Space-Time Trellis-Coded Transmissions With Parallel Sequences Over Time-Varying Channels

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***Abstract*—In this letter, we investigate the performance of space-time trellis codes in a time-varying channel of a multiple-access wireless system where symbols of a user are transmitted using parallel sequences. Using rank and determinant criteria, it is shown that space-time trellis codes originally designed for quasi-static channels are efficient codes for this system as well. Simulation results demonstrate that the proposed transmission scheme can exploit spatial and temporal diversities to achieve performance gains at practical decoding complexity levels.**

***Index Terms*—Parallel-sequence transmission, space-time code, temporal diversity.**

I. Introduction

FUTURE wireless systems are expected to use multiple antennas at the transmitter and/or receiver to enhance rate and reliability. As an example, space-time trellis codes (STTCs), invented in [1], incorporate jointly-designed channel coding, modulation, transmit diversity and optional receive diversity to achieve maximum spatial diversity in addition to coding gain without sacrificing bandwidth over quasi-static channels.

High-mobility conditions, as in the forthcoming 4G systems, result in rapid channel variations that, if properly exploited, can result in additional time-diversity gains. To address this issue, the pioneering work in [1] proposed the use of smart-greedy space-time codes where a Reed–Solomon code is used as an outer code. The use of an outer code with STTC improves the frame error rate at the cost of reduced code rate. In this letter, we adopt an approach based on a novel combination of STTC and a parallel-symbol transmission scheme proposed in [2] and referred to as the *System with Parallel Sequences* or *SYPS* [3]. In SYPS, a user simultaneously transmits all symbols within the same frame in parallel and exploits temporal variations efficiently. Coding techniques such as convolutional codes, trellis codes, and turbo codes, can be implemented in SYPS where the parallel symbol transmission combats multiplicative fading effects, while channel coding mitigates additive interference and noise effects.

II. System Model

We start by describing the system model for space-time trellis-coded SYPS using vector notation. The space-time encoder selects a transition branch depending on the state of the encoder and the input bits at time u. We denote this transition branch of user k as $\left\{c_{k,j}^{(u)}\right\}_{j=1}^{n}$, where $c_{k,j}^{(u)}$ is the transmitted symbol of user k from antenna j and n is the total number of transmit antennas. Each data frame consists of N symbols, where N is assumed to be an integer multiple of n. Each symbol of duration T_s is stretched over a frame duration NT_s/n; see Fig. 1. The stretched symbols $\left\{c_{k,j}^{(u)}\right\}_{j=1}^{n}$ at different antennas are processed using the same long spreading sequences $\mathbf{s}_k^{(u)}$ of length NL/n where $L = T_s/T_c$ is the processing gain and T_c is the chip duration.

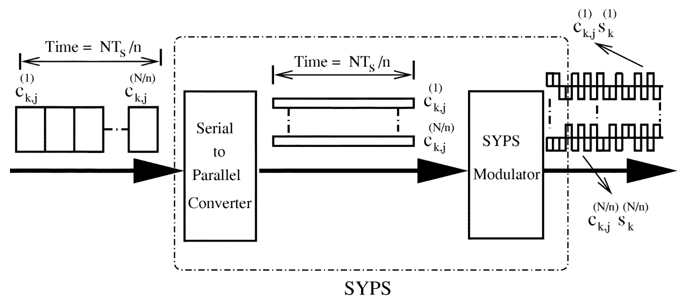

Fig. 1. The SYPS transmitter for antenna j.

Let Δ_k denote the coherence time of user k on each channel. Assuming that the channel obeys the simple block-fading law, the number of independent fades over a frame is $NT_s/n\Delta_k \triangleq \eta_k$ which is assumed to be an integer for simplicity. Different portions of the spreading code $\mathbf{s}_k^{(u)}$ are affected by the channels differently. If the spreading sequences which are affected by the ith fade on each channel form a column vector $\mathbf{s}_k^{(i,u)}$ of length $NL/n\eta_k$, then[1]

$$\mathbf{s}_k^{(u)} \triangleq \left[\left(\mathbf{s}_k^{(1,u)}\right)^\top \left(\mathbf{s}_k^{(2,u)}\right)^\top \ldots \left(\mathbf{s}_k^{(\eta_k,u)}\right)^\top\right]^\top. \quad (1)$$

