An Optimized Cost-Effective Multicarrier Transceiver for Two-Ray Channels

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Abstract

The optimum set of modulation filters for multicarrier modulation on two–ray channels is derived in closed–form. These filters are shown to be sinusoidal even for a finite number of carriers.

The proposed modulation scheme optimizes the performance of two–ray channels corrupted by additive white Gaussian noise without incurring the data rate loss associated with FFT–based multicarrier transceivers. The case of multiple–ray channels is also considered.

I. Introduction

Multicarrier modulation (MCM) has attracted considerable attention recently as a viable transmission technology on spectrally–shaped channels [7, 3]. It involves the use of an orthogonal set of modulation basis vectors at the transmitter to partition the channel response into a large number of parallel, independent, and memoryless subchannels that can be individually decoded.

The optimum modulation basis vectors require the computation of the complete eigen–decomposition of a channel–dependent matrix, which becomes a formidable task even for a moderate number of subchannels [7]. In a suboptimum but computationally–efficient implementation of MCM known as the discrete multitone transceiver (DMT), a *cyclic prefix* is added to each input block to make the the channel matrix circulant, and hence diagonalizable by the FFT basis vectors [4]. This cyclic prefix represents a loss in the achievable data rate that becomes significant when a moderate FFT size is used (to reduce complexity and latency).

In this letter, we show that for any two–ray channel, the optimum modulation vectors become sinusoidal, and hence can be computed efficiently without incurring the cyclic prefix overhead of DMT. This is achieved by exploiting the special structure of the finite–blocklength MCM transceiver that was ignored in previous works. To apply the proposed modulation scheme for multiple–ray channels, a low–complexity equalization scheme is proposed.

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II. Problem Formulation

A. Optimum Modulation/Demodulation Basis Vectors

We shall start by assuming that the channel impulse response (CIR) consists of $(\nu+1)$ rays, i.e., $h(D) = h_0+h_1D+\cdots+h_\nu D^\nu$. In the "vector coding" implementation of MCM [7], each input block of size N is padded with ν zeros to eliminate interblock interference, thus making successive block transmissions independent.

In this case, the input–output relationship can be expressed in matrix form as follows

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \\ y_{N+1} \\ \vdots \\ y_{N+\nu-1} \end{bmatrix} = \begin{bmatrix} h_0 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ h_\nu & \cdots & h_0 & 0 & \vdots \\ 0 & h_\nu & & h_0 & \vdots \\ 0 & \cdots & h_\nu & \cdots & h_0 \\ \vdots & \cdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & h_\nu \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_N \\ n_{N+1} \\ \vdots \\ n_{N+\nu-1} \end{bmatrix} , \tag{1}$$

or more compactly

$$\mathbf{y} \stackrel{def}{=} \mathbf{Hx} + \mathbf{n} \, .$$

For the case when both $\mathbf{x}$ and $\mathbf{n}$ are complex Gaussian vectors, the channel block throughput (or equivalently the mutual information between the input and output blocks) is given by [1]

$$\mathbf{I}(\mathbf{x}\,;\mathbf{y}) = \log_2 |\mathbf{I}_N + \mathbf{H}^t\mathbf{R}_{nn}^{-1}\mathbf{H}\mathbf{R}_{xx}| \, , \tag{2}$$

where $\mathbf{I}_N$ is the identity matrix of size N, $(.)^t$ denotes the transpose, and $\mathbf{R}_{nn}$ and $\mathbf{R}_{xx}$ denote the noise and input correlation matrices, respectively.

