A Distributed Opportunistic Access Scheme and its Application to OFDMA Systems

Dandan Wang, Student Member, IEEE, Hlaing Minn, Senior Member, IEEE, and Naofal Al-Dhahir, Fellow, IEEE

Abstract—Multiuser systems can provide multiuser diversity gains by assigning channels to users with higher channel gains. To avoid the extensive information exchange with the access point for the uplink access in centralized approaches, we propose in this paper a distributed opportunistic access scheme. Through a judicious design of a novel backoff mechanism to utilize the channel information and reduce collisions, significant multiuser diversity gains are achieved. To a user, the higher the channel gain is, the smaller the backoff time-slot and, hence, the higher the access priority of that user is. In addition, for heterogeneous systems, our proposed scheme can realize multiuser diversity gains and achieve fairness among the users at the same time. Finally, we design two distributed opportunistic access schemes for OFDMA systems. Users contend on all sub-channels in the first scheme and only on several strongest sub-channels in the second scheme. Compared with traditional centralized OFDMA systems and other distributed access schemes, our proposed schemes reduce overhead and achieve a higher throughput.

Index Terms—Opportunistic, OFDMA, MAC, CSI, Slotted Aloha, Multiuser Diversity

I. INTRODUCTION

A fundamental characteristic of wireless mobile communication systems is random channel fading. Traditionally, diversities in the time, space, and frequency domains are exploited to combat the detrimental effects of channel fading. However, instead of mitigating unreliability of each user’s fading channel, multiuser diversity has been proposed to maximize the total information-theoretic capacity in the context of multiuser communications [1][2]. The basic idea of multiuser diversity is to exploit the randomness of fading channels among different users. The larger the dynamic range of channel fluctuations and the number of users, the larger the available multiuser diversity gain is. In a centralized downlink system, the access point assigns the channel to the user with the best instantaneous channel gain as in [1],[3] to achieve the multiuser diversity gain. For an up-link (multiple access) model, this requires the centralized scheduler to acquire estimates of each user’s channel state information (CSI) before making the scheduling decision. The overhead and delay incurred in doing this may limit the system’s performance, particularly if the number of active users is large or the channels change rapidly. Thus, a distributed access scheme reducing this information exchange but still utilizing the multiuser diversity is very desirable.

In distributed systems, all users only know their own channels (also called decentralized CSI). Several recently proposed access schemes utilize this decentralized CSI. A binary distributed scheduling scheme is derived in [4] which asymptotically achieves a fraction ($\frac{1}{2}$) of the centralized throughput obtained with multiuser diversity. To resolve collisions, an opportunistic splitting algorithm is proposed in [5]-[6]. Although this algorithm can guarantee access to the user with the best channel gain when the contention length is unlimited, its overhead is not minimized since its design is mainly based on contentions. Thus, when the contention length is limited (which is the case in practical systems), there may be some frames on which no user successfully accesses the channel. Another problem of this algorithm is that it requires frequent handshakes between the access point and users. When the channel is not good, these handshaking signals can be detected incorrectly which further increases the collision probability. Other schemes utilizing CSI to enhance the capture effects are reported in [7]-[8].

In this paper, we propose a novel distributed access scheme with a carefully-designed backoff mechanism to utilize the decentralized CSI and reduce the collision probability. All users estimate their channel gains through a periodically-transmitted beacon signal from the access point and compare their channel power gains with predefined backoff thresholds to decide on which mini-slot to send their contention packets. In homogeneous systems where all users have the same statistical channel characteristics, the design criterion of the backoff thresholds is the maximization of the sum throughput of all the users. However, in practical systems, some users could be far away from the access point creating heterogeneous systems, and we design backoff thresholds for them as well. Our proposed scheme achieves fairness among users and provides multiuser diversity gains.

The next issue we address is distributed opportunistic access schemes for orthogonal frequency division multiplexing (OFDM) systems. OFDM is a well-established transmission technology for broadband wireless communication systems [9]-[10]. There are mainly three multiple access schemes in OFDM systems: OFDM/TDMA (time division multiple access), OFDM/FDMA (frequency division multiple access), and OFDMA (orthogonal frequency division multiplex access). In TDMA or FDMA schemes, a single user transmits on all sub-carriers of OFDM symbols within a certain time slot or frequency band. However, in a typical wireless transmission environment, the channel responses of different users are
different. Some sub-carriers might be in deep fading for one user while they might experience high gains for others, hence, providing a diversity source for capacity enhancement. This multiuser and multicarrier diversity can hardly be exploited in TDMA and FDMA systems but is beneficial to OFDMA systems which allow multiple users to transmit simultaneously on different sub-carriers. Since the probability that all users experience a deep fade on a particular sub-carrier is very low, it would be beneficial if the sub-carriers are assigned to the users who experience good channel gains on them.

There are several works on sub-carrier allocation for OFDMA systems in the literature [11]-[12]. However, most of them are centralized algorithms. The access point has to collect channel information from all users to allocate the sub-carriers among different users. Furthermore, this information should be received correctly and with no delay. This requires prohibitive overhead for the practical implementation of OFDMA systems with centralized access and (sub)optimal resource allocation.

