Sample-Final Problem

Problem 3: (45 Points) Small Scale Fading: (Estimated Time to Solve: 35 minutes)

1. (25 points) A police officer provides a ticket to the driver of a vehicle, if his/her speed is over 100 km/h. The police officer’s radar (that detects the speed of a vehicle) radiates a sinusoidal frequency of 900 MHz. This sinusoidal signal hits Dr. S.’s vehicle and returns to the officer’s radar as a sinusoid of frequency 900.000200 MHz. Note that doppler effect depends on the relative velocity of the transmitter and the receiver.

(a) (15 points) Based on the above information, do you think Dr. S. will receive a ticket? Why? Please also tell us whether Dr. S. is driving towards the police officer or not. Justify your answers. Note that the police officer is stationary.

(b) (10 points) Assume that Dr. S.’s vehicle is equipped with a radar detector and the channel is Rayleigh fading. Calculate the level-crossing rate and average fade duration at a level of -10 dB for the radar detector.

2. (20 points) A vehicle receives a 900 MHz transmission over a Rayleigh fading channel while traveling at a velocity \(v_1\) for 10 s and then at \(v_2\) for the next 10 s. The average fade duration for a signal level 10 dB below the rms level is 1 ms and 1.5 ms for velocity \(v_1\) and \(v_2\), respectively. How far does the vehicle travel during that 20 s interval? How many fades does the signal undergo at the above threshold level during the last 15 s interval? Assume that local mean remains constant during the travel. Assume the speed of propagation for electromagnetic waves = \(3 \times 10^8\) m/s.
Answer to Problem 3:

(a) \[ t_{d_1} = \frac{v}{\lambda} \quad \Rightarrow \quad t_2 = t_1 + t_{d_2} \]
\[ = \frac{v}{\lambda} + \frac{v}{\lambda} \]
\[ = \frac{2v}{\lambda} \]
\[ \therefore \quad t_2 - t_1 = 2 \frac{v}{\lambda} \]
\[ 0.000200 \times 10^6 = 2 \frac{v}{\lambda} \]

\[ \lambda = \frac{v}{\lambda} = \frac{3 \times 10^8}{700 \times 10^6} \]
\[ \lambda = \frac{1}{3} \] m
\[ v = \frac{100}{3} \text{ m/s} \]
\[ = 33.33 \text{ m/s} \]

100 km/h = \[ \frac{100 \times 10^3}{3600} = 27.78 \text{ m/s} \]

Yes, Dr. S will receive a ticket
Dr. S is driving towards the police officer.
(b) \(-10 \text{ dB} = 10^{-\frac{1}{2}} = 0.316\)

Level Crossing rate = \(\sqrt{2\pi} \int_{-\infty}^{\infty} p e^{-p^2} \, dp\)

\[f_m = \frac{\nu f_c}{c} = \frac{33.33 \times 9 \times 10^8}{3 \times 10^8} = 100 \text{ Hz}\]

\[N_c = \sqrt{2\pi} \times 100 \times 0.316 \times e^{-\left(0.316\right)^2}\]

\[= 79.209 \times e^{-\left(0.316\right)^2} = 79.209 \times 0.904\]

\[= 71.68 \text{ crossing /sec}\]

Average fade duration

\[\tau = \frac{e^{p^2} - 1}{p f_m \sqrt{2\pi}} = \frac{0.105}{79.209} = 1.33 \text{ ms}\]
\( P = -10 \text{ dB} = 0.316 \)

\[ Z = \frac{e^{\frac{r^2}{2}} - 1}{e^{\frac{r^2}{2m}} \sqrt{2\pi}} \quad f_m = \frac{e^{\frac{r^2}{2}} - 1}{\sqrt{2\pi} 2P} \]

For first LOS velocity is \( v_1 \), average fade duration \( Z_1 = 1 \text{ ms} \) and let frequency be \( f_{m1} \)

\[ f_{m1} = \frac{e^{\frac{r^2}{2}} - 1}{\sqrt{2\pi} Z_1 \rho} = \frac{0.105}{1 \times 10^{-3} \times 0.316 \times \sqrt{2\pi}} = 132.56 \text{ Hz} \]

\[ v_1 = \frac{c}{f_c} \quad f_{m1} = \frac{3 \times 10^8}{9 \times 10^8} \ f_{m1} = 41.19 \text{ m/s} \]

For next LOS velocity is \( v_2 \), average fade duration \( Z_2 = 1.5 \text{ ms} \) and let the frequency be \( f_{m2} \)
\[
\frac{\epsilon_{m_2}}{\sqrt{2\pi} \cdot \sigma_x \cdot \rho} = \frac{0.105}{1.5 \times 10^{-3} \times \sqrt{2\pi} \times 0.316}
\]

\[\Rightarrow \quad \epsilon_{m_2} = 88.37 \quad \text{Hz}\]

\[\therefore \quad \frac{c}{f_c} = \frac{1}{3} \times 88.37 = 29.46 \text{ m/s}\]

The vehicle travels within 20 sec.

\[v_1 \times 10 + v_2 \times 10 = 49.19 \times 10 + 29.46 \times 10\]

\[\Rightarrow \quad 736.5 \text{ m}\]
Let the level crossing be $NR_1$ and $NR_2$ for velocity $v_1$ and $v_2$ respectively.

$$NR_1 = \sqrt{2x} \cdot f_m \cdot P \cdot e^{-\frac{P^2}{2}} = f_m \times 0.7161 = 94.93 \text{ crossing/sec}$$

$$NR_2 = \sqrt{2x} \cdot f_m \cdot P \cdot e^{-\frac{P^2}{2}} = f_m \times 0.7161 = 63.28 \text{ crossing/sec}$$

Total crossing for last 15 sec

$$= NR_1 \times 5 + NR_2 \times 10$$

$$= 474.65 + 632.8$$

$$= 1107 \text{ crossing}$$