CS 5333: Spring 2007 : UTD: Assignment #5

Exercises (Ch 6) (p398)

Do all 11.

1. What is the probability that a card selected from a deck is anan ace?
2. What is the probability that a die comes up six when it is rolled?
3. What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?
4. What is the probability that a randomly selected day of the year (from the 366 possible days) is in April?
5. What is the probability that the sum of the numbers on two dice is even when they are rolled?
6. What is the probability that a card selected from a deck is an ace or a heart?
7. What is the probability that when a coin is flipped six times in a row, it lands heads up every time?
8. What is the probability that a five-card poker hand contains the ace of hearts?
9. What is the probability that a five-card poker hand does not contain the queen of hearts?
10. What is the probability that a five-card poker hand contains the two of diamonds and the three of spades?
11. What is the probability that a five-card poker hand contains the two of diamonds, the three of spades, the six of hearts, the ten of clubs, and the king of hearts?

Exercises (Ch 7) (p.482) Do circled Problems.

1. How many comparisons are needed for a binary search in a set of 64 elements?
2. How many comparisons are needed to locate the maximum and minimum elements in a sequence with 128 elements using the algorithm in Example 2?
3. Multiply $(1110)_2$ and $(1010)_2$ using the fast multiplication algorithm.
4. Express the fast multiplication algorithm in pseudocode.
5. Determine a value for the constant C in Example 4 and use it to estimate the number of bit operations needed to multiply two 64-bit integers using the fast multiplication algorithm.
6. How many operations are needed to multiply two 32×32 matrices using the algorithm referred to in Example 4?
7. Suppose that $f(n) = f(n/3) + 1$ when n is divisible by 3, and $f(1) = 1$. Find
   a) $f(3)$. b) $f(27)$. c) $f(729)$.
8. Suppose that $f(n) = 2f(n/2) + 3$ when n is even, and $f(1) = 5$. Find
   a) $f(2)$. b) $f(8)$. c) $f(64)$. d) $f(1024)$.
9. Suppose that $f(n) = f(n/5) + 3n^2$ when n is divisible by 5, and $f(1) = 4$. Find
   a) $f(5)$. b) $f(125)$. c) $f(3125)$.
10. Find $f(n)$ when $n = 2^k$, where f satisfies the recurrence relation $f(n) = f(n/2) + 1$ with $f(1) = 1$.
11. Estimate the size of f in Exercise 10 if f is an increasing function.
12. Find $f(n)$ when $n = 3^k$, where f satisfies the recurrence relation $f(n) = 2f(n/3) + 4$ with $f(1) = 1$.
13. Estimate the size of f in Exercise 12 if f is an increasing function.
14. Suppose that there are $n = 2^k$ teams in an elimination tournament, where there are $n/2$ games in the first round, with the $n/2 = 2^{k-1}$ winners playing in the second round, and so on. Develop a recurrence relation for the number of rounds in the tournament.
15. How many rounds are in the elimination tournament described in Exercise 14 when there are 32 teams?
16. Solve the recurrence relation for the number of rounds in the tournament described in Exercise 14.
17. Suppose that the votes of n people for different candidates (where there can be more than two candidates) for a particular office are the elements of a sequence. A person wins the election if this person receives a majority of the votes.
    a) Devise a divide-and-conquer algorithm that determines whether a candidate received a majority and, if so, determine who this candidate is. [*Hint:* Assume that n is even and split the sequence of votes into two sequences, each with $n/2$ elements. Note that a candidate could not have received a majority of votes without receiving a majority of votes in at least one of the two halves.]
    b) Use the Master Theorem to estimate the number of comparisons needed by the algorithm you devised in part (a).
18. Suppose that each person in a group of n people votes for exactly two people from a slate of candidates to fill two positions on a committee. The top two finishers both win positions as long as each receives more than $n/2$ votes.
    a) Devise a divide-and-conquer algorithm that determines whether the two candidates who received the most votes each received at least $n/2$ votes and, if so, determine who these two candidates are.
    b) Use the Master Theorem to estimate the number of comparisons needed by the algorithm you devised in part (a).
19. a) Set up a divide-and-conquer recurrence relation for the number of multiplications required to compute x^n, where x is a real number and n is a positive integer, using the recursive algorithm from Exercise 26 in Section 4.4.
    b) Use the recurrence relation you found in part (a) to construct a big-O estimate for the number of multiplications used to compute x^n using the recursive algorithm.
20. a) Set up a divide-and-conquer recurrence relation for the number of modular multiplications required to compute a^n **mod** m, where a, m, and n are positive integers, using the recursive algorithms from Example 3 in Section 4.4.
    b) Use the recurrence relation you found in part (a) to construct a big-O estimate for the number of modular multiplications used to compute a^n **mod** m using the recursive algorithm.

