Extensions to Time Series Models for Event Counts

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1. Remaining dynamic count data issues

2. Multivariate count models
What if you have multiple count series?

Examples:

- Threats and actions between two belligerents in a dyad
- Different types of events over time
- Number of events in different categories over time
- Events for different geographic units over time

These are all examples of dynamic, multivariate counts.
Issues with dynamic, multivariate counts

- There is no unrestricted (i.e., allowing positive and negative correlation) multivariate analog for a count distribution. Most only admit correlations in one direction.
- King’s (1989) Seemingly Unrelated Poisson Regression (SUPREME) method only applies to positively correlated bivariate data and is not dynamic.
- Unrestricted covariance multivariate count models are generally not dynamic: Aitchison and Ho (1989) and Chib and Winkelmann (2001).
Suppose that we have a series of $m$ grouped counts for time periods $t = 1, \ldots, T$, denoted by $y_{tj}$. The observed data are assumed to have marginal Poisson distributions of the form:

$$y_{tj} | b_t, \beta_j \sim Psn(\mu_{tj})$$  \hspace{1cm} (1)

$$\mu_{tj} = \exp(x_{tj}' \beta_j + b_{tj}) \quad \forall j \leq m, t \leq T,$$  \hspace{1cm} (2)

where $y_{tj}$ is the $t^{th}$ observation of the $j^{th}$ count variable; $b_t = (b_{t1}, \ldots b_{tm})$ are a set of observation and equation specific latent effects (which capture the contemporaneous correlations among the series); $\mu_{tj}$ is the mean of the (conditionally) independent Poisson distributions for observation $t$ in equation $j$; and $x_{tj}'$ and $\beta_j$ are the regression components.
In this model, the $\exp(b_{tm}) = \nu_{tm} \sim \text{log N}(0, D)$.

The complication is that the probabilities for these log normal error terms are expensive to compute.

For $m > 3$ the rounding error in computing these will be large.

So rather than directly compute the maximum likelihood estimator, most of the estimation for this model will need to be simulated.
Chib and Winkelmann (2001) propose a Bayesian Markov chain Monte Carlo (MCMC) estimator for this model. Starting from the likelihood function,

\[ \Pr(y_{t} | \beta, D) = \int \prod_{j=1}^{m} f(y_{tj} | \beta_j, b_{tj}) \phi_m(b_t, 0, D) \, db_t, \]

they propose prior distributions for \( \beta \) (normal), \( b \) (normal), and \( D \) (inverse Wishart) terms. This model can easily be estimated with MCMC sampling software (e.g., BUGS or JAGS). Congdon (2005, 2007) presents further examples.
In this paper we want to model the targeting choices of transnational terrorists using disaggregated, monthly data (1968-2008). The multivariate count model needs to satisfy several criteria:

- Must allow us to model more than three series
- Include dynamics (like the PAR(p) model)
- Allow for dynamic inferences so we can assess complementarity and substitution across the target types.

The new model is a Bayesian Poisson vector autoregression (BaP-VAR).
Bayesian Poisson VAR

- Paper develops a Bayesian Poisson VAR model (BaP-VAR).
- Extends the Bayesian Poisson log-normal model of Chib and Winkelmann (2001) to allow for autoregressive dynamics.
- Prior for the dynamic coefficients follows Sims and Zha (1998): shrinkage toward low order dynamics.
- Posterior is non-standard — use MCMC simulation to estimate the parameters.
- Posterior sample can then be used to construct impulses responses and decompositions of the forecast error variance.
- Provides a direct test of dynamic complementarity and substitution.
Model setup — Poisson log-normal VAR

\[ y_{tj} | b_t, \beta_j, A \sim \text{Poisson}(\mu_{tj}), \quad b_t | D \sim N_j(0, D) \quad \forall t, j = 1, \ldots m \]

\[
\mu_t = \begin{pmatrix}
a_{11}y_{t-1,1} + a_{12}y_{t-1,2} + \cdots + a_{1m}y_{t-1,m} \\
a_{21}y_{t-1,1} + a_{22}y_{t-1,2} + \cdots + a_{2m}y_{t-1,m} \\
\vdots \\
a_{m1}y_{t-1,1} + a_{m2}y_{t-1,2} + \cdots + a_{mm}y_{t-1,m}
\end{pmatrix} + \begin{pmatrix}
\exp(x_{t1}' \beta_1 + b_{t1}) \\
\exp(x_{t2}' \beta_2 + b_{t2}) \\
\vdots \\
\exp(x_{tm}' \beta_m + b_{tm})
\end{pmatrix}
\]

\[ = A \begin{pmatrix}
y_{t-1,1} \\
y_{t-1,2} \\
\vdots \\
y_{t-1,m}
\end{pmatrix} + \exp(x' \beta + b) \]