The spreading sequence is normalized such that $\left(\mathbf{s}_k^{(u)}\right)^H \mathbf{s}_k^{(u)} = 1$. During the ith fade, the contribution of the transmitted symbols of user k from its jth transmit antenna, which is $\left\{c_{k,j}^{(u')}\right\}_{u'=1}^{N/n}$, to the received signal vector at the lth receive antenna is

$$\mathbf{r}_{k,j,l}^{(i)} = \sqrt{E_k}\alpha_{k,j,l}^{(i)} \sum_{u'=1}^{N/n} c_{k,j}^{(u')} \mathbf{s}_k^{(i,u')} \quad (2)$$

where $\alpha_{k,j,l}^{(i)}$ denotes the fading coefficient on the channel between transmit antenna j and receive antenna l. It is modeled as a sample of a complex Gaussian random variable with zero-

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[1]In this letter, the notation $\mathbf{X}^\top$ is used as the transpose of the matrix $\mathbf{X}$, $\mathbf{X}^H$ denotes the Hermitian of the matrix $\mathbf{X}$, and $(\mathbf{X})^*$ is the complex conjugate of $\mathbf{X}$.

mean and variance 0.5 per dimension. Note that the constellation points are scaled by a factor of $\sqrt{E_k}$ such that the average energy of the constellation points is unity. Considering K users in the system where each user transmits to its own receiver which is equipped with m receive antennas, we can write the total received signal at receive antenna l for the ith fade as $\mathbf{r}_l^{(i)} = \sum_{k=1}^{K}\sum_{j=1}^{n}\mathbf{r}_{k,j,l}^{(i)} + \mathbf{n}_l^{(i)}$. Equivalently

$$\mathbf{r}_l^{(i)} = \sum_{k=1}^{K}\sqrt{E_k}\sum_{u'=1}^{N/n}\sum_{j=1}^{n}\alpha_{k,j,l}^{(i)}c_{k,j}^{(u')}\mathbf{s}_k^{(i,u')} + \mathbf{n}_l^{(i)} \tag{3}$$

where $l = 1,\ldots,m$ and $i = 1,\ldots,\eta_k$. The term $\mathbf{n}_l^{(i)}$ in (3) represents a complex white Gaussian noise vector.

A. Symbol Decoding

The optimum maximum-likelihood multiuser detector has a formidable complexity [4]. This has prompted us to implement *single-user* Viterbi detection by lumping the multiple access and self-interference terms together and assuming them white Gaussian noise. By multiplying $\left(\mathbf{s}_k^{(i,u)}\right)^H$ with the received signal in (3), we obtain the decision statistic for symbols $\left\{c_{k,j}^{(u)}\right\}_{j=1}^{n}$ at receive antenna l for the ith fade as

$$r_{k,l}^{(i,u)} = \frac{\sqrt{E_k}}{\eta_k}\sum_{j=1}^{n}\alpha_{k,j,l}^{(i)}c_{k,j}^{(u)} + v_{k,l} + w_{k,l} + \left(\mathbf{s}_k^{(i,u)}\right)^H\mathbf{n}_l^{(i)} \tag{4}$$

where $v_{k,l}$ and $w_{k,l}$, respectively, denote the self interference and multiple access interference, and can be written as

$$v_{k,l} = \sqrt{E_k}\sum_{j=1}^{n}\sum_{\substack{u'=1\\u'\neq u}}^{N/n}\alpha_{k,j,l}^{(i)}c_{k,j}^{(u)}\left(\mathbf{s}_k^{(i,u)}\right)^H\mathbf{s}_k^{(i,u')} \tag{5}$$

$$w_{k,l} = \sum_{\substack{k'=1\\k'\neq k}}^{K}\sqrt{E_{k'}}\sum_{j=1}^{n}\sum_{u'=1}^{N/n}\alpha_{k',j,l}^{(i)}c_{k',j}^{(u')}\left(\mathbf{s}_k^{(i,u)}\right)^H\mathbf{s}_{k'}^{(i,u')}. \tag{6}$$

