Single-Carrier Frequency-Domain Equalization for Space-Time Block-Coded Transmissions over Frequency-Selective Fading Channels

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Abstract

We propose an Alamouti–like scheme for combining space–time block–coding with single–carrier frequency–domain equalization. With 2 transmit antennas, the scheme is shown to achieve significant diversity gains at low complexity over frequency–selective fading channels.

1 Introduction

Single–carrier minimum–mean–square–error frequency–domain equalization (SC MMSE–FDE) was shown in [8, 2] to be an attractive equalization scheme for broadband wireless channels which are characterized by their long impulse response memory. Under these conditions, SC MMSE–FDE has lower complexity, due to its use of the computationally–efficient Fast Fourier Transform (FFT), than time–domain equalization whose complexity grows exponentially with channel memory and spectral efficiency (trellis–based schemes) or require very long FIR filters to achieve acceptable performance (e.g. decision feedback equalizers). Furthermore, the SC MMSE–FDE was shown in [8] to have two main advantages over Orthogonal Frequency Division Multiplexing (OFDM), namely, lower peak–to–average ratio (PAR) and reduced sensitivity to carrier frequency errors.

Diversity transmission using Alamouti's space–time block–coding (STBC) scheme [1] has been proposed in several wireless standards due to its many attractive features. First, it achieves full spatial diversity at full transmission rate for 2 transmit antennas and any signal constellation. Second, it does not require channel state information at the transmitter. Third, maximum likelihood decoding of STBC requires only simple linear processing.

The SC MMSE–FDE was first combined with receive diversity in [4]. There has been some recent work on combining the Alamouti scheme with OFDM [6] and with time–domain equalization [5]. To the best of our knowledge, no previous work has been reported in the literature on combining STBC and SC MMSE–FDE to realize the benefits of both schemes, which is the objective of this paper.

The rest of this paper is organized as follows. We start in Section II by describing our model and assumptions and reviewing the SC MMSE–FDE. In Section III, we propose an Alamouti–like scheme for combining STBC and SC MMSE–FDE. Simulation results for the EDGE environment are given in Section IV and the paper is concluded in Section V.

2 Background

2.1 Channel Model and Assumptions

We consider *single-carrier* block transmission over an additive–noise frequency–selective channel with memory ν. Each block of length N is appended with a length–ν *cyclic prefix* to eliminate inter–block interference (IBI). This is achieved by discarding the first ν received symbols corresponding to the cyclic prefix. Hence, out of every $(N+\nu)$ received symbols, only N symbols are processed. The input–output relationship can be expressed in matrix form as follows

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$, $\mathbf{x}$, and $\mathbf{n}$ are length–N blocks of received, input, and noise symbols, respectively. The input and noise symbols are assumed complex, zero–mean, and uncorrelated with variances σ_x^2 and σ_n^2, respectively. The $N \times N$ channel matrix $\mathbf{H}$ is *circulant* with first column equal to the channel impulse response (CIR) appended by $(N-\nu-1)$ zeros.

Since $\mathbf{H}$ is a circulant matrix, it has the eigen–decomposition

$$\mathbf{H} = \mathbf{Q}^*\mathbf{\Lambda}\mathbf{Q} \tag{2}$$

where $(.)^*$ denotes complex–conjugate transpose, $\mathbf{Q}$ is the orthonormal Discrete Fourier Transform (DFT) matrix whose (l,k) element is given by $\mathbf{Q}(l,k) = \frac{1}{\sqrt{N}}e^{-j\frac{2\pi}{N}lk}$ where $0 \le l,k \le N-1$, and $\mathbf{\Lambda}$ is a *diagonal* matrix whose (k,k) entry is equal to the k^{th} DFT coefficient of the CIR.