Exercises (ch. 7) (p496) Do circled Problems

1. Find the generating function for the finite sequence 2, 2, 2, 2, 2, 2.
2. Find the generating function for the finite sequence 1, 4, 16, 64, 256.

In Exercises 3–8, by a **closed form** we mean an algebraic expression not involving a summation over a range of values or the use of ellipses.

3. Find a closed form for the generating function for each of these sequences. (For each sequence, use the most obvious choice of a sequence that follows the pattern of the initial terms listed.)
   - **a)** 0, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, ...
   - **b)** 0, 0, 0, 1, 1, 1, 1, 1, 1, ...
   - **c)** 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, ...
   - **d)** 2, 4, 8, 16, 32, 64, 128, 256, ...
   - **e)** $\binom{7}{0}, \binom{7}{1}, \binom{7}{2}, \ldots, \binom{7}{7}, 0, 0, 0, 0, 0, \ldots$
   - **f)** 2, −2, 2, −2, 2, −2, 2, −2, ...
   - **g)** 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, ...
   - **h)** 0, 0, 0, 1, 2, 3, 4, ...
4. Find a closed form for the generating function for each of these sequences. (Assume a general form for the terms of the sequence, using the most obvious choice of such a sequence.)
   - **a)** −1, −1, −1, −1, −1, −1, −1, 0, 0, 0, 0, 0, 0, ...
   - **b)** 1, 3, 9, 27, 81, 243, 729, ...
   - **c)** 0, 0, 3, −3, 3, −3, 3, −3, ...
   - **d)** 1, 2, 1, 1, 1, 1, 1, 1, 1, ...
   - **e)** $\binom{7}{0}, 2\binom{7}{1}, 2^2\binom{7}{2}, \ldots, 2^7\binom{7}{7}, 0, 0, 0, 0, \ldots$
   - **f)** −3, 3, −3, 3, −3, 3, ...
   - **g)** 0, 1, −2, 4, −8, 16, −32, 64, ...
   - **h)** 1, 0, 1, 0, 1, 0, 1, 0, ...
5. Find a closed form for the generating function for the sequence $\{a_n\}$, where
   - **a)** $a_n = 5$ for all $n = 0, 1, 2, \ldots$.
   - **b)** $a_n = 3^n$ for all $n = 0, 1, 2, \ldots$
   - **f)** $a_n = \binom{10}{n+1}$ for $n = 0, 1, 2, \ldots$.
7. For each of these generating functions, provide a closed formula for the sequence it determines.
   - **a)** $(3x - 4)^3$
   - **b)** $(x^3 + 1)^3$
   - **c)** $1/(1 - 5x)$
   - **d)** $x^3/(1 + 3x)$
   - **e)** $x^2 + 3x + 7 + (1/(1 - x^2))$
   - **f)** $(x^4/(1 - x^4)) - x^3 - x^2 - x - 1$
   - **g)** $x^2/(1 - x)^2$
   - **h)** $2e^{2x}$
8. For each of these generating functions, provide a closed formula for the sequence it determines.
   - **a)** $(x^2 + 1)^3$
   - **b)** $(3x - 1)^3$
   - **c)** $1/(1 - 2x^2)$
   - **d)** $x^2/(1 - x)^3$
   - **e)** $x - 1 + (1/(1 - 3x))$
   - **f)** $(1 + x^3)/(1 + x)^3$
   - ***g)** $x/(1 + x + x^2)$
   - **h)** $e^{3x^2} - 1$
9. Find the coefficient of x^{10} in the power series of each of these functions.
   - **a)** $(1 + x^5 + x^{10} + x^{15} + \cdots)^3$
   - **b)** $(x^3 + x^4 + x^5 + x^6 + x^7 + \cdots)^3$
   - **c)** $(x^4 + x^5 + x^6)(x^3 + x^4 + x^5 + x^6 + x^7)(1 + x + x^2 + x^3 + x^4 + \cdots)$
   - **d)** $(x^2 + x^4 + x^6 + x^8 + \cdots)(x^3 + x^6 + x^9 + \cdots)(x^4 + x^8 + x^{12} + \cdots)$
   - **e)** $(1 + x^2 + x^4 + x^6 + x^8 + \cdots)(1 + x^4 + x^8 + x^{12} + \cdots)(1 + x^6 + x^{12} + x^{18} + \cdots)$
10. Find the coefficient of x^9 in the power series of each of these functions.
    - **a)** $(1 + x^3 + x^6 + x^9 + \cdots)^3$
    - **b)** $(x^2 + x^3 + x^4 + x^5 + x^6 + \cdots)^3$
    - **c)** $(x^3 + x^5 + x^6)(x^3 + x^4)(x + x^2 + x^3 + x^4 + \cdots)$
    - **d)** $(x + x^4 + x^7 + x^{10} + \cdots)(x^2 + x^4 + x^6 + x^8 + \cdots)$
    - **e)** $(1 + x + x^2)^3$
11. Find the coefficient of x^{10} in the power series of each of these functions.
    - **a)** $1/(1 - 2x)$
    - **b)** $1/(1 + x)^2$
    - **c)** $1/(1 - x)^3$
    - **d)** $1/(1 + 2x)^4$
    - **e)** $x^4/(1 - 3x)^3$