Equation (4) yields the following path metric of user k for the uth transition in the Viterbi algorithm

$$\sum_{l=1}^{m}\sum_{i=1}^{\eta_k}\left|r_{k,l}^{(i,u)} - \frac{\sqrt{E_k}}{\eta_k}\sum_{j=1}^{n}\alpha_{k,j,l}^{(i)}c_{k,j}^{(u)}\right|^2 : u = 1,2,\ldots,\frac{N}{n}.$$

B. Code Design

Now we design a suitable STTC for SYPS under the assumption that the interference plus noise in (3) is a complex white Gaussian noise process. Let the variance of complex white Gaussian noise (which is the sum of interference plus Gaussian noise) in (4) be $\tilde{N}_0/2$ per dimension. For simplicity, we drop the subscript k. It is well-known that the conditional probability of transmitting

$$\mathbf{c} = c_1^{(1)}c_2^{(1)}\ldots c_n^{(1)}c_1^{(2)}c_2^{(2)}\ldots c_n^{(2)}\ldots c_1^{(N/n)}c_2^{(N/n)}\ldots c_n^{(N/n)}$$

and deciding in favor of

$$\mathbf{e} = e_1^{(1)}e_2^{(1)}\ldots e_n^{(1)}e_1^{(2)}e_2^{(2)}\ldots e_n^{(2)}\ldots e_1^{(N/n)}e_2^{(N/n)}\ldots e_n^{(N/n)}$$

is upper-bounded by

$$P\left(\mathbf{c}\to\mathbf{e}|\alpha_{j,l}^{(i)}\right) \le \exp\left(-d^2(\mathbf{c},\mathbf{e})\frac{E_s}{4\eta^2\tilde{N}_0}\right) \tag{7}$$

where

$$d^2(\mathbf{c},\mathbf{e}) = \sum_{l=1}^{m}\sum_{u'=1}^{N/n}\sum_{i=1}^{\eta}\left|\sum_{j=1}^{n}\alpha_{j,l}^{(i)}\left(c_j^{(u')} - e_j^{(u')}\right)\right|^2. \tag{8}$$

Denoting $\mathbf{A}_{pq} \triangleq \sum_{u'=1}^{N/n}\left(c_p^{(u')} - e_p^{(u')}\right)\left(c_q^{(u')} - e_q^{(u')}\right)^*$ as a (p,q) element of an $n\times n$ Hermitian matrix $\mathbf{A}(\mathbf{c},\mathbf{e})$, and letting $\mathbf{\Omega}_l^{(i)} \triangleq \left[\alpha_{1,l}^{(i)},\alpha_{2,l}^{(i)},\ldots,\alpha_{n,l}^{(i)}\right]$, we can rewrite (8) as

$$d^2(\mathbf{c},\mathbf{e}) = \sum_{l=1}^{m}\sum_{i=1}^{\eta}\mathbf{\Omega}_l^{(i)}\mathbf{A}(\mathbf{c},\mathbf{e})(\mathbf{\Omega}_l^{(i)})^*. \tag{9}$$

Defining $\tilde{\mathbf{\Omega}}_l \triangleq \left[\mathbf{\Omega}_l^{(1)},\mathbf{\Omega}_l^{(2)},\ldots,\mathbf{\Omega}_l^{(\eta)}\right]$ and $\tilde{\mathbf{A}}(\mathbf{c},\mathbf{e}) \triangleq I_{\eta\times\eta}\otimes\mathbf{A}(\mathbf{c},\mathbf{e})$, where $\otimes$ denotes the Kronecker product and $I_{\eta\times\eta}$ is the identity matrix of size η, we simplify (9) as

$$d^2(\mathbf{c},\mathbf{e}) = \sum_{j=1}^{m}\tilde{\mathbf{\Omega}}_j\tilde{\mathbf{A}}(\mathbf{c},\mathbf{e})\tilde{\mathbf{\Omega}}_j^*. \tag{10}$$