2.2 SC MMSE-FDE

After discarding the cyclic prefix, the received time–domain block $\mathbf{y}$ is transformed to the frequency domain by applying the DFT

$$\begin{aligned} \mathbf{Y} &\stackrel{def}{=} \mathbf{Q}\mathbf{y} = \mathbf{\Lambda}\mathbf{Q}\mathbf{x} + \mathbf{Q}\mathbf{n} \ : \ \text{Using (1) and (2)} \\ &\stackrel{def}{=} \mathbf{\Lambda}\mathbf{X} + \mathbf{N} \end{aligned} \tag{3}$$

The SC MMSE–FDE is represented by the $N \times N$ *diagonal* matrix $\mathbf{W}$ whose (i,i) element is given by

$$\mathbf{W}(i,i) = \frac{\mathbf{\Lambda}^*(i,i)}{|\mathbf{\Lambda}(i,i)|^2 + \frac{1}{SNR}} \tag{4}$$

where $SNR \stackrel{def}{=} \frac{\sigma_x^2}{\sigma_n^2}$ [1]. The output of the SC MMSE–FDE, denoted by $\mathbf{Z} \stackrel{def}{=} \mathbf{W}\mathbf{Y}$, is transformed back to the time–domain resulting in a length–N vector $\mathbf{z}$ given by

$$\begin{aligned} \mathbf{z} &\stackrel{def}{=} \mathbf{Q}^*\mathbf{Z} = \mathbf{Q}^*\mathbf{W}\mathbf{Y} \\ &= \mathbf{Q}^*\mathbf{\Lambda}^*(\mathbf{\Lambda}\mathbf{\Lambda}^* + \frac{1}{SNR}\mathbf{I}_N)^{-1}\mathbf{\Lambda}\mathbf{Q}\mathbf{x} + \tilde{\mathbf{n}} \end{aligned}$$

where $\tilde{\mathbf{n}} \stackrel{def}{=} \mathbf{Q}^*\mathbf{W}\mathbf{Q}\mathbf{n}$. Finally, hard decisions are made on $\mathbf{x}$ by applying $\mathbf{z}$ to a slicer. As pointed out in [8], the key difference between FDE and OFDM is that decisions are made in the time domain in the former and in the frequency domain in the latter.

[1] Note that the noise vector $\mathbf{n}$ and its DFT $\mathbf{N}$ have the same variance since they are related through the orthonormal transformation $\mathbf{Q}$.

3 Main Results

3.1 Proposed Transmit Diversity Scheme

Denote the n^{th} symbol of the k^{th} transmitted block from antenna i by $\mathbf{x}_i^{(k)}(n)$. At times $k = 0, 2, 4, \cdots$, pairs of length–N blocks $\mathbf{x}_1^{(k)}(n)$ and $\mathbf{x}_2^{(k)}(n)$ (for $0 \le n \le N-1$) are generated by an information source. Inspired by the Alamouti STBC [1], we propose the following transmit diversity scheme[2] (c.f. Figure (1))

$$\mathbf{x}_1^{(k+1)}(n) = -\bar{\mathbf{x}}_2^{(k)}((-n)_N) \ \text{ and } \ \mathbf{x}_2^{(k+1)}(n) = \bar{\mathbf{x}}_1^{(k)}((-n)_N) \ : \ \text{for } n = 0, 1, \cdots, N-1 \ \text{ and } \ k = 0, 2, 4, \cdots \tag{5}$$

where $\bar{(.)}$ and $(.)_N$ denote complex conjugation and modulo–N operations, respectively. In addition, a cyclic prefix of length ν is added to each transmitted block to eliminate IBI and make all channel matrices *circulant*. Finally, the transmitted power from each antenna is half its value in the single–transmit case so that total transmitted power is fixed.

3.2 SC MMSE-FDE for Proposed Transmit Diversity Scheme

With 2 transmit and 1 receive antenna [3], received blocks k and $k+1$ are given by

$$\mathbf{y}^{(j)} = \mathbf{H}_1^{(j)}\mathbf{x}_1^{(j)} + \mathbf{H}_2^{(j)}\mathbf{x}_2^{(j)} + \mathbf{n}^{(j)} \ : \ \text{for } j = k, k+1 \tag{6}$$

where $\mathbf{H}_1^{(j)}$ and $\mathbf{H}_2^{(j)}$ are the *circulant* channel matrices from transmit antennas 1 and 2, respectively, over block j, to the receive antenna. Analogous to the single–transmit case (c.f. (3)), by applying the DFT to $\mathbf{y}^{(j)}$, we get