Exercises (ch 7) (p504) Do all 7.

1. How many elements are in $A_1 \cup A_2$ if there are 12 elements in A_1, 18 elements in A_2, and
   - **a)** $A_1 \cap A_2 = \emptyset$?
   - **b)** $|A_1 \cap A_2| = 1$?
   - **c)** $|A_1 \cap A_2| = 6$?
   - **d)** $A_1 \subseteq A_2$?
2. There are 345 students at a college who have taken a course in calculus, 212 who have taken a course in discrete mathematics, and 188 who have taken courses in both calculus and discrete mathematics. How many students have taken a course in either calculus or discrete mathematics?
3. A survey of households in the United States reveals that 96% have at least one television set, 98% have telephone service, and 95% have telephone service and at least one television set. What percentage of households in the United States have neither telephone service nor a television set?
4. A marketing report concerning personal computers states that 650,000 owners will buy a modem for their machines next year and 1,250,000 will buy at least one software package. If the report states that 1,450,000 owners will buy either a modem or at least one software package, how many will buy both a modem and at least one software package?
5. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in each set and if
   - **a)** the sets are pairwise disjoint.
   - **b)** there are 50 common elements in each pair of sets and no elements in all three sets.
   - **c)** there are 50 common elements in each pair of sets and 25 elements in all three sets.
   - **d)** the sets are equal.
6. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in A_1, 1000 in A_2, and 10,000 in A_3 if
   - **a)** $A_1 \subseteq A_2$ and $A_2 \subseteq A_3$.
   - **b)** the sets are pairwise disjoint.
   - **c)** there are two elements common to each pair of sets and one element in all three sets.
7. There are 2504 computer science students at a school. Of these, 1876 have taken a course in Pascal, 999 have taken a course in Fortran, and 345 have taken a course in C.