In this paper, we propose two approaches based on the backoff mechanism for distributed access in OFDMA systems. In the first approach, each user contends on all sub-channels (CAC). In the second approach, each user contends on his strongest sub-channels (CSC), where a sub-channel is defined as a group of sub-carriers. When the number of users in the system is known, CAC exhibits better performance than CSC since more users can introduce more multiuser diversity gain.

The rest of this paper is organized as follows. The proposed distributed opportunistic scheme for single-carrier systems is presented in Section II. Novel backoff thresholds design for homogeneous and heterogeneous systems are presented in Section III. In Section IV, applications to OFDMA systems are discussed. Simulation results are given in Section V and the paper is concluded in Section VI.

II. THE PROPOSED OPPORTUNISTIC SCHEME FOR SINGLE-CARRIER SYSTEMS

In this section, we propose a distributed opportunistic access scheme for single-carrier systems, in which a novel backoff mechanism is designed to utilize CSI and reduce collisions. We consider saturated time division duplex (TDD) systems, i.e., all the users always have data to send to the access point in a TDD mode. Thus, the channel gains between users and the access point can be estimated at each user through a periodically-transmitted beacon signal from the access point. Each user is assumed to know his own channel information only. We do not consider the capture effect, i.e., we assume that only one user can be decoded correctly on each channel. Also, the transmission powers for all users are assumed fixed.

Data are transmitted in frames and each frame is divided into three sub-frames: contention period, acknowledgment (ACK) period, and data transmission period. The contention period consists of $K$ mini-slots. The key idea of our proposed backoff scheme is to encode knowledge of the channel power gain into the backoff time. A user with higher channel power gain is designed to have higher probability to access the system so that the overall throughput of the system is improved. All users compare their channel power gains with a set of backoff thresholds $\eta = \{\eta_0, \eta_1, ..., \eta_K, 0 \leq \eta_K < \eta_{K-1} < ... < \eta_1 \leq \eta_0(=\infty)\}$ associated with backoff mini-slots.

Note that in homogeneous systems, all users have the same set of thresholds, while in heterogeneous systems, different users have different sets of thresholds as described in the next section. Assume that there are $N_u$ users in the system. Let $G_j, j = 1, 2, ..., N_u$ denote the channel power gain of the $j$th user and $f_j(\cdot), j = 1, 2, ..., N_u$ denote the probability density function (pdf) of $G_j$. Moreover, let $B(G_j)$ be the adaptive data rate function which takes full advantage of the channel power gain $G_j$ and thus is a nondecreasing function of $G_j$. Then, according to the frame structure of the system, our proposed protocol proceeds as follows:

- Contention period: The length of this period is set much smaller than the length of the data transmission period to reduce the contention overhead. Each user generates a backoff time (in mini-slots) by comparing his channel power gains with the set of backoff thresholds (Next section describes the thresholds design). For the $j$th user, if $\eta_i \leq G_j < \eta_{i-1}, i = 1, 2, ..., K$, then his backoff time-slot is the $i$th mini-slot. Each user waits until his backoff mini-slot has elapsed and then sends a contention packet on the $i$th mini-slot, which contains his address information. The access point decodes the received contention packets and keeps the address information of the first successfully-decoded user. Note that there may be several successfully-decoded contention packets on different mini-slots. However, since we do not consider the capture effect, there is at most one user that can transmit on this channel successfully. The successful user is the earliest one whose contention packet is correctly decoded. Only if there is no user or more than one user contending on any mini-slot in the first $(i-1)$ mini-slots, we can have a successful user on the $i$th mini-slot.

- ACK period: This period consists of one mini-slot which is used to transmit the ACK message from the access point. The address of the successful user, which is obtained from the contention period, is broadcast through the ACK message. All unsuccessful users will try to access the network in the next frame.

- Data transmission period: In this period, the successful user transmits his packets.

III. BACKOFF THRESHOLDS DESIGN FOR SINGLE-CARRIER SYSTEMS

In this section, the backoff thresholds design for both homogeneous and heterogeneous systems are described. In the network, system throughput is a useful design criterion as it considers the effects of both physical channels (physical layer) and access schemes (MAC layer). Thus, in this paper, we adopt this cross-layer view to design our backoff scheme.

A. Backoff Thresholds Design for Homogeneous Systems

For homogeneous systems, $\{f_j(\cdot)\}$ are the same for all users and hence we omit the user index $j$ for simplicity in this subsection. For each mini-slot, only if there is exactly one user sending his contention packet, then that contention packet can...
be decoded correctly. The probability that any user sends his contention packet on the \( i \)-th mini-slot is

\[
q_i = \left( \int_{q_i}^{\eta_{i-1}} f(x) \, dx \right). \tag{1}
\]

