Dynamics of discrete systems with nonregularized impact and friction

Alain Léger
CNRS - Laboratoire de Mécanique et d'Acoustique
31, chemin Joseph Aiguier
13402, Marseille cedex 20, France

We are studying the dynamics of discrete systems made of particles or rigid bodies possibly connected to each others or to a rigid frame by some elastic devices, and such that their trajectories may involve shocks and friction with some obstacles. The simplest example of such systems is given by a single particle attached by springs to a rigid frame, moving in the plane and constrained to remain in a half-plane. We shall keep strictly non-regularized contact and friction conditions, but for the sake of simplicity we shall concentrate on a simple example and just comment on the points which either can be extended to any finite dimensional systems or remain difficult open problems in general.

The lecture will concentrate on three points.

1. The first point is the foundation of the analysis: we give the convenient mathematical statement of the problem, in which the reaction of the obstacle and the acceleration are both measures so that the velocity of the particle is a function of bounded variation. The result of this part is that, given initial data, a trajectory exists as soon as the external forces are integrable functions but in general the Cauchy problem is ill-posed and uniqueness of the trajectory is recovered only if the external forces are extremely smooth.

2. The second point deals with computational aspects, describing an algorithm of the time stepping type, which is shown to converge no matter the solution involves isolated shocks or infinitely many shocks in finite time.

3. The last point is closer to classical analyses of dynamical systems, but the results are very different. Choosing external data such that the initial value problem is well-posed, we study the dynamics under oscillating excitation. The $\{$frequency - amplitude$\}$ plane of the excitation appears to be divided into parts where there exist only equilibrium points, and other parts where there exist only periodic solutions among which some are of small or very small amplitude while others are of large amplitude. Moreover, in some transition ranges there exist inifinitely many equilibrium points or infinitely many periodic solutions.