$$\begin{aligned} \mathbf{Y}^{(j)} &\stackrel{def}{=} \mathbf{Q}\mathbf{y}^{(j)} \\ &\stackrel{def}{=} \mathbf{\Lambda}_1^{(j)}\mathbf{X}_1^{(j)} + \mathbf{\Lambda}_2^{(j)}\mathbf{X}_2^{(j)} + \mathbf{N}^{(j)} \ : \ \text{for } k = k, k+1 \end{aligned} \tag{7}$$

where $\mathbf{X}_i^{(j)} \stackrel{def}{=} \mathbf{Q}\mathbf{x}_i^{(j)}$ and $\mathbf{N}^{(j)} \stackrel{def}{=} \mathbf{Q}\mathbf{n}^{(j)}$. Using the encoding rule in (5) and properties of the DFT [7], we have

$$\mathbf{X}_1^{(k+1)}(m) = -\bar{\mathbf{X}}_2^{(k)}(m) \ \text{ and } \ \mathbf{X}_2^{(k+1)}(m) = \bar{\mathbf{X}}_1^{(k)}(m) \ : \ \text{for } m = 0, 1, \cdots, N-1 \ \text{ and } \ k = 0, 2, 4, \cdots \tag{8}$$

Furthermore, we assume that the channels are fixed over 2 consecutive blocks [4], i.e.

$$\begin{aligned} \mathbf{H}_i^{(k+1)} &= \mathbf{H}_i^{(k)} \stackrel{def}{=} \mathbf{H}_i \\ \Leftrightarrow \mathbf{\Lambda}_i^{(k+1)} &= \mathbf{\Lambda}_i^{(k)} \stackrel{def}{=} \mathbf{\Lambda}_i \end{aligned} \tag{9}$$

Combining (7), (8), and (9), we get

$$\begin{aligned} \mathbf{Y} &\stackrel{def}{=} \begin{bmatrix} \mathbf{Y}^{(k)} \\ \bar{\mathbf{Y}}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \mathbf{\Lambda}_1 & \mathbf{\Lambda}_2 \\ \mathbf{\Lambda}_2^* & -\mathbf{\Lambda}_1^* \end{bmatrix} \begin{bmatrix} \mathbf{X}_1^{(k)} \\ \mathbf{X}_2^{(k)} \end{bmatrix} + \begin{bmatrix} \mathbf{N}^{(k)} \\ \bar{\mathbf{N}}^{(k+1)} \end{bmatrix} \\ &\stackrel{def}{=} \mathbf{\Lambda}\mathbf{X} + \mathbf{N} \end{aligned} \tag{10}$$

Since $\mathbf{\Lambda}$ is an orthogonal matrix, we can (without loss of optimality) multiply both sides of (10) by $\mathbf{\Lambda}^*$ to decouple the two signals $\mathbf{X}_1^{(k)}$ and $\mathbf{X}_2^{(k)}$ resulting in

$$\tilde{\mathbf{Y}} \stackrel{def}{=} \begin{bmatrix} \tilde{\mathbf{Y}}^{(k)} \\ \tilde{\mathbf{Y}}^{(k+1)} \end{bmatrix} = \mathbf{\Lambda}^*\mathbf{Y} = \begin{bmatrix} \tilde{\mathbf{\Lambda}} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{\Lambda}} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1^{(k)} \\ \mathbf{X}_2^{(k)} \end{bmatrix} + \tilde{\mathbf{N}} \tag{11}$$

[2] Alamouti's STBC was designed for flat-fading channels and the encoding rule was applied to two consecutive *symbols* not *blocks* as it is the case here.

[3] Extension to multiple receive antennas is straightforward and similar to the approach followed in [1] for the flat–fading case.

[4] This is a valid assumption when the channel coherence time is much longer than the block length, which is the case in EDGE [3].

where $\tilde{\mathbf{\Lambda}} \stackrel{def}{=} |\mathbf{\Lambda}_1|^2 + |\mathbf{\Lambda}_2|^2$ is an $N \times N$ diagonal matrix with (i, i) element equal to $|\mathbf{\Lambda}_1(i, i)|^2 + |\mathbf{\Lambda}_2(i, i)|^2$ which is also equal to the sum of the squared i^{th} DFT coefficients of first and second CIRs. This quantifies the order–two transmit diversity gain achieved by this scheme. The filtered noise vector $\tilde{\mathbf{N}}$ has a diagonal auto–correlation matrix equal to $diag(\tilde{\mathbf{\Lambda}}, \tilde{\mathbf{\Lambda}})$. Therefore, the i^{th} coefficient of the MMSE–FDE in this case is equal to $\frac{1}{(\tilde{\mathbf{\Lambda}}(i,i)+\frac{1}{SNR})}$ for $0 \leq i \leq N-1$.