Hence, the optimum STTC for SYPS is obtained by maximizing the rank and determinant [1] of $\tilde{\mathbf{A}}(\mathbf{c},\mathbf{e})$, where $\operatorname{rank}\left(\tilde{\mathbf{A}}(\mathbf{c},\mathbf{e})\right) = \eta\times\operatorname{rank}(\mathbf{A}(\mathbf{c},\mathbf{e}))$ and $\det\left(\tilde{\mathbf{A}}(\mathbf{c},\mathbf{e})\right) = \det(\mathbf{A}(\mathbf{c},\mathbf{e}))^{\eta}$. These two equalities imply that an optimum STTC based on the rank and determinant criteria of $\mathbf{A}(\mathbf{c},\mathbf{e})$ (as derived in [1]) is also optimum based on those criteria of $\tilde{\mathbf{A}}(\mathbf{c},\mathbf{e})$. The intuition behind this result is that due to the parallel transmission in SYPS, temporal variations of the channel affect all symbols transmitting from the same antenna in the same fashion. Thus, one can view that every channel in SYPS is a quasi-static channel.

C. Simulation Results

Computer simulations were performed to evaluate the performance of space-time trellis-encoded SYPS. The information data generated is 128 bits/frame. The carrier frequency is 2 GHz and the duration of a frame is 10 ms. In all simulations, we implement a 4-PSK STTC for two transmit antennas from [1] which was designed for quasi-static channels. Note that this code is also efficient based on rank and determinant criteria when channel coefficients are correlated [1]. Complex spreading sequences for all users are randomly generated. The cross-correlation of parallel sequences of a user within the coherence time is zero. It is assumed that all users move at the same speed v. We use $\Delta = (0.423/f_m)$ [5] to determine the coherence time of the channel, where f_m is the maximum Doppler frequency. For $v = 14$, 57, and 114 mph, the number of fades η per channel over one frame is approximately 1, 4, and 8, respectively. These speeds can be viewed as the speed of a user on a local road, on a high-way and onboard a train, respectively. Different frames of a user experience independent fades. We consider both known and estimated channel coefficients (amplitude and phase) scenarios. One pilot sequence per transmit antenna is sent in parallel with data sequences. The power of the pilot symbol is about 42% of the total transmitted power per transmit antenna. To decode the information symbols of a user, the receiver first eliminates the contribution of the pilot symbols from the received signal using the estimated channel coefficients and then executes the Viterbi algorithm.

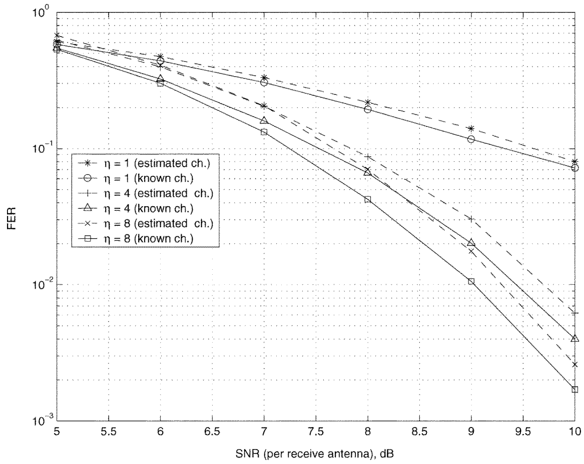

Fig. 2. FER of 4-state STTC with SYPS for both known and estimated channel (ch.) scenarios.

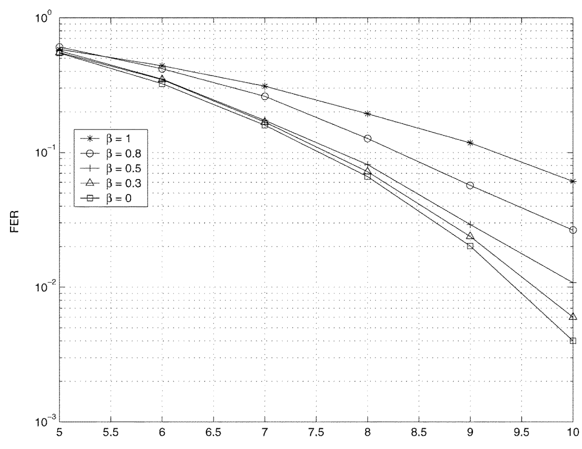

Fig. 3. FER of 4-state STTC with SYPS for different temporal channel correlations.