Denote \( E[n_1, n_2, ..., n_K] \) as the event that there are \( n_1, n_2, ..., n_K \) users contending on the 1st, 2nd, ..., \( K \)-th mini-slot, respectively. By the multinomial probability law, we have

\[
P(E[n_1, n_2, ..., n_K]) = \frac{N_u!}{n_1! n_2! ... n_K!} \left( \prod_{j=1}^{K} q_j^{n_j} \right) (1 - \sum_{j=1}^{K} q_j)^{N_u - \sum_{j=1}^{K} n_j}. \tag{2}
\]

To simplify the notation, we denote the convex set \( C_i = \{(n_1, n_2, ..., n_K) : 0 \leq n_j | j=1,2,...,i-1 \leq N_u, n_j | j=1,2,...,i-1 \neq 1, \sum_{j=1}^{i-1} n_j \leq N_u - 1, n_i = 1, \} \) and \( D_i = \{(n_1, n_2, ..., n_{K-1}) : 0 \leq n_j | j=1,2,...,i-1 \leq N_u, n_j | j=1,2,...,i-1 \neq 1, \sum_{j=1}^{i-1} n_j \leq N_u - 1 \}. \) The only difference between \( C_i \) and \( D_i \) is that \( D_i \) does not include \( n_i \). \( C_i \) and \( D_i \) include all the possible combinations of the users’ contention in the first \( i - 1 \) slots to make the successful user occur on the \( i \)-th mini-slot. Thus, the probability that the successful user occurs on the \( i \)-th mini-slot (successful access probability) is equal to (3). Then, the throughput of the system corresponding to the \( i \)-th mini-slot is defined as

\[
T_s(i) = \int_{q_i}^{\eta_{i-1}} R(x) f(x) \frac{P_v(i)}{q_i} \, dx. \tag{4}
\]

Thus, the throughput of the system over all \( K \) mini-slots is

\[
S = \sum_{i=1}^{K} T_s(i). \tag{5}
\]

Now, our design problem is reduced to solving the following optimization problem

\[
\max_{\eta_1, \eta_2, ..., \eta_K} \{ S = \sum_{i=1}^{K} T_s(i) \}
\]

\[
s.t. \quad 0 \leq \eta_1 < \eta_2 < ... < \eta_i \leq \eta_0 (= \infty), \tag{6}
\]

which is very difficult to solve, even numerically. To make the optimization problem tractable, we will consider a virtual system consisting of a total of \( N_u K \) users (there are \( N_u \) users in the actual system). These \( N_u K \) users are divided into \( K \) groups. Each group with \( N_u \) users contends on one mini-slot and the users contending on different time slots are independent. We aim to obtain the thresholds \( \{\eta_i\} \) by analyzing this virtual system.

In the virtual system, the probability that a contention packet can be decoded correctly at the \( i \)-th mini-slot is

\[
p_i = N_u q_i (1 - q_i)^{N_u - 1}, \quad i = 1, 2, \ldots, K. \tag{7}
\]

The successful access probability on the \( i \)-th mini-slot is

\[
P_v(i) = \prod_{j=1}^{i-1} (1 - p_j) N_u q_i (1 - q_i)^{N_u - 1}. \tag{8}
\]

Thus, the throughput of the virtual system corresponding to the \( i \)-th mini-slot is given by

\[
T_v(i) = \int_{q_i}^{\eta_{i-1}} R(x) f(x) \frac{P_v(i)}{q_i} \, dx, \quad i = 1, 2, \ldots, K, \tag{9}
\]

and its throughput over all \( K \) mini-slots is given by

\[
S_v = \sum_{i=1}^{K} T_v(i). \tag{10}
\]

Now, for the virtual system, the threshold design problem is reduced to solving the following optimization problem

\[
\max_{\eta_1, \eta_2, ..., \eta_K} \{ S_v = \sum_{i=1}^{K} T_v(i) \}
\]

\[
s.t. \quad 0 \leq \eta_1 < \eta_2 < ... < \eta_i \leq \eta_0 (= \infty). \tag{11}
\]

This optimization problem can be solved by setting the derivative of \( S_v \) with respect to all \( \eta_i, i = 1, 2, ... K \) to zeros,

\[
\frac{\partial S_v}{\partial \eta_i} = 0, \quad i = 1, 2, ..., K. \tag{12}
\]

The closed-form solution of (12) is still analytically intractable. However, several optimization packages can be used to compute its numerical solution, such as the \textit{fminsearch} function in MATLAB. In Appendix I, we show that when \( q_i \equiv \frac{1}{N_u} \), the optimization problem presented in (6) can be approximated by the optimization problem given in (11). Therefore, in this paper, we will use the thresholds designed for the virtual system as the solution for the actual system.

For the virtual system, we have the following theorem:

Theorem: If \( N_u > K \), the optimal set of thresholds \( \{\eta_i, i = 1, 2, ..., K\} \) for the virtual system for a constant rate (CR) function \( R(.) \) satisfies the relation

\[
\int_{q_i}^{\eta_{i-1}} f(x) \, dx = \frac{1}{N_u}, \quad i = 1, 2, ..., K. \tag{13}
\]

Proof: Refer to Appendix II. We denote the optimum thresholds for CR as the suboptimal solutions for variable rate (VR). In the simulation section, we will quantify the throughput loss introduced by the suboptimal thresholds.

