theta$ and outside $r = \sqrt{2}$.
   (b) Find the area of the common interior.

Date: February 5, 2008

8. (a) Find the area inside the inner loop of $r = 1 + 2\sin\theta$.
   (b) Find the area of the region between the inner and outer loops.

9. (a) Find the area of the region inside $r = 3\cos\theta$ and outside $r = 1 + \cos\theta$.
   (b) Find the area of the region inside $r = 1 + \cos\theta$ and outside $r = 3\cos\theta$.
   (c) Find the area of the common interior.

10. (a) Find the area of the region inside $r^2 = 8\cos 2\theta$ and outside $r = 2$.
    (b) Find the area of the common interior.

11. (a) Find the projection of **N** onto **L**. Then, compute the vector component of **N** orthogonal to **L**. Show that the answer you found is indeed orthogonal to **L**. Draw a picture to illustrate the significance of what you just computed.

$$\mathbf{L} = < 6, 3, 2 > \qquad \mathbf{N} = 4\mathbf{i} - 4\mathbf{j}$$

    (b) In words, what are the direction angles and direction cosines of **L**? Find the direction cosines of **L**.
    (c) Find the cosine of the angle between **N** and **L**.

12. (a) Verify that the points are the vertices of a parallelogram and find its area.

$$(2, -1, 1) \quad (5, 1, 4) \quad (0, 1, 1) \quad (3, 3, 4)$$

    (b) Pick three of the above points. Explain how you would find the area of the triangle determined by these three points. How would you find the three angles of the triangle? How would you find the lengths of the three sides?

13. Find two unit vectors orthogonal to both **u** and **v**

$$\mathbf{u} = \mathbf{i} + 6\mathbf{j} \qquad \mathbf{v} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$$

14. A parallelepiped has adjacent edges PQ, PR, and PS. Determine its volume.

$$P(1, 1, 1) \quad Q(2, 0, 3) \quad R(4, 1, 7) \quad S(3, -1, -2)$$

15. Find parametric equations and symmetric equations for the line.

On problems 7b, 8b, 9bc, and 10b, it's enough to just set up the integrals. However, you should work problems 6, 7a, 8a, 9a, and 10a all the way through.

(a) The line that passes through the point $(2, 4, -3)$ and is parallel to the vector $\mathbf{v} = 5\mathbf{i} - \mathbf{j}$

(b) The line that passes through the points $(2, 4, -3)$ and $(3, -1, 1)$

(c) The line that passes through the point $(-3, 5, 4)$ and is parallel to the line given by the symmetric equations

$$\frac{x-1}{3} = \frac{y+1}{-2} = z - 3$$

(d) The line that passes through the point $(-3, 5, 4)$ and is parallel to the line $x = 5 - 2t$, $y = -4 + 2t$, $z = 0$

(e) The line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$

16. Find an equation of the plane.

(a) The plane through the points $(3, -1, 2)$, $(8, 2, 4)$, and $(-1, -2, -3)$

(b) The plane through the point $(3, 2, 2)$ and perpendicular to the line $\dfrac{x-1}{4} = y+1 = \dfrac{z+3}{-3}$

(c) The plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$

(d) The plane containing the lines

$$\frac{x-2}{-3} = \frac{y-2}{6} = z - 3 \quad \text{and} \quad \frac{x-3}{2} = y + 5 = \frac{z+2}{4}$$

(e) The plane that passes through the points $(3, 2, 1)$ and $(3, 1, -5)$ and is perpendicular to the plane $6x + 7y + 2z = 10$

17. Verify that the planes $6x - 3y + z = 5$ and $-x + y + 5z = 5$ are not parallel. So, they intersect in a line. Find an equation of this line. If you don't want to do that, then at least give me a direction vector for the line. What can you say about the angle between the two planes? Quickly verify that the planes are not perpendicular.