If we define the following eigen–decomposition of the channel–and–noise description matrix $\mathbf{H}^t\mathbf{R}_{nn}^{-1}\mathbf{H}$

$$\mathbf{H}^t\mathbf{R}_{nn}^{-1}\mathbf{H} \stackrel{def}{=} \mathbf{U}\mathbf{\Lambda}\mathbf{U}^t \, , \tag{3}$$

then it can be shown [1] that for the case of Gaussian noise that is *independent* of the input, the optimum choice for $\mathbf{R}_{xx}$ that maximizes $\mathbf{I}(\mathbf{x}\,;\mathbf{y})$ has the eigen–decomposition

$$\mathbf{R}_{xx} \stackrel{def}{=} \mathbf{U}\mathbf{\Delta}\mathbf{U}^t \, . \tag{4}$$

For this choice of $\mathbf{R}_{xx}$, Equation (2) becomes

$$\begin{aligned} \mathbf{I}(\mathbf{x}\,;\mathbf{y}) &= \log_2 |\mathbf{I}_N + \mathbf{U}\mathbf{\Lambda}\mathbf{\Delta}\mathbf{U}^t| \\ &= \sum_{i=1}^{N} \log_2(1+\lambda_i\delta_i) \, , \end{aligned} \tag{5}$$

where δ_i and λ_i are the i^{th} diagonal elements of $\mathbf{\Lambda}$ and $\mathbf{\Delta}$, respectively.

In the sequel, we shall assume a two–ray channel model ($\nu = 1$) and white noise ($\mathbf{R}_{nn} = N_0\mathbf{I}_{N+\nu}$). This model is often used for magnetic recording (e.g. the $1 \pm D$ partial response channels) and some wireless channels; its simplicity and analytical tractability offers more insight into the problem. Section II-C shows how to accommodate the case of $\nu > 1$.

The two–ray channel assumption makes the symmetric Toeplitz[1] matrix $\mathbf{H}^t\mathbf{H}$ *tri-diagonal.* In this case, the eigenvalues and eigenvectors of $\mathbf{H}^t\mathbf{R}_{nn}^{-1}\mathbf{H}$ can be expressed in closed–form as follows [2]

$$\lambda_k = \frac{1}{N_0}(|h_0|^2 + |h_1|^2 + 2h_0h_1\cos(\frac{k\pi}{N+1})) \tag{6}$$

$$u_{kl} = \sqrt{\frac{2}{N+1}}\sin(\frac{kl\pi}{N+1}) \ : \ 1 \leq k, l \leq N \ , \tag{7}$$

where $u_{k,l}$ is the (k,l) element of $\mathbf{U}$.

We conclude this section by mentioning that this set of modulation vectors can be efficiently computed using the *discrete sine transform*, as described in [6]. Unlike the DMT implementation of MCM (that uses the FFT and incurs a data rate loss of $\frac{\nu}{N+\nu}$ due to the cyclic prefix), the proposed MCM transceiver incurs no such loss since it uses zero padding.

B. Bandwidth Optimization

Bandwidth optimization is concerned with determining the optimum number of subchannels, call it $\bar{N}(\leq N)$, that should be used to maximize the channel throughput, subject to a fixed energy budget $S_x \stackrel{def}{=} \sum_{i=1}^{N}\delta_i$. This can be achieved using standard Lagrange multiplier techniques and performing a "water-pour" procedure [5] on the channel eigenvalues given by (6).

Such a procedure also determines the optimum *energy allocation* to the $\bar{N}$ usable subchannels to be given by

$$\delta_i = \max(c - \frac{1}{\lambda_i}, 0) \ : 1 \leq i \leq N, \tag{8}$$

where c is a constant determined from the input energy constraint as follows

$$\begin{aligned} S_x &= \sum_{i=1}^{\bar{N}}(c - \frac{1}{\lambda_i}) \\ \Rightarrow c &= \frac{1}{\bar{N}}(S_x + \sum_{i=1}^{\bar{N}}\frac{N_0}{(|h_0|^2 + |h_1|^2 + 2h_0h_1\cos(\frac{i\pi}{N+1}))}) \ . \end{aligned}$$

[1] The (i,j) element of $\mathbf{H}^t\mathbf{H}$ is equal to $\sum_k h_k h_{k+|j-i|}$, which is a function of $(j-i)$.

[2] Without the two–ray channel assumption, the eigenvectors will still be symmetric (or anti–symmteric) but not sinusoidal in general.