B. Backoff Thresholds Design for Heterogeneous Systems

In the previous subsection, we described how to design the backoff thresholds in homogeneous systems. The design criterion is to maximize the total throughput of the system which also maximizes the throughput of individual users and guarantees fairness. However, in practice, some users may be close to the access point while the other users may be far away so that the channel statistics are not identical. In this scenario of heterogeneous systems, if the same set of thresholds as the homogeneous systems are used for all the users, the users close to the access point (with higher average channel gains) will always get access to the network which causes unfairness among users. Therefore, new approaches of designing backoff thresholds for heterogeneous systems have to be proposed.

Now, let us first look into how this problem is solved in centralized systems. In [13][3], proportional fair (PF) scheduling is proposed to realize multiuser diversity while ensuring fairness among users. The PF algorithm schedules a user when his instantaneous channel quality is high relative to his own average channel condition over a certain average time. Following the basic idea of [3], the authors in [14] designed a decentralized scheduler that maximizes the product of the users’ throughput. Both [3] and [14] show that the scheduler
for heterogeneous systems should be the one that views the other types of users as if they were of the same type, i.e. each user views himself in a homogeneous system.

Motivated by [3] and [14], we design the back-off thresholds for each user according to his own channel statistics while assuming the other users have the same channel statistics. Therefore, different users will have different sets of thresholds which are decided by the channel characteristics of that type of user. If one type of users always has better channel gain than others, the corresponding thresholds will also be larger than their counterparts. More specifically, the thresholds for the $i$th user are obtained by solving (11) in Section III. A, in which, $f(x)$ and $R(x)$ are replaced by $f_i(x)$ and $R_i(x)$, respectively. Note that $N_u$ is the total number of all types of users in the system. Then, user $i$ will compare his channel power gains with the thresholds to decide his backoff mini-slot as described in Section II. The fairness and multiuser diversity gain of this scheme will be corroborated by the simulation results in Section V.

IV. DISTRIBUTED OPPORTUNISTIC ACCESS SCHEMES FOR OFDMA SYSTEMS

In this section, we discuss the application of our proposed opportunistic access schemes to TDD OFDMA systems. The frame structure is illustrated in Fig. 2, where the contention periods on all sub-channels have the same length so that all data are transmitted simultaneously during the data transmission period which simplifies decoding at the access point. Here, a sub-channel refers to a group of sub-carriers in OFDMA systems. We describe two approaches based on the scheme proposed in the previous sections for single-carrier systems. In the first approach, each user contends on all the sub-channels (CAC) while in the second approach each user contends only on his $\beta$ strongest sub-channels (CSC). In this section, we only consider homogeneous systems. However, the development can be easily extended to heterogeneous systems using a similar approach to that given in Section III.B.

\[
P_s(i) = \sum_{\mathcal{C}_i} P(E[n_1, n_2, \ldots, n_i]) = \frac{N_u q_i}{n_1 n_2! \cdots n_i - 1! (N_u - \sum_{j=1}^{i-1} n_j - 1)!} \left( \prod_{j=1}^{i-1} q_j^{n_j} \right) (1 - \sum_{j=1}^{i} q_j)^{N_u - \sum_{j=1}^{i} n_j - 1}. \tag{3}
\]

A. OFDMA System Model

We consider an uplink OFDMA system with $N$ users where the adjacent sub-carriers are grouped into sub-channels. Since the access point periodically transmits beacon signals, the average sub-channel gains are known by the users. To simplify our analysis, we assume that all sub-channels have the same number of sub-carriers. Due to the correlation of the channel frequency response (or equivalently, due to the limited channel delay spread which is much less than the OFDM symbol duration), the sub-carriers within each sub-channel have highly-correlated channel gains. Let $N_s$ denote the total number of sub-carriers in the system. We consider a multipath Rayleigh fading channel consisting of $N_p$ independent taps and the average total energy of all these taps is $1$. Let $H_{i,j}$ denote the channel coefficient of user $i$ ($i = 1, 2, \ldots, N$) on the $j$-th ($j = 1, 2,\ldots,N_s$) sub-carrier. Then, $\{H_{i,j}\}$ are circularly-symmetric complex Gaussian random variables with zero mean and unit variance. Suppose all $N_s$ sub-carriers are divided into $N_c$ sub-channels. Then, each sub-channel has $N_c/N_s$ sub-carriers. The average channel power gain of the $n$-th ($n = 1, 2, \ldots, N_c$) sub-channel for user $i$ is given by

\[
G_{i,n} = \sum_{j=1}^{N_c} H_{i,j}^2 \overline{\sum_{j=1}^{N_c} |H_{i,j}|^2} \approx \overline{|H_{i,j}|^2} (n-1)+1, \tag{14}
\]

which can be estimated from the beacon signal periodically transmitted from the access point. From (14), we find that $G_{i,n}$ is an exponential random variable with mean 1. The cumulative distribution function (CDF) of $G_{i,n}$ is given by

\[
F_G(g) = (1 - e^{-g}) u(g), \tag{15}
\]

where $u(g)$ is the unit step function. Note that $\{G_{i,n}, n = 1, 2, \ldots, N_c\}$ may be dependent in practical systems. However, to simplify the design, we assume in this section that $\{G_{i,n}, n = 1, 2, \ldots, N_c\}$ are independent and identically distributed (i.i.d) random variables\(^1\). However, in the simulation, our channel model does not require this assumption.