Paper shows how to compute 1) impulse responses for shocks to \( b_t \) and 2) decompositions of the forecast error
Model setup — Poisson log-normal VAR

\[ \mu_t = \left( \begin{array}{c}
a_{11}y_{t-1,1} + a_{12}y_{t-1,2} + \cdots + a_{1m}y_{t-1,m} \\
a_{21}y_{t-1,1} + a_{22}y_{t-1,2} + \cdots + a_{2m}y_{t-1,m} \\
\vdots \\
a_{m1}y_{t-1,1} + a_{m2}y_{t-1,2} + \cdots + a_{mm}y_{t-1,m}
\end{array} \right) + \left( \begin{array}{c}
\exp(x_{t1}' \beta_1 + b_{t1}) \\
\exp(x_{t2}' \beta_2 + b_{t2}) \\
\vdots \\
\exp(x_{tm}' \beta_m + b_{tm})
\end{array} \right) \]

\[ = A \left( \begin{array}{c}
y_{t-1,1} \\
y_{t-1,2} \\
\vdots \\
y_{t-1,m}
\end{array} \right) + \exp(x' \beta + b) \]

Paper shows how to compute 1) impulse responses for shocks to \( b_t \) and 2) decompositions of the forecast error.
Use monthly ITERATE data, 1968–2008, aggregated across four target types:

- Officials
- Military
- Business
- Private Parties


Past 3 months counts are used for the BaP-VAR specification.
Data

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Just like with a Gaussian VAR model, we are interested in three quantities:

- **Error covariances**: tell us whether the count series are independent.
- **Impulse responses**: characterize the modal dynamics.
- **Decomposition of the forecast error variances**: how much of the variation is due to changes in each series.
## 1968:1-1973:1 Covariances

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Median estimates of the covariance parameters. 75% CI in parentheses.
### 1973:2-1979:12 Covariances

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### 1980:1:2-1989:11 Covariances

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### 1989:12-2001:9 Covariances

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### 2001:10-2008:12 Covariances

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Impulse Responses

(a) 1968:1–1973:1

(b) 1973:2–1979:12

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Impulse Responses

(c) 1980:1–1989:11

(d) 1989:12–2001:9
Impulse Responses

(e) 2001:10–2008:12
Decomposition of the Forecast Error Variance

Subsample

1968:1–1973:1
1973:2–1979:12
1989:12–2001:9
2001:10–2008:12

Response in

O
M
B
P
Decomposition of the Forecast Error Variance

Response in Subsample
Decomposition of the Forecast Error Variance

Response in Subsample

1989:12–2001:9

2001:10–2008:12
Conclusion

Several contributions from this analysis:

- We see that there is complementarity within regimes. This is driven by regime-specific factors.
- Substitution occurs dynamically: moving across the regimes there is substitution from attacks on officials driving the variation toward attacks on business and private parties driving the outcomes.
- BaP-VAR model can easily be fit to multiple count time series and used to make inferences about contemporaneous correlation and dynamics.
Final Comments

- Start simple: use one or two lag PAR(\(p\)) or PEWMA models before looking at more lags or dynamics.
- Diagnose and model the dynamics of the count series before introducing covariates into the models.
- Compare the event count time series model results to Poisson and negative binomial regressions, since these are nested.
- For the PAR(\(p\)) models, be sure you report the impact multipliers or dynamic effects. There is code in PESTS for these computations.
Most of the software for these models is available in R (not Stata!):

**PESTS**  Poisson Estimators for State-Space Time Series. R code available on my webpage:  
http://www.utdallas.edu/~pbrandt.

**MCMCpack**  Has functions for estimating the Bayesian Poisson Changepoint Regressions and plotting the results:  
mcmcpack.wustl.edu or your favorite CRAN mirror.

**RJMCMC Poisson changepoint**  Peter Green has Fortran code for this on his webpage that is pretty easy to modify and get working:  
http://www.maths.bris.ac.uk/~mapjg/. There is also a R package, bcp, that can fit some changepoint models using an earlier algorithm.

**BaP-VAR**  When the Brandt and Sandler (ND) paper is published, I will release code for this model.
References