Using (5), the maximum achievable throughput of the MCM transceiver is

$$I(\mathbf{x}\,;\mathbf{y})_{max} = \bar{N}\log_2(\frac{SNR}{\bar{N}}+(\frac{1}{\bar{N}}\sum_{m=1}^{\bar{N}}\frac{1}{(|h_0|^2+|h_1|^2+2h_0h_1\cos(\frac{m\pi}{N+1}))}))+\sum_{i=1}^{\bar{N}}\log_2(|h_0|^2+|h_1|^2+2h_0h_1\cos(\frac{i\pi}{N+1}))\ , \tag{9}$$

which can be readily evaluated given h_0, h_1, N, and the input SNR ($\stackrel{def}{=} \frac{S_x}{N_0}$).

C. Multiple-Ray Channels

For channels with memory $\nu > 1$, we propose the use of a linear equalizer, denoted by $\mathbf{w}$, of length N_f to equalize the CIR into a target impulse response (TIR), denoted by $\mathbf{b}$, of length 2 and combine it with the MCM transceiver of Section II-A.

More specifically, under the unit–energy constraint, $|\mathbf{b}| = 1$, it can be shown using our results in [2] that the optimum length–2 TIR is equal to the eigenvector of $\mathbf{R}_\Delta$ that corresponds to its *minimum* eigenvalue (which is equal to the minimum equalization mean square error (MSE)). The 2×2 matrix $\mathbf{R}_\Delta$ is given by

$$\mathbf{R}_\Delta \stackrel{def}{=} \begin{bmatrix} \mathbf{0}_{2\times\Delta} & \mathbf{I}_2 & \mathbf{0}_{2\times(N_f+\nu-\Delta-2)} \end{bmatrix} \mathbf{R}^{\perp}_{x/y} \begin{bmatrix} \mathbf{0}_{\Delta\times 2} \\ \mathbf{I}_2 \\ \mathbf{0}_{(N_f+\nu-\Delta-2)\times 2} \end{bmatrix} ,$$

where calculating $\mathbf{R}^{\perp}_{x/y} \stackrel{def}{=} S_x(\mathbf{I}_{N_f+\nu} - \mathbf{H}_1^t(\mathbf{H}_1\mathbf{H}_1^t + \frac{1}{SNR}\mathbf{I}_{N_f})^{-1}\mathbf{H}_1)$ requires the inversion of a symmetric *Toeplitz* matrix that can be done in $O(N_f^2)$ computations. The matrix $\mathbf{H}_1$ is a *fully–windowed* Toeplitz matrix of size $N_f \times (N_f+\nu)$ whose first row is $\begin{bmatrix} h_0 & h_1 & \cdots & h_\nu & \mathbf{0}_{1\times(N_f-1)} \end{bmatrix}$. The delay parameter Δ $(0 \le \Delta \le N_f+\nu-2)$ is optimized to result in the minimum equalization MSE.[3]

Once $\mathbf{b}_{opt}$ is computed, the optimum equalizer settings are given by [4]

$$\mathbf{w}^t_{opt} = \mathbf{b}^t_{opt}\mathbf{H}_1^t(\mathbf{H}_1\mathbf{H}_1^t + \frac{1}{SNR}\mathbf{I}_{N_f})^{-1} .$$

III. Conclusions

We showed that the optimum modulation basis vectors for a finite–blocklength multicarrier transceiver on two–ray channels is sinusoidal. These modulation vectors maximize the channel throughput and can be computed efficiently using the discrete sine transform. A closed–form expression for the maximum channel throughput in terms of the channel impulse response coefficients, number of input carriers, and input SNR was presented. For multiple–ray channels, the proposed modulation scheme can be combined with equalization to achieve desirable performance–complexity tradeoffs.

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[3] This minimum (but generally non–zero) MSE will cause the achievable channel throughput to be less than its maximum value in (9).

[4] Using the *Orthogonality Principle.*

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