B. Access Scheme I: Contending on All Sub-Channels (CAC)

This approach applies our proposed scheme for single-carrier systems in Section III. A on each sub-channel. All users estimate the average channel power gain on each sub-channel through the periodically transmitted beacon signal and compare them with the designed thresholds to obtain the backoff mini-slots on each sub-channel. Each sub-channel can be treated as a single-carrier system. Since the average channel power gains on different sub-channels may be different, a

\(^1\)If $N_s = N_p^2 = N_c^2$ and these $N_p$ taps are i.i.d with a uniform power delay profile, then $\overline{|H_{i,j}|^2 (n-1)+1}, n = 1, 2, \ldots, N_c$ are independent and thus $\{G_{i,n}, n = 1, 2, \ldots, N_c\}$ are independent due to (14). For other channel models, there may be some correlation among $G_{i,n}, n = 1, 2, \ldots, N_c$. For design purposes, we still use this assumption.
user may have different backoff mini-slots on different sub-channels and the successful users on different sub-channels may be different. In the data transmission period, the successful users will transmit their data on their associated sub-channels. The backoff thresholds can be numerically computed according to (12) with the channel statistics given in (15).

C. Access Scheme II: Contending on Strongest Sub-Channels (CSC)

In this approach, each user contends on his $\beta$ strongest channels based on his channel power gain $G_{i,n}$ defined in (14). Define

$$Z_{i,m} = G_{i,(m)},$$

where $\{G_{i,(m)} : m = 1, 2, ..., N_c\}$ is the ordered sequence of $\{G_{i,n} : n = 1, 2, ..., N_c\}$ such that $G_{i,(1)} \leq G_{i,(2)} \leq ... \leq G_{i,(N_c)}$. Under the assumption that $G_{i,n}$ are independent, the CDF of the $m$th order statistic $Z_{i,m}$ is [15]

$$F_{Z_{i,m}}(z) = \sum_{l=m}^{N_c} \binom{N_c}{l} F_{G}(z)[1 - F_{G}(z)]^{N_c-l},$$

where $F_{G}$ is given in (15). Since we consider homogeneous OFDMA systems, we simply ignore the index $i$ of $F_{Z_{i,m}}$. In this approach, a specific sub-channel and a particular frame may be chosen by some users as their best channel, while other users may view it as their $\beta$th strongest channel. Let $N_{m,j}, m = 1, 2, ..., N_c, j = 1, 2, ..., \beta$ denote the exact number of users choosing the $m$th sub-channel as their $j$th strongest contention channel at this frame. Then, on the $m$th sub-channel, there are $\beta$ types of users with $N_{m,j}$ users in the $j$th type. The channel power gain of the users in the $j$th type has the CDF $F_{Z_{N_c-j+1}}(z)$. This introduces a new problem of designing thresholds which maximize throughput for each sub-channel.

We design the thresholds for each sub-channel using the distribution of the average channel power gain on that sub-channel. We treat all the users contending on a specific sub-channel as one group and the average channel power gain is defined as the channel power gain of a randomly-selected user in this group with equal probability. Then, the distribution of the average channel power gain on the $m$th sub-channel conditioned on $N_{m,j}, m = 1, 2, ..., N_c, j = 1, 2, ..., \beta$ is

$$F_{H_{m|N_{m,1},N_{m,2}...N_{m,\beta}}}(z) = \frac{\beta}{\sum_{j=1}^{\beta} \sum_{i=1}^{N_{m,i}} N_{m,j}} F_{Z_{N_c-j+1}}(z).$$  

(18)

However, $N_{m,j}$ for $m = 1, 2, ..., N_c$ and $j = 1, 2, ..., \beta$ are random variables in practical systems, which may vary from one frame to another frame. Thus, the distribution of the average channel power gain is

$$F_{H_{m}}(z) = E[F_{H_{m|N_{m,1},N_{m,2}...N_{m,\beta}}}(z)],$$

where the expectation is over $N_{m,1}, N_{m,2}...N_{m,\beta}$. Using the assumption that $G_{i,n}, i = 1, 2, ..., N_c$ are i.i.d, the probability that one user chooses any sub-channel as his $j$th ($j = 1, 2, ..., \beta$) strongest sub-channel is $\frac{1}{N_c}$. Then, we have

$$P\{N_{m,j}\} = \binom{N}{N_{m,j}} \left(\frac{1}{N_c}\right)^{N_{m,j}} \left(1 - \frac{1}{N_c}\right)^{N - N_{m,j}},$$

and the expectation of $N_{m,j}$ is

$$\bar{N}_{m,j} = \frac{N}{N_c}, j = 1, 2, ..., \beta.$$  

(21)

For analytical tractability, we approximate the expectation of (18) over $\{N_{m,j}\}$ (i.e., (19)) with (18) conditioned on $\bar{N}_{m,j}$ instead of $\{N_{m,j}\}$. This gives

$$\bar{F}_{H_m}(z) = \frac{\beta}{\beta} \frac{\bar{N}_{m,j}}{\sum_{i=1}^{\beta} N_{m,i}} F_{Z_{N_c-j+1}}(z) = \frac{1}{\beta} \sum_{j=1}^{\beta} F_{Z_{N_c-j+1}}(z).$$

(22)

Finally, we can compute the thresholds from (11) using $N_u = \frac{N_c}{2}$ and the pdf associated with $\bar{F}_{H_m}(z)$.