Since both equations in (11) have the same form as (3), decoding proceeds in a similar fashion. Note that the same N SC MMSE–FDE taps are applied to blocks $\tilde{\mathbf{Y}}^{(k)}$ and $\tilde{\mathbf{Y}}^{(k+1)}$ (for $k = 0, 2, 4, \cdots$) since their equivalent channel gain matrix and SNR vector are the same.

4 Simulation Results

We consider the Typical Urban (TU) channel with a linearized GMSK transmit pulse shape, 8–PSK modulation, and a symbol duration of $3.69\mu sec$ as in the proposed 3^{rd} generation TDMA cellular standard EDGE [5][3]. The overall CIR memory is $\nu = 3$. We assume perfect CIR knowledge at the receiver and a DFT size $N = 64$ [6] which implies a 64–tap SC MMSE–FDE. Therefore, the cyclic prefix power penalty is $-10\log_{10}(\frac{64}{67}) = 0.2$ dB only.

Figure 3 shows the significant improvement achieved in SC MMSE–FDE performance when combining it with the proposed STBC scheme, especially at high SNR where effects of diversity are more pronounced.

5 Conclusions

We presented a low–complexity single–carrier transmit–diversity scheme for frequency–selective channels. The scheme combines the advantages of an Alamouti–like space–time block–coding scheme and FFT–based singler–carrier frequency–domain equalization. Significant performance gains over single–antenna transmission were demonstrated for the EDGE TU channel.

[5]EDGE stands for Enhanced Data Rates for GSM Evolution.

[6]It is preferable to choose N as a power of 2 to allow DFT computations using the Fast Fourier Transform (FFT) algorithm.

