Fall 11 EE 3350-001  Quiz 5A Name:

(i) Write the waveform equation for the FM signal (without any integral sign) with the following properties (25 points):

   (a) The carrier frequency is 1 MHz  
   (b) The modulating signal $m(t) = 4 \cos (2\pi \times 10^3 t)$  
   (c) The frequency deviation constant is 500 $\pi$ rad/sec/volt  
   (d) The power of the FM signal is 50 W

(ii) Also, determine the bandwidth of the FM signal. (25 points)

(iii) The FM signal passes through an ideal band pass filter with a center frequency of 1 MHz and a bandwidth of 300 Hz, to produce the output $y(t)$. What percentage of the FM signal power is present in the output, $y(t)$? (50 points)

(i) \[
m(t) = 4 \cos (2\pi \times 10^3 t).
\]

\[
k_f = \frac{500\pi \text{ rad/sec/volt}}{2\pi} = 500 \pi \times 10^3
\]

\[
\beta = \frac{k_f \cdot \beta}{m} = \frac{500 \pi \times 10^3}{2\pi \times 10^2} = 10.
\]

\[
\text{Power of FM} = \frac{A^2}{2} = 50 \text{ W} \implies A = 10 \text{ V}
\]

\[
\phi(t) = 10 \cos (2\pi \times 10^3 t + 10 \sin(2\pi \times 10^3 t))
\]

\[
\text{(ii) BW of FM signal} = 2(\beta + 1)m = 2(10 + 1) \times 10^2 = 2.2 \text{ kHz}
\]

\[
\phi(t) = 10 \sum_{n=-\infty}^{\infty} J_n(10) \cos(2\pi \times 10^3 t + n \times 10^3 t)
\]

Because the BPFF has a CE $= 1 \text{ MHz}$ and BW $= 300 \text{ Hz}$, $H(f)$

The filter will only pass $n \neq 0, n \neq \pm 1$ component.
1. An angle modulated (EM) signal is given as below:

\[ \phi_{EM}(t) = 10 \cos (2\pi 10^6 t + 2 \sin 2\pi 2000 t) \]

Determine (a) the frequency deviation, (b) the bandwidth estimate as per Carson's rule, and (c) the power in the carrier signal. (50 points)

(a) Frequency Deviation:

\[ \Delta f = \beta f_m \frac{1}{4} \approx 2 \times 2 \text{ kHz} = \frac{4 \times 2}{4} \text{ kHz} \]

(b) FM Bandwidth:

\[ BW_{fm} = 2(\beta+1)f_m = 2(2+1)2 \text{ kHz} = 12 \text{ kHz} \]

(c) Carrier Power:

\[ \phi_{EM}(t) = A \sum_{n=0}^{\infty} J_n(\beta) \cos((\omega_c + \omega_{m,n})t) \]

Carrier Component:

\[ A J_0(\beta) \cos(\omega_c t) \]

Carrier Power:

\[ \frac{1}{2} A^2 J_0^2(\beta) \approx \frac{1}{2} \times 100 \times (0.224)^2 \approx 2.506 \text{ Watts} \]
2. Shown below is the block diagram to generate the wideband FM signal $\phi_{\text{NBFM}}(t)$ with a carrier frequency of 120 MHz and a (peak) frequency deviation of 100 kHz from the narrowband FM signal:

$$\phi_{\text{NBFM}}(t) = 5 \cos (2\pi f_0 t + \beta \sin 2\pi \cdot 10^4 t).$$

The frequency deviation of the NBFM signal is 10 Hz. The local oscillator frequency, $f_{LO}$, is 880 MHz. Determine (1) the NBFM parameters, $f_0$ and $\beta$, (2) the frequency multiplier, $N$ and (3) the center frequency and bandwidth of the band pass filter. (50 points)

$$N = \frac{\Delta f}{\Delta f_0} = \frac{100 \times 10^3}{10} = 10^4$$

$$f_1 = N \cdot f_o \Rightarrow |f_1 - f_{LO}| = f_c \Rightarrow f_i + f_{LO} = f_c.$$

Using $|f_1 - f_{LO}| = \beta$ we have

$$|N f_o - 880 \times 10^6| = 120 \times 10^6$$

$$\Rightarrow \begin{cases} N f_o = 1000 \text{ MHz} \Rightarrow f_o = 100 \text{ kHz} \\ N f_o = 760 \text{ MHz} \Rightarrow f_o = 76 \text{ kHz} \end{cases}$$

(Either one will work)

$$\beta = \frac{\Delta f_0}{f_m} = \frac{100}{10^4} = 10^{-3}$$

$$CF = f_c = 120 \text{ MHz}$$

$$BW = 2(\Delta f + f_m) = 2(100 + 10) \text{ kHz} = 220 \text{ kHz}$$