V. NUMERICAL AND SIMULATION RESULTS

Simulation and theoretical results of both single-carrier and OFDMA systems are discussed in this section. The simulation results are obtained from $N_f = 100,000$ independent frames and we assume an operating point of BER=$10^{-5}$.

A. Single-Carrier Systems

1) Performance of homogeneous systems: In homogeneous single-carrier systems, the channel $h_i$ between user $i$ and the access point has a zero-mean unit-variance circularly-symmetric complex Gaussian distribution. Hence, the channel power gain $g_i = |h_i|^2$ has an exponential distribution with mean “1”. The transmission power is “1” and the received Signal-to-Noise-Ratio (SNR) is 15dB. The continuous rate function from [16] is defined as

$$R(g_i) = \log_2(1 + \frac{\gamma g_i}{\sigma_i^2})$$

(23)

where $\sigma_i^2$ is the noise power and $\gamma$ is the SNR gap given by

$$\gamma = \frac{1.5}{\ln(5 \text{ BER})}.$$  

(24)

In the opportunistic splitting algorithm [5], all users have to wait for the feedback from the access point after they send the contention packets. Therefore, one mini-slot in [5] is equivalent to two mini-slots in our proposed algorithm. Since we use one mini-slot to transmit ACK after the contention period, $K$ mini-slots in the opportunistic splitting algorithm is equivalent to $2K - 1$ mini-slots in our contention period.

In Fig. 2, we compare the throughput of the proposed scheme and the opportunistic splitting scheme. Also included for comparison, as an upper bound, is the performance of the centralized approach which always assigns channels to the user with the best channel gain. Our proposed scheme performs better than the opportunistic splitting scheme and approaches the upper bound as $K$ increases. Another advantage of our proposed scheme over the opportunistic splitting scheme is that we do not need the frequent handshakes between users and the access point as these handshakes could be possibly decoded.
incorrectly which will further reduce the throughput. Fig. 2 also illustrates that our theoretical throughput expressions of the virtual system (eq. (10)) match very well with the simulation results, verifying that the throughput of the virtual system is a good approximation to that of the actual system. Fig. 2 also shows the effect of inaccurately estimating the number of users on the throughput via the curve labeled as ‘est. err.’ where the actual number of users ($N_u$) varies from 10 to 100. The suboptimal thresholds due to inaccuracies in $N_u$ cause only a small throughput degradation unless $N_u$ is significantly smaller than $N_u$. This shows the robustness of our scheme. We can also draw the same conclusion as in Section III. A that it is better to overestimate than underestimate $N_u$.

Fig. 3 shows that the successful access probabilities of both the opportunistic splitting and proposed schemes do not vary too much with the number of users. The reason is that the thresholds in both schemes change with the number of users in the system to achieve the maximum throughput. Fig. 4 shows the throughput comparison for the suboptimal thresholds defined by (13) and the optimal thresholds of the virtual system. The suboptimal solutions experience only small degradation but they simplify the system design in practical systems.

2) Performance of heterogeneous systems: For heterogeneous systems, we choose 5 types of users whose SNR values are evenly spaced in dB scale over the range of 10 dB to 22 dB. Each type has the same number of users. Fig. 5 gives the throughput of different schemes while Fig. 6 shows the average access probability of each type, i.e., the ratio of the number of frames successfully accessed by each type over the total number of frames, where it can be seen that our proposed scheme achieves much higher multiuser diversity gain compared with the TDMA scheme. In the TDMA scheme, all users gain access to the system in different frames based on the round-robin scheduling scheme. As a benchmark, an upper bound is also given for which the channel is always assigned to the user with the best channel gain without any fairness constraint. Although the throughput of our proposed scheme is lower than the upper bound, our scheme guarantees fairness among different users which can be seen from Fig. 6.

B. OFDMA Systems

In OFDMA systems, the channels of different users are modeled as independent 3-tap Rayleigh fading channels with an exponential power delay profile. Since the access point periodically transmits beacon signals, we assume that the average sub-channel gains are known by the users.

