

## The Exponential Distribution

- *Setting.* The *exponential* distributions are a particular kind of the *continuous type* that take values  $0 < x < \infty$  and serve as models for *lifetimes*, *waiting times*, and *radioactive decay*, for example.
- *The Exponential( $\lambda$ ) Probability Distribution.* For parameter  $\lambda > 0$ , the probability density function

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0,$$

defines the exponential( $\lambda$ ) distribution. The corresponding *cumulative distribution function* is

$$F(x) = 1 - e^{-\lambda x}, \quad x > 0.$$

- *Mean and Variance.* Let  $Y$  be an exponential( $\lambda$ ) random variable. Then

$$E(Y) = \frac{1}{\lambda}$$

and

$$\text{Var}(Y) = \frac{1}{\lambda^2}.$$

- *Alternate Parameterization.* Sometimes the exponential density is given in the equivalent form

$$f(x) = \frac{1}{\mu} e^{-x/\mu}, \quad x > 0.$$

In this parameterization, the parameter  $\mu$  is the mean instead of the reciprocal of the mean.

- *Illustrative Application.* Suppose that the time  $Y$  it takes to process an order at a fast-food restaurant is exponentially distributed with mean 1.5 minutes, that is, exponential( $\lambda$ ) with  $\lambda = 1/1.5 = 2/3$ . What is the probability that it takes more than 3 minutes to process an order? Answer:  $P(Y > 3) = 1 - P(Y \leq 3) = 1 - (1 - e^{-(2/3)3}) = e^{-(2/3)3} = 0.135$ .
- *The Exponential Distribution has a Memory-Free Property.* The *conditional probability* that  $Y > t + x$ , given that  $Y > x$ , is the same as the unconditional probability that  $Y > t$ :

$$P(Y > t + x | Y > x) = \dots = e^{-\lambda t} = P(Y > t),$$

for all  $t > 0$  and  $x > 0$ .

*Interpretation.* The future performance of the device satisfies the “*memory-free property*” that the failure probabilities for the device after use for time  $x$  are exactly the same as for starting out with a *brand new* device. That is, a used device is neither better nor worse than a new one. *Which would you rather be given – a brand new light bulb or one which has been used without failure for 10 hours?*