EXAM 2 REVIEW

MATH 2419 – SPRING 2008

1. Find $\mathbf{r}(t)$ for the given conditions.

   (a) $\mathbf{r}''(t) = 25e^{-5t}\,\mathbf{i} + 12t\,\mathbf{j} - 16\cos 2t\,\mathbf{k}$
   $\mathbf{r}'(0) = 3\mathbf{j} - 4\mathbf{k}$
   $\mathbf{r}(0) = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$

   (b) $\mathbf{r}'(t) = \dfrac{t}{t^2 - 3}\mathbf{i} - e^{-3t}\mathbf{k}$
   $\mathbf{r}(2) = -\mathbf{i} + \mathbf{j}$

2. Find the unit vectors, $\mathbf{T}(t)$ and $\mathbf{N}(t)$, and the curvature $K(t)$ for the following space curve.
$$\mathbf{r}(t) = 4t\mathbf{i} + \cos 5t\mathbf{j} + \sin 5t\mathbf{k}$$

3. Given $\mathbf{r}(t) = t\mathbf{i} + \frac{\sqrt{3}}{2}t^2\mathbf{j} + t^3\mathbf{k}$, find the unit tangent vector, the tangential component of acceleration, the normal component of acceleration, and the curvature.

4. For the following space curve $\mathbf{r}(t)$, find the unit tangent vector, the tangential component of acceleration, the normal component of acceleration, and the curvature at the given value of t.
$$\mathbf{r}(t) = (t - \cos t)\mathbf{i} + \ln(t+1)\mathbf{j} + \frac{e^{-5t}}{5}\mathbf{k} \qquad t = 0$$
   Then, find a set of parametric equations for the line tangent to the curve at $t = 0$.

5. Find the length of the curve.

   (a) $\mathbf{r}(t) = \frac{2}{3}t^{3/2}\mathbf{i} + \sqrt{2}t\mathbf{j} + 2\sqrt{t}\mathbf{k} \qquad 4 \le t \le 9$

   (b) $\mathbf{r}(t) = 6\mathbf{i} + 3\sqrt{2}t^2\mathbf{j} + 2t^3\mathbf{k} \qquad 1 \le t \le \sqrt{2}$

   (c) $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k} \qquad 0 \le t \le 1$

6. (a) The dimensions of a closed rectangular box are measured to be 80 cm, 60 cm, and 50 cm, respectively, with a possible error of 1% in each dimension. Use differentials to estimate the maximum error in calculating the surface area of the box. What is the maximum percent error?

   (b) The box begins moving. The length and width are increasing at a rate of 2 cm/s, while the height is decreasing at a rate of 5 cm/s. Calculate the rate at which the volume is changing at the instant the length is 84 cm, the width is 64 cm, and the height is 40 cm.

7. Given

$$\begin{aligned} w &= (x + x^2y^3 + y^6 + 1)^5 \\ x &= \sin r \cos(\pi s - t) \\ y &= \frac{t}{s^2} + r^2 \end{aligned}$$

(a) Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$. Leave your answer in terms of x, y, r, s, t.

(b) Find $\frac{\partial w}{\partial r}$ at $s = 1$, $t = 0$, $r = 0$.

(c) $z = \ln(x - yz) - y^3 z \cos z$ implicitly defines z as a function of x and y. Find the first partial derivatives of z. That is, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

8.

$$f(x, y, z) = e^{\sin(xy)} + z^2$$

(a) Find the directional derivative of f at the point $P(\frac{\pi}{2}, -2, 0)$ in the direction of the point $Q(\frac{\pi}{2}, -1, 6)$.

(b) Find the maximum value of the directional derivative at the point $P(\frac{\pi}{2}, -2, 0)$.

(c) In which direction does this occur?

9. (a) Consider the surface given by the equation $xy^2 + zy^2 + 4y - xz^2 = 18$. Find an equation of the tangent plane and an equation of the normal line to the surface at the point $(-2, 0, 3)$.

(b) Now, consider the surface described by the function $f(x, y) = x \tan y$. Find an equation of the tangent plane and an equation of the normal line to the surface at the point $(2, \pi/4, 2)$.

10. The temperature in a region of space is given by

$$T(x, y, z) = e^{\sin(xy)} + z^2$$

A bee is flying through the air. Its position at time t is given by

$$\mathbf{r}(t) = \langle t, 2\sin 3t, 2\cos 3t \rangle$$

(a) In what direction is the bee heading as it passes through the point $P(\frac{\pi}{2}, -2, 0)$?

(b) How fast is the temperature around the bee changing as it passes through that point? (i.e. What is the directional derivative of T at the point P in the direction you found in part (a)?)

(c) Suppose the bee is hovering at the point $P(\frac{\pi}{2}, -2, 0)$. The bee feels cold. In what direction should it fly to get warm? (i.e. In what direction from the point P does the temperature increase most rapidly?)

(d) Not knowing any calculus, the bee flies to the point $Q(\frac{\pi}{4}, -3, 1)$. Use differentials to estimate the change in temperature as the bee flies from the point P to the point Q.

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