We consider OFDM/OFDMA systems with $N_s = 256$ sub-carriers and each frame contains 48 OFDM symbols. In our proposed schemes, one sub-channel has 64 sub-carriers and, hence, there are $N_c = 4$ sub-channels for $N_s = 256$. For the contention period, each mini-slot is composed of the
transmission time of one contention packet and the maximum propagation delay of the system. The contention packet contains the address information of the transmitting user. Only 32 sub-carriers are used to transmit this contention packet. We set one mini-slot to be 1/2 of one OFDM symbol duration. According to the frame structure given in Fig. 1, we consider $K = 7$ mini-slots in the contention period. Thus, the length of the data transmission period is $L = 2 \times 48 - 7 - 1 = 88$ mini-slots. The following four schemes are given as benchmarks.

- **OFDM/TDMA**: Users access the system according to the TDMA round-robin scheme and all the sub-carriers are allocated to one user within one frame.
- **Ideal centralized OFDMA**: Each sub-channel is allocated to the user with the best average sub-channel gain. This is an upper bound of all centralized OFDMA schemes.
- **Opportunistic splitting scheme CAC**: The scheme in [5] is directly applied on each sub-channel.

For the VR service, according to the required BER performance, successful users can use adaptive modulation to send higher data rates at better channel conditions to improve the overall throughput. The rate function $R_{i,n}$ achieved by a successful user $i$ on the $n$-th sub-channel is

$$R_{i,n} = \frac{\sum_{j=1}^{N_s} \log_2(1 + \frac{2|h_{i,j}|^2}{\sigma_j^2}) N_s / N_c}{N_f(K + L + 1)N_c}$$

(25)

where $h_{i,j}$ is the channel gain of user $i$ on the $j$-th sub-carrier, $\sigma_j^2$ is the noise power at the $j$-th sub-carrier and $\gamma$ is the SNR gap defined in (24).

Let $M_n, n = 1, 2, \ldots, N_c$ denote the number of busy frames on the $n$-th sub-channel and $c_{n,j}$ denote the corresponding rate on the $j$th busy frame of the $n$th sub-channel which is calculated as in (25) for the successful user on the $n$-th sub-channel. Then, the throughput of the system is given by

$$S = \frac{\sum_{n=1}^{N_c} \sum_{j=1}^{M_n} c_{n,j} L}{N_f(K + L + 1)N_c}$$

(26)

The results in Fig. 7 show that our proposed scheme performs much better than OFDM/TDMA which does not have multiuser diversity gain. In Fig. 7, even if we do not include the huge overhead of the ideal centralized OFDMA scheme, our proposed scheme is still comparable with it. Note that the mismatch between the simulation and theoretical results is due to the approximation in (14). Fig. 8 shows the performance of CSC for different $\beta$ where at $\beta = 4$, CSC is equivalent to CAC. The simulation results show that at low traffic load, CAC outperforms CSC while at medium and high traffic loads, the throughput of CAC and CSC with larger $\beta$ are almost the same. Intuitively, CSC could outperform CAC at high traffic load since the number of users contending on each sub-channel is smaller for CSC, resulting in a larger successful access probability. However, with the contention length of $K = 7$ for our proposed schemes and equivalently $K = 4$ for the opportunistic splitting algorithm, the successful access probability is already very high (about 0.94) even for a very large number of users as shown in Fig. 3. Hence, for $K = 7$, CSC’s marginal improvement in the successful access probability does not outweigh its loss in multiuser diversity gain over CAC, resulting in almost the same throughput for CAC and CSC with larger $\beta$ at high traffic load.

**VI. CONCLUSIONS**

In this paper, novel distributed opportunistic access schemes for single-carrier and OFDMA systems are proposed to achieve multiuser diversity gains through the judicious design of a novel backoff mechanism which utilizes the channel information in reducing collisions. For homogeneous systems, the backoff design criterion is to maximize the sum throughput...
of all users. For the heterogeneous systems, the backoff design is performed for each user type separately as in a homogeneous system. For OFDMA systems, we propose two distributed opportunistic access schemes analogous to those designed for single-carrier systems to utilize multiuser and multichannel diversity gains. Users contend on all sub-channels in CAC scheme and on the $\beta$ strongest sub-channels in CSC scheme. CAC outperforms CSC since more users can introduce more diversity gain. Compared with the existing distributed or centralized schemes, our proposed scheme reduces overhead and achieves a better spectral efficiency as corroborated by both simulation results and theoretical analysis.

**APPENDIX I**

**SIMILARITY OF THE ACTUAL AND VIRTUAL SYSTEMS**

Since the rate function $R(.)$ is a non-decreasing function, the throughput defined in (4) can be bounded as follows

$$R(\eta) P_s(i) \leq T_s(i) \leq R(\eta-1) P_s(i), \quad (27)$$

where $P_s(i)$ is defined in (3). Similarly, the throughput of the virtual system defined in (9) can be bounded as follows

$$R(\eta) P_v(i) \leq T_s(i) \leq R(\eta-1) P_v(i), \quad (28)$$

where $P_v(i)$ is defined in (8). In the following, we will see the similarity of the virtual system and the actual system by comparing $P_s(i)$ with $P_v(i)$.