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List of Figures

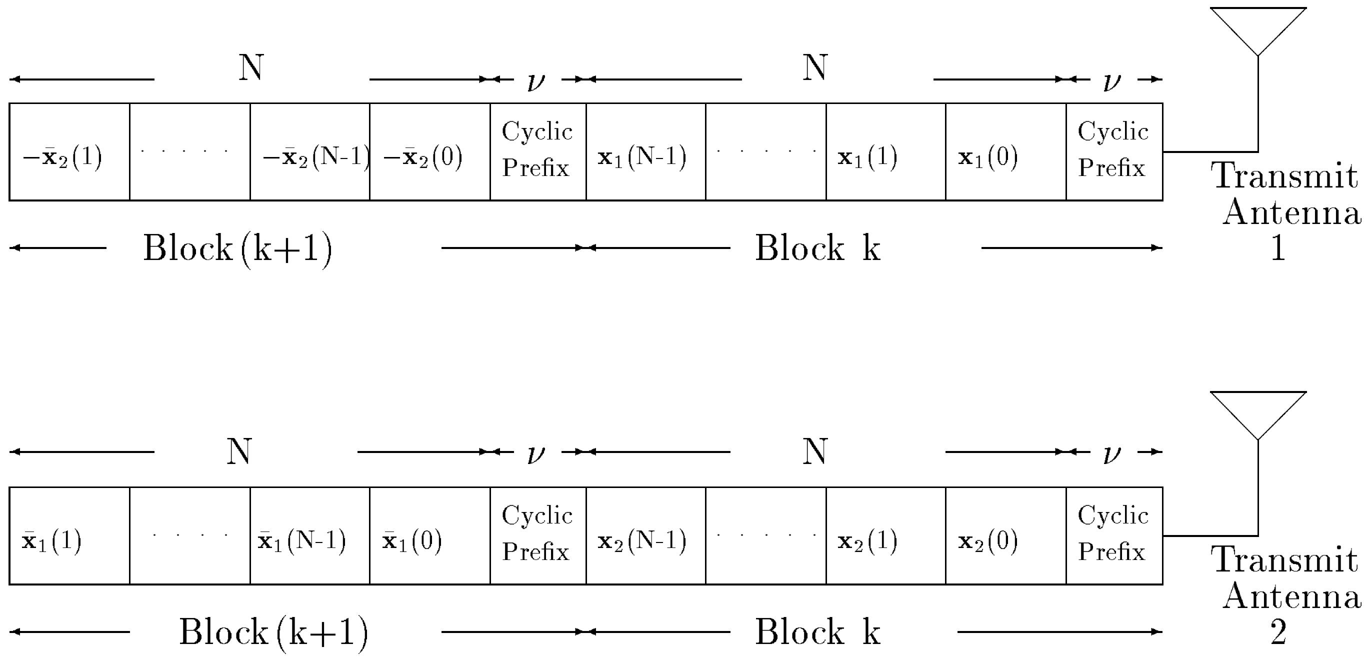

Figure 1: Block Format for Proposed Transmission Scheme

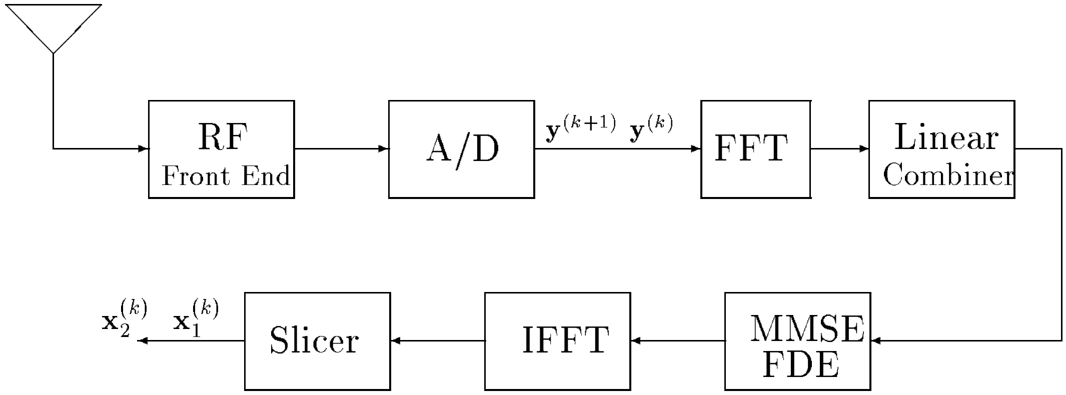

Figure 2: Receiver Block Diagram

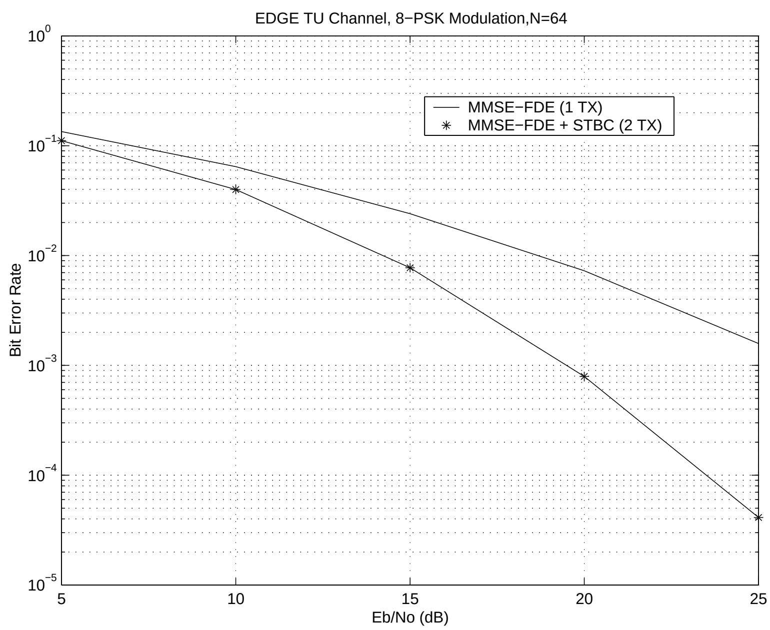

Figure 3: BER of SC MMSE-FDE w/ and w/o STBC for EDGE TU Channel with 8-PSK Modulation and $N = 64$