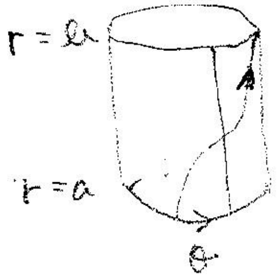
These notes of the Rafael Ortega lecture have been kindly provided by Kazuyuki Yagasaki

Rafael Ortega   Joint work with M. Kunze

Twist mappings $M:(\theta,r)\mapsto(\theta_1,r_1)$ $\theta_1=F(\theta,r)$, $r_1=G(\theta,r)$

$\theta\equiv\theta+2\pi$, $r\in[a,b]$ $\frac{\partial F}{\partial r}>0 \Rightarrow F(\theta,\cdot)\nearrow$

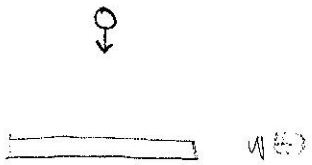


$\dot q=\frac{\partial H}{\partial p}$, $\dot p=-\frac{\partial H}{\partial q}$,

$H(q,p)$, $q,p\in\mathbb{R}^2$

$H(t,q,p)$, $q,p\in\mathbb{R}$

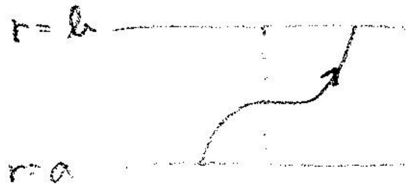
2π-periodic



t_0 time of impact ⋯ θ

v_0 velocity ⋯ r

$\theta\in\mathbb{R}$, $r\in[a,b]$

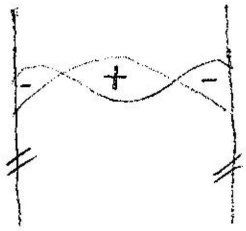


twist + Exact Symplectic

Exact Symplectic on the cylinder

Area preserving = symplectic

Exact symplectic



Area$_+$ = Area$_-$

ES $(\theta,r)\mapsto(\theta+\omega,r)$

S $(\theta,r)\mapsto(\theta,r+1)$

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$\omega = r_1 d\theta_1 - r\, d\theta$ $\qquad$ ES $\quad \omega$: exact

$\qquad\qquad\qquad\qquad\qquad\qquad$ S $\quad \omega$: closed

Generating functions $\quad (\theta, r) \mapsto (\theta_1, r_1)$

$h = h(\theta, \theta_1) \qquad r = \partial_1 h(\theta, \theta_1),\ r_1 = -\partial_2 h(\theta, \theta_1)$

ES + Twist on the cylinder $\exists h$

$\dfrac{\partial^2 h}{\partial\theta\,\partial\theta_1} > 0$

$h(\theta + 2\pi, \theta_1 + 2\pi) = h(\theta, \theta_1)$

Example

$(\theta, r) \longmapsto (\theta + r, r + \lambda) \qquad h(\theta, \theta_1) = -\frac{1}{2}(\theta_1 - \theta)^2 - \lambda\theta_1$

$\forall\lambda \Rightarrow S \quad ES \Rightarrow \lambda = 0$ $\qquad$ For $\lambda = 0$ $h(\theta, \theta_1)$ bounded in $\delta \leq \theta_1 - \theta \leq \Delta$

Perturbation of the integrable twist map

$(\theta, r) \mapsto (\theta + \gamma r, r),\ \gamma > 0$

$h(\theta, \theta_1) = -\frac{\gamma}{2}(\theta_1 - \theta)^2$

Th $\quad h \in C^1 \quad \{(\theta, \theta_1) : \delta \leq \theta_1 - \theta \leq \Delta\} \quad 0 < \delta < \Delta$

$\underline{\alpha}(\theta_1 - \theta_2)^2 \leq h(\theta, \theta_1) \leq \overline{\alpha}(\theta_1 - \theta)^2, \quad \overline{\alpha} \leq \frac{3}{2}\underline{\alpha}$

If $\frac{\Delta}{\delta}$ is large enough then

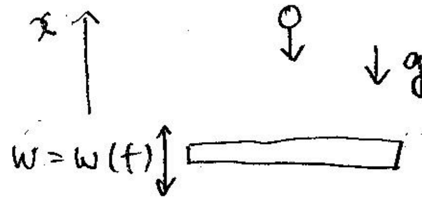
$\exists \{(\theta_n, r_n)\}_{n\in\mathbb{Z}},\ \delta \leq \theta_{n+1} - \theta_n \leq \Delta,\ \forall n \in \mathbb{Z}$

Proof Aubry–Mather theory

$$\Theta = \{\theta_n\}_{n\in\mathbb{Z}},\quad \delta \le \theta_{n+1}-\theta_n \le \Delta$$

$$S(\Theta) = \sum_{n\in\mathbb{Z}} h(\theta_n,\theta_{n+1})$$ Terracini–Verzini

Ping-pong model



Pustylnikov

w periodic $\|w\|$ small C^5

all motions are bounded

$\exists w$ periodic large norm & unbounded motions

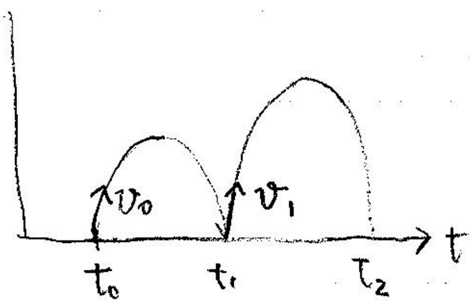
Th $w \in C^2(\mathbb{R})$, $\|w\|_\infty + \|\ddot{w}\|_\infty < \infty$

$\Rightarrow$ $\exists$ infinitely many bounded orbits with arbitrary large energy

Proof 1. Change $y = x - w(t)$

$$\ddot{y} = -(g+\ddot{w}(t)),\quad y \ge 0$$

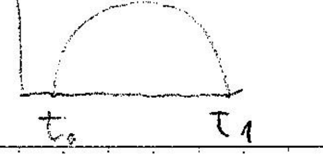
$$y(t_0^+) = -y(t_0^-)$$



$\theta = t_0$, $r = \frac{1}{2}v_0^2$

Selected Chapters in the Calculus of Variations, J. Moser

$$h(t_0,t_1) = \int_{t_0}^{t_1} L(t, y(t;t_0,t_1), \dot{y}(t;t_0,t_1))\,dt$$



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$$h(t_0,t_1) = -\frac{g^2}{24}(t_1-t_0)^3 - \frac{g}{2}(w(t_1)+w(t_0))(t_1-t_0) + \frac{(w(t_1)-w(t_0))^2}{2(t_1-t_0)}$$

$$+\, g\int_{t_0}^{t_1} w(t)\,dt + \frac{1}{2}\int_{t_0}^{t_1} w(t)^2\,dt$$

Th. changes to

$$\underline{\alpha}(\theta_1-\theta_2)^{2k} \le h(\theta,\theta_1) \le \overline{\alpha}\,(\theta_1-\theta_2)^{2k}$$

$$\overline{\alpha} < \left(\frac{1}{2}+\frac{1}{2^k}\right)^{-1}\underline{\alpha}$$