In the first experiment, we consider a single-user system. The number of trellis states is 4. In Fig. 2, we plot the frame error rate (FER) as a function of SNR per receive antenna for $\eta = 1, 4,$ and 8 where the number of receive antennas $m = 2$. Here it is assumed that correlation among η fades of each antenna is zero. Notice that the STTC performance improves with the increase in the number of fades per channel due to temporal diversity. For a given FER, the performance degradation in terms of SNR due to channel estimation error is within 0.5 dB.

In the second experiment, we study the effect of the temporal correlation of the η fades of each antenna. The correlation between two consecutive fades is denoted by β and $\beta = 0.0, 0.3, 0.5, 0.8,$ and 1.0. Here $\eta = 4, m = 2$ and the number of trellis states is 4. We assume known channel scenario at the receiver. Notice from Fig. 3 that the performance degradation becomes gradual when $\beta \leq 0.5$ and becomes noticeable for $\beta \geq 0.8$.

In the next experiment, our goal is to observe the tradeoff between performance gain and decoding complexity. In Fig. 4, we plot the performance of STTCs with different numbers of trellis states as a function of SNR per receive antenna for $\eta = 1$ and 4, $K = 1$, and $m = 1$ in known channel scenario. Notice that increasing the decoding complexity from 4 to 8 states results in significant performance gains when $\eta = 4$. Additional gains as the number of states is increased from 8 to 16 are modest. Hence, 8 states represents a favorable performance-complexity tradeoff point.

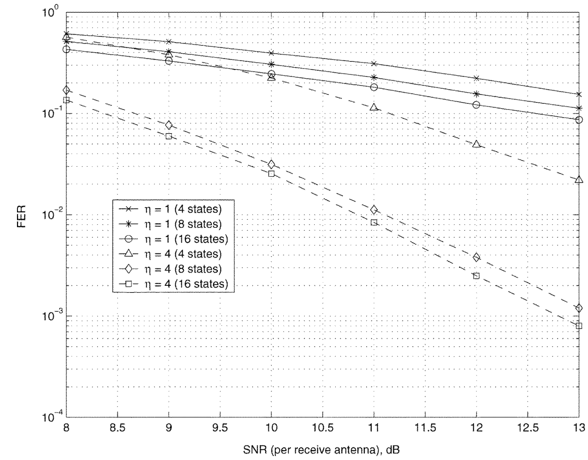

Fig. 4. FER of STTC with SYPS for different numbers of trellis states.

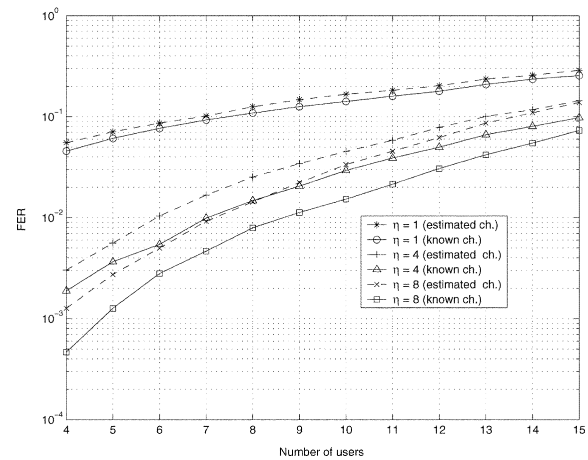

Fig. 5. FER of 4-state STTC with SYPS as a function of the number of users for both known and estimated channel scenarios.

Finally, we study the variation of capacity (the number of supportable users at a given FER) with the increase in the number of fades per channel in uplink scenario considering both known and estimated channel coefficients. In Fig. 5, we plot the FER of the 4-state STTC as a function of the number of users in the system when $\mathrm{SNR} = 12$ dB per receive antenna and $m = 2$. The capacity gain is more pronounced when the number of fades increases from $\eta = 1$ to $\eta = 4$ than from $\eta = 4$ to $\eta = 8$. This result suggests that space-time trellis-coded SYPS can support more users at high-way and train speeds than at local road speeds. Notice that due to channel estimation errors the capacity degrades by 0–3 users depending on η and the target FER.

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