MATH 2413 Fall 2012, Lectures 26-30

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November, 2012

Outline of Sections 4.2–4.5

1. Rolle's Theorem;
2. The Mean Value Theorem;
3. Increase/Decrease test;
4. First derivative test (for local max/min);
5. Concavity test.
6. Second derivative test (for local max/min);
7. Indeterminate forms $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, ∞^0, 1^∞.
8. L'Hospital's rule for indeterminate forms $\frac{0}{0}$, $\frac{\infty}{\infty}$.
9. Indeterminate product/difference/powers;
10. Slant asymptotes $y = mx + b$: $m = \lim_{x \to \pm\infty} \frac{f(x)}{x}$, $b = \lim_{x \to \pm\infty} f(x) - mx$.

9. Curve sketching: A: Domain/ B: Intercepts/C: Symmetry/D: Asymptotes/E: Intervals of increase or decrease/F: local max and or min/G: Concavity.

Exercises:

1. Is it true that $f(x) = x^3 + \sin x$ has only one zero in $[-1, 1]$?
2. Let f be a continuous and differentiable function on $[a, , b]$. Is it true that if $f(a) \cdot f(b) < 0$ and $f'(x) \neq 0$ for every $x \in (a, b)$, then there is a unique number $c \in [a, b]$ such that $f(c) = 0$? Hint: Use the Intermediate Value Theorem and Rolle's Theorem.
3. Let $f(x)$ be a differentiable function with domain $\mathbb{R}$. If $x = c$ is the unique solution of $f'(x) = 0$ and $f''(x) > 0$ for all $x \in \mathbb{R}$, is it true that $f(c)$ is the unique minimum of f? Draw an exemplary graph and justify your answer.
4. Is it true that for every $n \in \mathbb{N}$, we have $\lim_{x \to +\infty} \frac{x^n}{e^x} = 0$? Hint: use L'Hospital's rule.
5. Is it true that for every $n \in \mathbb{N}$, we have $\lim_{x \to +\infty} \frac{(\ln x)^n}{x} = 0$? Hint: use L'Hospital's rule and mathematical induction.