First, let us check $P_s(i)$. As the successful user in the $i$th mini-slot has a higher transmission rate than the successful user in the $j$th mini-slot for $j > i$, we aim to assign higher access probability for lower-indexed mini-slots, i.e., $P_s(1) \geq P_s(2) \geq \ldots \geq P_s(K)$. For the first mini-slot, $P_s(1) = N_u q_1 (1 - q_1)^{N_u - 1}$, which is the probability obtained when there are $N_u$ contending users. The maximum is achieved when $q_1 = 1/N_u$. With $q_1 = 1/N_u$, using the Poisson law when $N_u \gg 1$, $P(E[n_1 = 2]) \approx e^{-1} = 0.1839$ and $P(E[n_1 \geq 3]) \leq e^{-1} = 0.0613$. Then, for the second mini-slot, we simplify $D_2$ as $D_2' = \{(n_1, n_2) : n_1 = 0, 2\}$ by neglecting $n_1 \geq 3$ and $P_s(2) = N_u q_2 (1 - q_2)(1 - q_1)(1 - q_2)^{N_u - 3} = (N_u - 2)q_2(1 - q_1 - q_2)^{N_u - 3}$. Since $N_u q_2 (1 - q_1 - q_2)^{N_u - 3} \leq N_u (1 - q_1 - q_2)^{N_u - 3}$, we have $P_s(2) < N_u q_2 (1 - q_1 - q_2)^{N_u - 3}$. By optimizing the upper bound of $P_s(2)$, we have $q_2 \approx 1/N_u$. Then $P(E[n_1 \leq 2, n_2 \geq 3]) \leq e^{-1}/3 \approx 0.0613$. Therefore, when considering the third mini-slot, $D_3$ can be simplified as $D_3' = \{(n_1, n_2, n_3) : n_1 = 0, 2, 2\}$ by neglecting $n_2 \geq 3$. Similarly, we can obtain $D_i' = \{(n_1, n_2, \ldots, n_i-1) : n_j \in \{0, 2\}, \sum_{j=1}^{i-1} n_j \leq N_u - 1\}$ for the $i$th mini-slot.

Now, let us check $P_v(i)$. We have

$$P_v(i) = \sum_{j=1}^{i-1} (1 - p_j)^{N_u - 1} \left[ \prod_{j=1}^{i-1} (1 - N_u q_j (1 - q_j)^{N_u - 1}) \right]$$

$$= N_u q_1 (1 - q_1)^{N_u - 1} \left[ \prod_{j=1}^{i-1} (1 - N_u q_j (1 - q_j)^{N_u - 1}) \right]$$

$$= N_u q_1 (1 - q_1)^{N_u - 1} \left[ \prod_{j=1}^{i-1} \sum_{n_j \neq 1} N_u! / n_j! (N_u - n_j)! q_j^{n_j} (1 - q_j)^{N_u - n_j} \right]$$

(29)

when $N_u$ is small, the thresholds can be calculated from the actual system.
Substituting (35) into (30) gives

\[ T_V(i) = R \left( \prod_{j=1}^{K-1} (1 - p_j) \right) \left( \frac{N_u}{1} \right) \left( \int_{\eta_i}^{\eta_i-1} f(x) dx \right) \left( 1 - \int_{\eta_i}^{\eta_i-1} f(x) dx \right)^{N_u-1} = R \left( \prod_{j=1}^{K-1} (1 - p_j) \right) p_i. \] (30)

\[ -f(\eta_K) \left( 1 - \int_{\eta_K}^{\eta_K-1} f(x) dx \right)^{N_u-1} + (N_u - 1) \left( 1 - \int_{\eta_K}^{\eta_K-1} f(x) dx \right)^{N_u-2} f(\eta_K) \int_{\eta_K}^{\eta_K-1} f(x) dx = 0. \] (31)

Substituting (36) into (10) and setting the gradient of \( S \) with respect to \( \eta_{K-2} \) to zero, we get

\[ \frac{\partial S_V}{\partial \eta_{K-2}} = \frac{\partial (T_V(K - 2) + T_V(K - 1) + T_V(K))}{\partial \eta_{K-2}} = 0 \Rightarrow \int_{\eta_{K-2}}^{\eta_{K-3}} f(x) dx = \frac{1}{N_u}. \] (37)

Repeating the same process for other gradients of \( S \) given

\[ \int_{\eta_i}^{\eta_i-1} f(x) dx = \frac{1}{N_u}, \quad i = 1, 2, \ldots, K. \] (38)

**REFERENCES**


**TABLE I**

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Dandan Wang (S’2005) received her B.S. and M.S. degrees in electrical engineering from Beijing University of Posts & Telecommunications, Beijing, China, in 2000 and 2003, respectively. She worked at China radio research lab, Ericsson from 2003 to 2004. She is currently a Ph.D student at the Department of Electrical and Computer Engineering, University of Texas at Dallas. Her research interests include wireless communications and networking, signal processing for communications and sensor networks and cooperative/relay systems.

Haung Minn (S’99-M’01-SM’07) for photograph and biography, see vol. 56, no. 3, pp. 474-484, Mar. 2008 in this journal.

Naofal Al-Dhahir (F’08) for photograph and biography, see vol. 55, no. 5, pp. 864-877, May, 2007 in this journal.